

SCALAR MESONS AND GLUEBALLS IN A CHIRAL $U(3) \times U(3)$ QUARK MODEL WITH 'T HOOFT INTERACTION

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In a $U(3) \times U(3)$ quark chiral model of the Nambu–Jona-Lasinio (NJL) type with the 't Hooft interaction, the ground scalar isoscalar mesons and a scalar glueball are described. The glueball (dilaton) is introduced into the effective meson Lagrangian written in a chirally symmetric form on the base of scale invariance. The singlet-octet mixing of scalar isoscalar mesons and their mixing with the glueball are taken into account. Mass spectra of the scalar mesons and glueball and their strong decays are described.

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1 Introduction

The self-interaction of gluons, a peculiarity of QCD, gave an idea that gluons can form bound states that can propagate particlelike in the space. Unfortunately, because of theoretical problems, there is still no exact answer on the question: "Do these states really exist or not?". However, from recent lattice simulations [1–3] one can conclude that it is most probably that glueballs are the reality of our world. Having assumed that glueballs exist, one can try to construct a model to describe their interaction with other mesons, their properties, such as, e.g., the mass and the decay width, and to identify them with observed resonances.

An exact microscopic description of bound gluon states cannot be done systematically in the framework of QCD. In this situation, QCD-motivated phenomenological models are the tool which can help to deal with glueballs as well as with quarkonia which form the most of observed meson states. However, using these models to describe glueballs, we encounter many difficulties concerning, e.g., the ambiguity of the ways of including glueballs into models and identification of experimentally observed meson states. This justifies the variety of points of view on this problem.

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First of all, we do not know the exact mass of a glueball. From the quenched QCD lattice simulations, Weingarten (see e.g. [1, 3]) concluded that the lightest scalar glueball is expected around 1.7 GeV. Amsler [4] considered the state $f_0(1500)$ as a candidate for the scalar glueball. QCD sum rules [5] and K-matrix method [6] showed that both $f_0(1500)$ and $f_0(1710)$ are mixed states with large admixture of the glueball component.

All the bound isoscalar $q\bar{q}$ states are subject to mixing with glueballs, and their spectrum has many interpretations made by different authors. For instance, Palano [7] suggested a scenario, in which the states $a_0(980)$, $K_0^*(1430)$, $f_0(980)$, and $f_0(1400)$ form a nonet. The state $f_0(1500)$ is considered as the scalar glueball. Törnqvist *et al.* [8] looked upon the states $f_0(980)$ and $f_0(1370)$ as manifestations of the ground and excited $s\bar{s}$ states, and the state $f_0(400 - 1200)$ as the ground $u\bar{u}$ state. Eef van Beveren *et al.* [9] considered the states $f_0(400 - 1200)$ and $f_0(1370)$ as ground $u\bar{u}$ states, and the states $f_0(980)$ and $f_0(1500)$ as ground $s\bar{s}$ states. Two states for each $q\bar{q}$ system occur due to pole doubling, which takes place for scalar mesons in their model. Shakin *et al.* [10] obtained from a nonlocal confinement model that the $f_0(980)$ resonance is the ground $u\bar{u}$ state and $f_0(1370)$ is the ground $s\bar{s}$ state. The state $f_0(1500)$ is considered as a radial excitation of $f_0(980)$. They believe the mass of scalar glueball to be 1770 MeV.

In our recent papers [11–13], following the methods given in Refs. [14–17], we showed that all experimentally observed scalar meson states with masses in the interval from 0.4 GeV till 1.7 GeV can be interpreted as members of two scalar meson nonets — the ground meson nonet and its first radial excitation. We considered all scalar mesons as $q\bar{q}$ states and took into account the singlet-octet mixing caused by the 't Hooft interaction. However, we did not describe the scalar state $f_0(1500)$. We supposed that this state contained a significant component of the scalar glueball (see [5, 6]).

In this work, we introduce the glueball field into our effective Lagrangian as a dilaton and describe it and its mixing with the scalar isoscalar $q\bar{q}$ states. We consider only ground $q\bar{q}$ states. As a result, we describe three scalar mesons $f_0(400 - 1200)$, $f_0(980)$, and $f_0(1500)$ (or $f_0(1710)$). Let us note that the model we present here is just a tentative one. In this model, we probe one of the possible ways of including the scalar glueball into a model that would describe the glueball state and quarkonia simultaneously.

In various works (see e.g. [18–21]), the authors introduced the glueball into meson Lagrangians using the principle of scale invariance and the dilaton model. For example, $SU(2) \times SU(2)$ models were considered [18, 19, 21] and the $U(3) \times U(3)$ model, however, without the 't Hooft interaction was investigated in [20]. Here we follow these works and introduce the scalar glueball state into the Lagrangian constructed in [14, 15, 22].

In this paper, we introduce the dilaton field into the phenomenological meson Lagrangian obtained from the effective quark interaction of the NJL type after bosonization. We take the meson Lagrangian in a chirally symmetric form (before the spontaneous breaking of chiral symmetry), where only quadratic meson terms are of scale-noninvariant form, therefore, we introduce the dilaton field only into these (massive) terms. To this Lagrangian, we add the potential describing the dilaton self-interaction in the form given in Ref. [23] and dilaton kinetic term.

After introducing the glueball into our effective Lagrangian we describe the mixing of three scalar mesons: two scalar isoscalar ($q\bar{q}$) states and the glueball. As a result, we obtain the mass spectrum of these scalar meson states and also describe their main strong decays.

We emphasize once more that we do not claim this model to give a quantitative description

of experimental data. We do not take into account the states $f_0(1370)$ and one of the states $f_0(1500)$, $f_0(1710)$ which could not be neglected if the real meson spectrum were our goal. We intend to obtain rather qualitative estimates that will allow us to choose a more reasonable way of including the scalar glueball field into the effective meson Lagrangian.

In Section 2, we discuss an effective meson Lagrangian with the 't Hooft interaction. In Section 3, we introduce a dilaton field into the Lagrangian. In Section 4, we derive gap equations for the masses of $u(d)$ and s quarks. We also deduce a set of equations allowing us to bind two of the three new parameters connected with the dilaton field. In Section 5, we obtain mass formulae for scalar mesons and the dilaton and describe the singlet-octet mixing among quarkonia and the mixing of the dilaton with two isosinglet scalar mesons. Here we fix the new model parameters and give their numerical estimates as well as the estimates for the masses of scalar mesons and the glueball. In Section 6, we calculate the widths of main strong decays of the scalar mesons and the glueball. In the Conclusion, we discuss obtained results.

2 Chiral effective Lagrangian with 't Hooft interaction

A $U(3) \times U(3)$ chiral Lagrangian with the 't Hooft interaction was investigated in paper [22]. It consists of three terms as shown in formula (1). The first term represents the free quark Lagrangian, the second is composed of four-quark vertices as in the NJL model, and the last one describes the six-quark 't Hooft interaction that is necessary to solve the $U_A(1)$ problem.

$$L = \bar{q}(i\hat{\partial} - m^0)q + \frac{G}{2} \sum_{a=0}^8 [(\bar{q}\lambda_a q)^2 + (\bar{q}i\gamma_5 \lambda_a q)^2] - K \{ \det[\bar{q}(1 + \gamma_5)q] + \det[\bar{q}(1 - \gamma_5)q] \}. \quad (1)$$

Here λ_a ($a = 1, \dots, 8$) are the Gell-Mann matrices and $\lambda_0 = \sqrt{2/3} \mathbf{1}$, with $\mathbf{1}$ being the unit matrix; m^0 is a current quark mass matrix with diagonal elements m_u^0, m_d^0, m_s^0 ($m_u^0 \approx m_d^0$).

The standard treatment for local quark models consists in replacing the four-quark vertices with bosonic fields (bosonization). The final effective bosonic Lagrangian appears as a result of the calculation of the quark determinant. First of all, using the method described in [22, 24, 25], we go from Lagrangian (1) to a Lagrangian which contains only four-quark vertices⁴

$$L = \bar{q}(i\hat{\partial} - \bar{m}^0)q + \frac{1}{2} \sum_{a,b=1}^9 [G_{ab}^{(-)}(\bar{q}\tau_a q)(\bar{q}\tau_b q) + G_{ab}^{(+)}(\bar{q}i\gamma_5 \tau_a q)(\bar{q}i\gamma_5 \tau_b q)], \quad (2)$$

where

$$\begin{aligned} \tau_a &= \lambda_a \quad (a = 1, \dots, 7), \quad \tau_8 = (\sqrt{2}\lambda_0 + \lambda_8)/\sqrt{3}, \\ \tau_9 &= (-\lambda_0 + \sqrt{2}\lambda_8)/\sqrt{3}, \\ G_{11}^{(\pm)} &= G_{22}^{(\pm)} = G_{33}^{(\pm)} = G \pm 4K m_s I_1^\Lambda(m_s), \\ G_{44}^{(\pm)} &= G_{55}^{(\pm)} = G_{66}^{(\pm)} = G_{77}^{(\pm)} = G \pm 4K m_u I_1^\Lambda(m_u), \end{aligned} \quad (3)$$

⁴In addition to the one-loop corrections of constants of four-quark vertices, we allow for two-loop contributions from the 't Hooft interaction that modify the current quark masses. Thus, we avoid the problem of double counting of the 't Hooft contribution in gap equations which was encountered by the author in [25].

$$\begin{aligned}
G_{88}^{(\pm)} &= G \mp 4Km_s I_1^\Lambda(m_s), \quad G_{99}^{(\pm)} = G, \\
G_{89}^{(\pm)} &= G_{98}^{(\pm)} = \pm 4\sqrt{2}Km_u I_1^\Lambda(m_u) \\
G_{ab} &= 0 \quad (a \neq b; \quad a, b = 1, \dots, 7),
\end{aligned}$$

$$\overline{m}_u^0 = m_u^0 - 32Km_u m_s I_1^\Lambda(m_u) I_1^\Lambda(m_s), \quad (4)$$

$$\overline{m}_s^0 = m_s^0 - 32Km_u^2 I_1^\Lambda(m_u)^2. \quad (5)$$

Here m_u and m_s are constituent quark masses and the integrals

$$I_n^\Lambda(m_a) = \frac{N_c}{(2\pi)^4} \int d^4k \frac{\theta(\Lambda^2 - k^2)}{(k^2 + m_a^2)^n} \quad (6)$$

are calculated (for simplicity) in the Euclidean metric and regularized by a simple $O(4)$ -symmetric ultra-violet cutoff Λ . The mass m_a can be m_u or m_s .

Now it is necessary to bosonize Lagrangian (2). After bosonization and renormalization of the meson fields, we obtain [14, 15, 22]

$$\begin{aligned}
L(\sigma, \phi) &= - \sum_{a,b=1}^9 \frac{g_a g_b}{2} \left(\sigma_a (G^{(-)})_{ab}^{-1} \sigma_b + Z \phi_a (G^{(+)}_{ab})^{-1} \phi_b \right) \\
&\quad - i \operatorname{Tr} \ln \left\{ 1 + \frac{1}{i\hat{\partial} - m} \sum_{a=1}^9 g_a \tau_a (\sigma_a + i\gamma_5 \sqrt{Z} \phi_a) \right\},
\end{aligned} \quad (7)$$

where $(G^{(\pm)})^{-1}$ is the inverse of $G^{(\pm)}$, and ϕ_a and σ_a are the pseudoscalar and scalar fields, respectively.

$$g_1^2 = g_2^2 = g_3^2 = g_8^2 = g_u^2 = [4I_2^\Lambda(m_u)]^{-1}, \quad (8)$$

$$g_4^2 = g_5^2 = g_6^2 = g_7^2 = [4I_2^\Lambda(m_u, m_s)]^{-1},$$

$$g_9^2 = g_s^2 = [4I_2^\Lambda(m_s)]^{-1},$$

$$\begin{aligned}
I_2^\Lambda(m_u, m_s) &= \frac{N_c}{(2\pi)^4} \int d^4k \frac{\theta(\Lambda^2 - k^2)}{(k^2 + m_u^2)(k^2 + m_s^2)} = \\
&= \frac{3}{(4\pi)^2(m_s^2 - m_u^2)} \left[m_s^2 \ln \left(\frac{\Lambda^2}{m_s^2} + 1 \right) - m_u^2 \ln \left(\frac{\Lambda^2}{m_u^2} + 1 \right) \right], \\
Z &= \left(1 - \frac{6m_u}{M_{A_1}^2} \right)^{-1} \approx 1.44,
\end{aligned} \quad (9)$$

where M_{A_1} is the mass of the axial-vector meson.

In the one-loop approximation the meson Lagrangian looks as follows⁵

$$L(\sigma, \phi) = \frac{1}{2} \sum_{a=1}^9 [(\partial_\mu \sigma_a)^2 + (\partial_\mu \phi_a)^2] + \quad (10)$$

⁵The expression under Tr in (10) is written formally for the reason of being concise and is not ready for practical use. One will find which g_a should be taken, considering the corresponding quark-loop diagrams.

$$\begin{aligned}
& + \left[(m_u - \bar{m}_u^0)(G^{(-)})_{88}^{-1} - \frac{m_s - \bar{m}_s^0}{\sqrt{2}}(G^{(-)})_{98}^{-1} - 8m_u I_1^\Lambda(m_u) \right] g_8 \sigma_8 - \\
& + \left[(m_u - \bar{m}_u^0)(G^{(-)})_{89}^{-1} - \frac{m_s - \bar{m}_s^0}{\sqrt{2}}(G^{(-)})_{99}^{-1} + 8\frac{m_s}{\sqrt{2}} I_1^\Lambda(m_s) \right] g_9 \sigma_9 - \\
& - \frac{1}{2} \sum_{a,b=1}^9 g_a g_b \left[\sigma_a (G^{(-)})_{ab}^{-1} \sigma_b + Z \phi_a (G^{(+)})_{ab}^{-1} \phi_b \right] + \\
& \frac{1}{4} \text{Tr} \left\{ g^2 \left[8I_1^\Lambda(m)(\sigma^2 + Z\phi^2) - \right. \right. \\
& \left. \left. - \left(\sigma^2 - \frac{\{M, \sigma\}_+}{g} + Z\phi^2 \right)^2 + \left[\left(\sigma - \frac{M}{g} \right), \phi \right]_-^2 \right] \right\}, \\
& C_a = 0 \quad (a = 1, \dots, 7), \quad C_8 = 1, \quad C_9 = -\frac{1}{\sqrt{2}}. \tag{11}
\end{aligned}$$

It is useful to notice that Lagrangian (10) can be given a chirally symmetric form. For this purpose, we introduce fields $\sigma' = \sigma - mg^{-1}$ that have nonzero expectation values, $\langle \sigma' \rangle_0 \neq 0$ [15]. Here

$$g = g_8 \tau_8 - \frac{g_9 \tau_9}{\sqrt{2}}; \quad m = m_u \tau_8 + \frac{m_s \tau_9}{2}. \tag{12}$$

This is the form of the effective Lagrangian in the case of unbroken chiral symmetry:

$$L(\sigma', \phi) = \frac{1}{2} \sum_{a=1}^9 [(\partial_\mu \sigma'_a)^2 + (\partial_\mu \phi_a)^2] \tag{13}$$

$$\begin{aligned}
& - \frac{1}{2} \sum_{a,b=1}^9 \left[(g_a \sigma'_a + \mu_a) (G^{(-)})_{ab}^{-1} (g_b \sigma'_b + \mu_b) + Z g_a g_b \phi_a (G^{(+)})_{ab}^{-1} \phi_b \right] + \\
& + \frac{1}{4} \text{Tr} \left\{ 8g^2 (I_1^\Lambda(m) + m^2 I_2^\Lambda(m)) (\sigma'^2 + Z\phi^2) - g^2 [(\sigma'^2 + Z\phi^2)^2 - Z[\sigma', \phi]_-^2] \right\}, \\
& \mu_a = 0 \quad (a = 1, \dots, 7), \quad \mu_8 = \bar{m}_u^0, \quad \mu_9 = -\frac{\bar{m}_s^0}{\sqrt{2}}, \tag{14}
\end{aligned}$$

where $\sigma' = \sum_{a=1}^9 \sigma_a \tau^a$ and $\phi = \sum_{a=1}^9 \phi_a \tau^a$. From (13), one can see that, in the one-loop approximation, our Lagrangian does not violate chiral symmetry if $m^0 = 0$, and the 't Hooft interaction is switched off. Let us require the chiral and scale invariance both to be properties of the effective Lagrangian. We remind that the QCD Lagrangian satisfies this requirement in the chiral limit ($m^0 = 0$). The scale invariance of the effective meson Lagrangian is restored by means of a dilaton field [18–20] introduced into the Lagrangian so that it provides a proper dimension for each Lagrangian term. The dilaton field is introduced only into the mass terms. We also introduce the dilaton potential intended to reproduce QCD scale anomaly [21, 26, 27].

3 Nambu–Jona-Lasinio model with dilaton

Following earlier works devoted to dilatons, we introduce a color singlet dilaton field χ that experiences a potential

$$V(\chi) = B \left(\frac{\chi}{\chi_0} \right)^4 \left[\ln \left(\frac{\chi}{\chi_0} \right)^4 - 1 \right]. \quad (15)$$

It has a minimum at $\chi = \chi_0$, and the parameter B represents the vacuum energy, when there are no quarks. The curvature of the potential at its minimum determines the bare glueball mass

$$m_g = \frac{4\sqrt{B}}{\chi_0}. \quad (16)$$

To introduce the dilaton field into the effective meson Lagrangian (13), we use the following principle. Insofar as the QCD Lagrangian, with the current quark masses equal to zero, is chiral and scale invariant, we suppose that our effective meson Lagrangian, motivated by QCD, has also to be chiral and scale invariant in the case when the current quark masses and the 't Hooft interaction are equal to zero. Then, instead of Lagrangian (13), we obtain

$$\begin{aligned} L(\sigma', \phi, \chi) = & \frac{1}{2} \sum_{a=1}^9 [(\partial_\mu \sigma'_a)^2 + (\partial_\mu \phi_a)^2] \\ & + \left\{ \frac{1}{2} \sum_{a,b=1}^9 \left[(g_a \sigma'_a + \mu_a) (G^{(-)})_{ab}^{-1} (g_b \sigma'_b + \mu_b) + Z g_a g_b \phi_a (G^{(+)}_{ab})^{-1} \phi_b \right] \right. \\ & - \frac{1}{4} \text{Tr} \left\{ 8g^2 (I_1^\Lambda(m) + m^2 I_2^\Lambda(m)) (\sigma'^2 + Z \phi^2) \right\} \left. \right\} \left(\frac{\chi}{\chi_c} \right)^2 \\ & - \frac{1}{4} \text{Tr} \left\{ g^2 [(\sigma'^2 + Z \phi^2)^2 - Z [\sigma', \phi]_-^2] \right\} + \frac{1}{2} (\partial_\mu \chi)^2 - V(\chi), \end{aligned} \quad (17)$$

where χ_c is the vacuum expectation value of dilaton fields $\chi = \bar{\chi} + \chi_c$, $\langle \chi \rangle_0 = \chi_c$ and $\langle \bar{\chi} \rangle_0 = 0$.⁶

By rewriting this Lagrangian in terms of quantum fields σ and $\bar{\chi}$ with vacuum expectations equal zero, we finally obtain

$$L(\sigma, \phi, \bar{\chi}) = L(\sigma, \phi) + \Delta L(\sigma, \phi, \bar{\chi}), \quad (18)$$

where $L(\sigma, \phi)$ equals Lagrangian (10), and $\Delta L(\sigma, \phi, \bar{\chi})$ has the form

$$\begin{aligned} \Delta L(\sigma, \phi, \bar{\chi}) = & \frac{1}{2} (\partial_\mu \bar{\chi})^2 - V'(\bar{\chi} + \chi_c) + \frac{\bar{\chi}}{\chi_c} \left(2 + \frac{\bar{\chi}}{\chi_c} \right) \\ & \times \left\{ -8g_8 m_u^3 I_2^\Lambda(m_u) \sigma_8 + 8g_9 \frac{m_s^3 I_2^\Lambda(m_s)}{\sqrt{2}} \sigma_9 \right. \\ & - \frac{1}{2} \sum_{a,b=1}^9 g_a g_b \left[\sigma_a (G^{(-)})_{ab}^{-1} \sigma_b + Z \phi_a (G^{(+)}_{ab})^{-1} \phi_b \right] \end{aligned} \quad (19)$$

⁶Since the mass of current quark explicitly breaks the scale invariance of the model, there is no need to make these terms scale invariant, using the dilaton fields.

$$+4 \sum_{a=1}^9 g_a^2 \mathcal{J}_a (\sigma_a^2 + Z \phi_a^2) \Big\},$$

where

$$\begin{aligned} \mathcal{J}_a &= I_1^\Lambda(m_u) + m_u^2 I_2^\Lambda(m_u), \quad (a = 1, 2, 3, 8), \\ \mathcal{J}_a &= \frac{1}{2} \left(I_1^\Lambda(m_u) + I_1^\Lambda(m_s) + \frac{(m_u + m_s)^2}{4} I_2^\Lambda(m_u, m_s) \right), \quad (a = 4, 5, 6, 7), \\ \mathcal{J}_9 &= I_1^\Lambda(m_s) + m_s^2 I_2^\Lambda(m_s), \end{aligned} \quad (20)$$

and we used gap equations (25) in the terms linear over σ_a . The potential $V'(\chi) = V(\chi) + (\chi/\chi_c)^2 A$, where

$$A = \frac{1}{2} \sum_{a,b=1}^8 \langle \sigma_a \rangle (G^{(-)})_{ab}^{-1} \langle \sigma_b \rangle - 4 \mathcal{J}_8 m_u^2 - 2 \mathcal{J}_9 m_s^2. \quad (21)$$

From this Lagrangian, one can obtain the system of equations determining constituent quark masses (gap equations) and dilaton potential parameters χ_0 , χ_c and B .

4 Equations for the quark masses (gap equations) and dilaton potential parameters χ_0 , χ_c and B

The conditions for linear terms being absent in our Lagrangian

$$\left. \frac{\delta L}{\delta \sigma_8} \right|_{\phi, \sigma, \bar{\chi}=0} = 0, \quad \left. \frac{\delta L}{\delta \sigma_9} \right|_{\phi, \sigma, \bar{\chi}=0} = 0, \quad \left. \frac{\delta L}{\delta \chi} \right|_{\phi, \sigma, \bar{\chi}=0} = 0 \quad (22)$$

lead us to the following equations

$$(m_u - \bar{m}_u^0)(G^{(-)})_{88}^{-1} - \frac{m_s - \bar{m}_s^0}{\sqrt{2}}(G^{(-)})_{89}^{-1} - 8m_u I_1^\Lambda(m_u) = 0, \quad (23)$$

$$(m_s - \bar{m}_s^0)(G^{(-)})_{99}^{-1} - \sqrt{2}(m_u - \bar{m}_u^0)(G^{(-)})_{98}^{-1} - 8m_s I_1^\Lambda(m_s) = 0, \quad (24)$$

$$-4B \left(\frac{\chi_c}{\chi_0} \right)^3 \frac{1}{\chi_0} \ln \left(\frac{\chi_c}{\chi_0} \right)^4 - \frac{2A}{\chi_c} = 0. \quad (25)$$

An additional equation follows from the relation between the divergence of dilaton current S_μ and the gluon condensate

$$\langle \partial_\mu S^\mu \rangle_0 = \left(\chi \frac{\partial V'}{\partial \chi} - 4V' \right) \Big|_{\chi=\chi_c} = \left(\frac{11}{24} N_c - \frac{1}{12} N_f \right) \left\langle \frac{\alpha}{\pi} G_{\mu\nu}^2 \right\rangle_0 = \mathcal{C}_g, \quad (26)$$

$$4B \left(\frac{\chi_c}{\chi_0} \right)^4 - 2A = \mathcal{C}_g. \quad (27)$$

	σ_I	σ_{II}	σ_{III}	χ_c	χ_0	$B[\text{GeV}^4]$	m_g
(a)	516	1027	1500	208	217	0.007	1430
(b)	518	1042	1710	180	187	0.007	1660

Table 1. The masses (in MeV) of physical scalar meson states σ_I , σ_{II} , σ_{III} and parameter χ_c for two cases: (a) $M_{\sigma_{III}} = 1500$ MeV and (b) $M_{\sigma_{III}} = 1710$ MeV.

Using (4) and (5), one can rewrite the gap equations (23) and (24) in a well-known form [25]

$$m_u^0 = m_u - 8Gm_u I_1^\Lambda(m_u) - 32Km_um_s I_1^\Lambda(m_u)I_1^\Lambda(m_s), \quad (28)$$

$$m_s^0 = m_s - 8Gm_s I_1^\Lambda(m_s) - 32K(m_u I_1^\Lambda(m_u))^2. \quad (29)$$

To define all three parameters of the dilaton potential (χ_0, χ_c, B), we have to use, in addition to equations (23)–(26), the equation for the bare glueball mass to be given in the next Section.

5 Mass formulae for scalar isoscalar mesons and glueball

The free part of the Lagrangian (18) has the form

$$\begin{aligned} L^{(2)}(\sigma, \phi, \bar{\chi}) = & -\frac{1}{2}g_8^2[(G^{(-)})_{88}^{-1} - 8I_1^\Lambda(m_u) + 4m_u^2]\sigma_8^2 \\ & - \frac{1}{2}g_9^2[(G^{(-)})_{99}^{-1} - 8I_1^\Lambda(m_s) + 4m_s^2]\sigma_9^2 \\ & - g_8g_9(G^{(-)})_{89}^{-1}\sigma_8\sigma_9 - 8(\mathcal{C}_g + A)\left(\frac{\bar{\chi}}{\chi_c}\right)^2 \\ & - \frac{16}{\chi_c}\left[g_8m_u^3I_2^\Lambda(m_u)\sigma_8 - g_9\frac{m_s^3I_2^\Lambda(m_s)}{\sqrt{2}}\sigma_9\right]\bar{\chi}. \end{aligned} \quad (30)$$

The dilaton and its interaction with quarkonia does not change the model parameters m_u , m_s , Λ , G , and K fixed in our earlier paper [22]

$$\begin{aligned} m_u &= 280 \text{ MeV}, \quad m_s = 420 \text{ MeV}, \quad \Lambda = 1.25 \text{ GeV}, \\ G &= 4.38 \text{ GeV}^{-2}, \quad K = 11.2 \text{ GeV}^{-5}. \end{aligned} \quad (31)$$

After the dilaton field is introduced into our model, there appear three new parameters: χ , χ_c , and B . To determine these parameters, we use two equations (25) and (26) and the bare glueball mass

$$m_g^2 = 16\frac{(\mathcal{C}_g + A)}{\chi_c^2}. \quad (32)$$

We adjust it so that, in the output, the mass of the heaviest meson would be 1500 MeV or 1710 MeV and thereby fix χ_c . For the glueball condensate, we use the value $(390 \text{ MeV})^4$ [28]. The result of our fit is presented in Table 1 where we show the spectrum of three physical scalar isoscalar states σ_I , σ_{II} and σ_{III} . The parameters χ_0 and B are fixed by the gluon condensate

and constituent quark masses

$$\chi_0 = \chi_c \exp \left(\frac{A}{2(\mathcal{C}_g + 2A)} \right), \quad (33)$$

$$B = \frac{\mathcal{C}_g + 2A}{4} \exp \left(\frac{2A}{\mathcal{C}_g + 2A} \right). \quad (34)$$

It is worth noting that the mixing of the glueball with quarkonia shifts the quarkonia mass spectrum. They become lighter, whereas the glueball becomes heavier. This is good for our model because, in previous paper [22], the mass of σ_{II} associated with $f_0(980)$ was exaggerated.

6 Decay widths

Once all parameters are fixed, we estimate decay widths for the major strong decay modes of the scalar mesons: $\sigma_a \rightarrow \pi\pi$ and $\sigma_a \rightarrow KK$. We neglect decays into $\eta\eta$ and $\eta\eta'$ as they are small. The results are displayed below. The state σ_I that we identify with $f_0(400-1200)$ decays mostly into a pair of pions, and this process determines the width of σ_I :

$$\Gamma_{\sigma_I \rightarrow \pi\pi} \approx 630 \text{ MeV}. \quad (35)$$

The decay of the state σ_{II} that we identify with $f_0(980)$ into pions is noticeably enhanced by the glueball component because of mixing with the $s\bar{s}$ quarkonium. We obtain

$$\Gamma_{\sigma_{II} \rightarrow \pi\pi} = 116 \text{ MeV} \quad (36)$$

if $\sigma_{III} \equiv f_0(1500)$ and

$$\Gamma_{\sigma_{II} \rightarrow \pi\pi} = 96 \text{ MeV} \quad (37)$$

if $\sigma_{III} \equiv f_0(1710)$. From experiment, we know that its decay width lies within the interval from 40 MeV to 100 MeV.

The decay width of σ_{III} is slightly different for both cases. In case σ_{III} is $f_0(1500)$, we have

$$\Gamma_{\sigma_{III} \rightarrow \pi\pi} = 83 \text{ MeV}, \quad \Gamma_{\sigma_{III} \rightarrow KK} = 152 \text{ MeV}, \quad (38)$$

and in the other case ($\sigma_{III} \equiv f_0(1710)$)

$$\Gamma_{\sigma_{III} \rightarrow \pi\pi} = 105 \text{ MeV}, \quad \Gamma_{\sigma_{III} \rightarrow KK} = 136 \text{ MeV}. \quad (39)$$

As one can see, we obtained reasonable values for the states which are mostly quarkonia and overestimated decay width for the state σ_{III} . This can be explained by that the mixing of $s\bar{s}$ quarkonium with the glueball in this type of model is too large (it is proportional to the cubed mass of strange quark, see (30)). As a result, the decay $\sigma_{III} \rightarrow KK$ is large. This mixing becomes a bit less, when we fit the paramers for a higher glueball mass.

Let us note also that we do not include the decay into 4π . This process is not dominant in our model (contrary to Ref. [10]). Our estimates are based on the assumption that the process $\sigma_{III} \rightarrow 4\pi$ occurs through two intermediate σ resonances $\sigma_{III} \rightarrow \sigma\sigma \rightarrow 4\pi$. We found that $\Gamma_{\sigma_{III} \rightarrow 4\pi}$ does not exceed 20 MeV. Therefore, in both the cases, the total width of σ_{III} is approximately 260 MeV.

7 Conclusion

In this work, we investigated a possible way of including the glueball into an effective chirally symmetric meson Lagrangian. This Lagrangian was studied in Ref. [22] where the masses and strong decay widths of the ground scalar meson states were estimated in the NJL model with the 't Hooft interaction taken into account. Now, following Ref. [22], we considered the interaction of the glueball with the ground scalar isoscalar $q\bar{q}$ states $f_0(400 - 1200)$ and $f_0(980)$. The mixing of the glueball with radially excited states $f_0(1370)$ and $f_0(1710)$ (if $\sigma_{III} \equiv f_0(1500)$) was not taken into account. However, their mixing is very important and will be considered in subsequent works. Therefore, the results obtained here are tentative and are not claimed for a quantitative explanation of experimental data on scalar resonances.

As it was mentioned in the Introduction, nowadays there are many papers devoted to the description of the scalar glueball in the framework of an effective meson Lagrangian [18–21]. The way, we introduce the scalar glueball, is closer to that used in Ref. [20], but our work differs in two points. First, we take into account the 't Hooft interaction leading to the singlet-octet mixing of the scalar isoscalar quarkonia. The glueball is also involved in this mixing and changes it. Next, we introduce the glueball as a dilaton field into the Lagrangian written in a chirally symmetric form corresponding to the phase with chiral symmetry not broken spontaneously. Thereby, the Lagrangian is given a highly symmetric form that keeps both the chiral symmetry and scale invariance. In this form the dilaton fields are introduced only into the mass terms of meson Lagrangian. The rest of Lagrangian terms are scale invariant except the term with the current quark mass which explicitly breaks both the chiral symmetry and scale invariance.

As a result, we obtain reasonable estimates for the masses of the scalar mesons $f_0(400 - 1200)$, $f_0(980)$ ⁷, and the glueball $f_0(1500)$ (or $f_0(1710)$) and also for their strong decay widths. But we have to point out that the width of the $f_0(1500)$ (or $f_0(1710)$) resonance is, possibly, overestimated (> 200 MeV), however, it can change after the states $f_0(1370)$ and $f_0(1710)$ (or $f_0(1500)$) are included into the whole picture.

The results that we obtained in this work, as one can see, are not enough to answer the question: which of the states $f_0(1500)$ and $f_0(1710)$ is the scalar glueball? We hope to make closer towards the solution of this problem in our further works, where alternative ways of including the glueball into an effective meson Lagrangian will be investigated, radially excited meson states will be considered as well as the ground states, and the mixing of five scalar isoscalar states $f_0(400 - 1200)$, $f_0(980)$, $f_0(1370)$, $f_0(1500)$, and $f_0(1710)$ will be taken into account.

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⁷In Ref. [20], the ground $q\bar{q}$ scalar states were presented by $f_0(980)$ and $f_0(1300)$ as $u\bar{u}$ and $s\bar{s}$ quarkonia mixed with the glueball.

References

- [1] J. Sexton, A. Vaccarino, and D. Weingarten: *Phys. Rev. Lett.* **75** (1995) 4563
- [2] W. Lee and D. Weingarten: *Phys. Rev. D* **59** (1999) 094508
- [3] A. Vaccarino and D. Weingarten: *hep-lat/9910007*
- [4] C. Amsler *et al.*: *Phys. Lett. B* **353** (1995) 425; C. Amsler and F. Close: *Phys. Lett. B* **353** (1995) 385
- [5] S. Narison: *Nucl. Phys. B* **509** (1998) 312
- [6] V. V. Anisovich, D. V. Bugg and A. V. Sarantsev: *Phys. Rev. D* **58** (1998) 111503
- [7] A. Palano: *Nucl. Phys. Proc. Suppl.* **39 BC** (1995) 287
- [8] N.A. Törnqvist and M. Roos: *Phys. Rev. Lett.* **76** (1996) 1575
- [9] Eef van Beveren and G. Rupp: *hep-ph/9806246*
- [10] L. S. Celenza, B. Huang, H. Wang, and C. M. Shakin: preprint BCCNT:99/111/283, Brooklin Coll., N.Y., 1999
- [11] M. K. Volkov and V. L. Yudichev: *Int. J. Mod. Phys. A* **14** (1999) 4621
- [12] M. K. Volkov and V. L. Yudichev: *Yad. Fiz.* **63(10)** (2000) 1924 [*hep-ph/9905368*]
- [13] M. K. Volkov and V. L. Yudichev: *Fiz. Elem. Chast. At. Yadra* **31(3)** (2000) 576 [*hep-ph/9906371*]
- [14] M. K. Volkov and D. Ebert: *Sov. J. Nucl. Phys.* **36** (1982) 736; *Z. Phys. C* **16** (1983) 205; M. K. Volkov: *Ann. Phys.* **157** (1984) 282
- [15] M. K. Volkov: *Sov. J. Part. and Nucl.* **17** (1986) 186; D. Ebert and H. Reinhardt: *Nucl. Phys. B* **271** (1986) 188; D. Ebert, H. Reinhardt, and M.K. Volkov: *Prog. Part. Nucl. Phys.* **33** (1994) 1
- [16] M. K. Volkov and C. Weiss: *Phys. Rev. D* **56** (1997) 221
- [17] M. K. Volkov: *Phys. At. Nucl.* **60** (1997) 1920
- [18] K. Kusaka, M. K. Volkov, and W. Weise: *Phys. Lett. B* **302** (1993) 145
- [19] A. A. Andrianov, V. A. Andrianov, V. Yu. Novozhilov and Yu. V. Novozhilov: *JETP Lett.* **43** (1986) 719; A. A. Andrianov, V. A. Andrianov, D. Ebert, and T. Feldmann: *Int. J. Mod. Phys. A* **12** (1997) 5589; A. A. Andrianov and V. A. Andrianov: *Z. Phys. C* **55** (1992) 435; A. A. Andrianov, V. A. Andrianov, Yu. V. Novozhilov, and V. Yu. Novozhilov: *Phys. Lett. B* **186** (1987) 401
- [20] J. Cugnon, M. Jaminon, and B. Van den Bosche: *Nucl. Phys. A* **598** (1996) 515; M. Jaminon and B. Van den Bosche: *Nucl. Phys. A* **619** (1997) 285
- [21] J. Ellis and J. Lánik: *Phys. Lett. B* **150** (1984) 289; J. Ellis and J. Lánik: *Phys. Lett. B* **175** (1986) 83; J. Lánik: *Acta Phys. Slov.* **35** (1985) 343; *JINR Rapid Comm.* **N20-86** (1986) 10
- [22] M. K. Volkov, M. Nagy, and V. L. Yudichev: *Nuovo Cim. A* **112** (1999) 225
- [23] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov: *Nucl. Phys. B* **120** (1977) 316
- [24] H. Vogl and W. Weise: *Progr. Part. Nucl. Phys.* **27** (1991) 195
- [25] S. P. Klevansky: *Rev. Mod. Phys.* **64** (1992) 649
- [26] J. Collins, A. Duncan, and S. D. Joglekar: *Phys. Rev. D* **16** (1977) 438; N. K. Nielsen: *Nucl. Phys. B* **120** (1977) 212
- [27] M. K. Volkov: *Phys. Part. Nucl.* **24** (1993) 35
- [28] D. J. Broadhurst *et al.*: *Phys. Lett. B* **329** (1994) 103; B. V. Geshkenbein: *Phys. At. Nucl.* **58** (1995) 1171; S. Narison: *Phys. Lett. B* **387** (1996) 162