ON THE DEPENDENCE OF QUANTUM PARTICLE PRODUCTION ON MAXIMAL ACCELERATION

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The introduction of an upper limit to the proper acceleration of point-like particles may lead to new as well as interesting results in the particle production scheme of QFT in curved space-times. Some examples are discussed.

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In last decades grooving specialization and diversification have brought a host of papers and textbooks dealing with problems and features of Quantum Gravity. This activity has led to the development of important theoretical concepts, the construction of many explicit examples and the discovery of several peculiar properties (see e.g. [1]). Among the others, an upper kinematical limit $A_{\rm max}$ for the proper acceleration felt by a point-like particle is expected to appear (see e.g. [2,3]). A maximal acceleration principle would play a significative role in several contexts (see e.g. [2-8]). Interestingly, it can be incorporated into a geometrically inspired model by considering the relativistic phase-space $M' = (x^\mu, A_{\rm max}^{-1}\dot{x}^\mu)$ as the basis manifold for the description of dynamics of physical systems [2,3]. Hence, the relevant invariant line element turns out to be the one on M', that is

$$d\tau^2 = G_{\mu\nu}\left(dx^\mu dx^\nu + A_{max}^{-2} d\dot{x}^\mu d\dot{x}^\nu\right) \tag{1}$$

where $G_{\mu\nu}$ is the space-time metric. Remarkably, the ansatz $d\tau^2 \geq 0$ corresponds to the request that upper kinematical limits for velocity and acceleration, c and A_{max} respectively, are simultaneously present. Furthermore, the representation of position and momentum operators as covariant derivatives on M' allows us to interpret the quantization as curvature of M' [2].

Although fashinating, studies based on the line element (1) turn out to be disadvantageous for practical purposes. For this reason Caianiello proposed to proceed by means of a proper *redefinition* of the space-time background metric. The idea, in particular, is to make use of space-time geodesic equations. Following this suggestion, the line element (1) can be rewritten in the approximate form [2] (see also [8])

$$d\tilde{s}^2 = \left(G_{\mu\nu} + A_{max}^{-2}\,\Gamma_{\alpha\beta\mu}\Gamma^{\alpha}_{\nu\gamma}\dot{x}^\beta\dot{x}^\gamma\right)dx^\mu dx^\nu = \tilde{G}_{\mu\nu}dx^\mu dx^\nu\ . \tag{2}$$

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In this way one thus gets an *effective* space-time metric $\tilde{G}_{\mu\nu}$ which can be decomposed into a *classical* term, $G_{\mu\nu}$, plus a *quantum* one, $A_{max}^{-2}\,\Gamma_{\alpha\beta\mu}\Gamma^{\alpha}_{\nu\gamma}\dot{x}^{\beta}\dot{x}^{\gamma}$. The classical limit, which is concerned with the vanishing of quantum fluctuation of gravitational field, is recovered in the formal limit $A_{max} \to \infty$. This strategy allows one to analyze low-order effects induced by $A_{\max}$ on the dynamics of arbitrary physical systems.

The aim of this short communication is to stimulate discussions towards the direction of the role of $A_{\max}$ within the particle production scheme of QFT in curved space-times (see e.g. [1]). We give some idea of the various features that may arise in studying, for instance, a massive scalar field minimally coupled to a gravitational field. Since they provide a good approximation to the large structure geometry of the Universe, we restrict our attention on space-time backgrounds of the Robertson-Walker (RW) type by selecting, in particular, few simple -but still elucidating-examples.

As it is well known, RW space-times are defined in terms of the metric [1]

$$ds^2 = dt^2 - a^2(t)\,h_{ij}dx^i\,dx^j = C(\eta)(d\eta^2 - \,h_{ij}dx^i\,dx^j) \tag{3}$$

where η is the so-called conformal time defined by $d\eta = dt/a(t)$, $C(\eta) = a(t)^2$ is the conformal scale factor and $h_{ij}dx^i\,dx^j$ is given by

$$\begin{aligned} h_{ij}dx^i\,dx^j &= [(1-Kr^2)^{-1}\,dr^2 + r^2(d\theta^2 + \sin^2\theta\; d\varphi^2)] \\ &= d\chi^2 + f^2(\chi)(d\theta^2 + \sin^2\theta\; d\varphi^2) \end{aligned}$$

with

$$f(\chi) = \begin{cases} \sin\chi\ , & 0 \le \chi \le 2\pi \qquad K=1 \\ \chi & 0 \le \chi < \infty \qquad K=0 \\ \sinh\chi\ , & 0 \le \chi < \infty \qquad K=-1\ . \end{cases}$$

By exploiting the geodesic velocity field for a comoving observer, $\dot{x}^{\mu} = (C(\eta)^{-\frac{1}{2}}, 0, 0, 0)$, it easily follows from (2) that effects of maximal acceleration in RW space-times can be subsumed into the definition of an effective conformal factor

$$\tilde{C}(\eta) = C(\eta) + \left[\frac{1}{2A_{max}C(\eta)}\frac{dC(\eta)}{d\eta}\right]^2\ . \tag{4}$$

As our first application of formula (4), we consider a Milne universe. It corresponds to $K = -1$ and

$$C(\eta) = \exp(2a\eta) \tag{5}$$

(i.e. $a(t) = at$) where a is a constant (Hubble parameter). By virtue of (4), the effective Milne background is defined by the conformal scale factor

$$\tilde{C}(\eta) = \exp(2a\eta) + A_{max}^{-2}\,a^2\ . \tag{6}$$

The singularity of the metric for $\eta \to -\infty$ thus disappears. The minimally coupled Klein-Gordon equation describing a scalar field into the space-time with the effective conformal factor (6) is similar to the one describing the scalar field into the standard Milne space-time (5). It is therefore a trivial matter to see that particle production is expressed via

$$|\tilde{\beta}_k|^2 = [\exp(2\pi a^{-1}\sqrt{k^2 + m^2a^2A_{max}^{-2}}) - 1]^{-1} \tag{7}$$

with $k^2 = \sum_{i=1}^{3} k_i{}^2$ [1]. That is, the Planckian-like spectrum is preserved but it is weakly depressed by the presence of $A_{\max}$. Further, an upper threshold appears so that zero momentum particle production is no longer infinitum. In the limit $\eta \to -\infty$, we thus get a zero mode Planckian spectrum with a finite temperature given by $T_{k=0} = A_{max}/(2\pi k_B)$. Analogous results occur when considering a spatially flat ($K = 0$) RW universe with the same conformal scale factor (5).

Although the Klein-Gordon equation over RW space-times with conformal factors of the type (4) may be not easy to handle in general, indicative results as well as consequences can be further investigated[2]. For instance, let us consider the RW space-time defined via

$$C(\eta) = a_0^2 \cosh^2(\eta) \tag{8}$$

(*i.e.* $a^2(t) = a_0^2 + t^2$) where a_0 is a nonvanishing constant. This universe is nonsingular and asymptotically tangent, in the distant future and in the distant past, to two distinct Milne universes [10]. In this case, we get

$$\tilde{C}(\eta) = a_0^2 \cosh^2(\eta) + A_{max}^{-2} \tanh^2(\eta) \; . \tag{9}$$

When is A_{max} great enough compared to a_0^{-1}, the quantum correction in (9) is a very slowly varying function with respect the conformal factor $C(\eta)$. This circumstance allows us to substitute the quantum correction with its mean value and to adopt the following approximate structure:

$$\tilde{C}(\eta) \cong a_0^2 \cosh^2(\eta) + A_{max}^{-2} \; . \tag{10}$$

The solution to equation the Klein-Gordon equation with effective conformal scale factor (10) can be expressed in terms of Mathieu functions. Accordingly to [10], when the Compton wavelength of created quanta is large compared to the minimum of conformal factor (10), particle production can be approximated by

$$|\tilde{\beta}_k^2| \simeq \frac{1 - \cos\left[4\sqrt{k^2 + m^2 A_{max}^{-2}}\, \log\left(\rho + m a_0/4\right)\right]}{2 \sinh\left(\pi\sqrt{k^2 + m^2 A_{max}^{-2}}\right)} \tag{11}$$

where ρ is an irrelevant phase. The zero momentum particle production is therefore no longer vanishing.

A comment is now in order. In deriving (10), we assumed parameter a_0 involved by the *classical* conformal factor (8) to be great enough when compared to A_{max}^{-1}. Since A_{max} can induce indeed either an increasing or a decreasing behavior of particle production, formula (11) suggests to make such an assumption more concrete. We do not enter in the question of characterizing A_{max} but simply recall that in literature two cases are discussed. In one case, A_{max} is an universal constant to be identified with A_{Planck} and which may be related, for instance, to a supersymmetry breaking mechanism [11]. The second possibility is based on a mass-dependent maximal acceleration, i.e. $A_{max} = m$ in natural units (see e.g. [2,5]). It therefore implies a weak violation of equivalence principle in the quantum regime.

Previous formulae suggest that as long as universe expands the presence of A_{max} introduces new particle production rates. On general basis, what one expects is that for a given momentum

[2]Note that (4) provides a way to generate a scalar field *dynamical* mass, $m(\eta)^2 = m^2\tilde{C}(\eta)/C(\eta)$. Perhaps, it may be interesting to look into this aspect in the light of discussion in [9].

k the amount of particle production in a given background is influenced by A_{max}. Nevertheless, it is worth to note that this seemingly natural prediction encounters one interesting exception, at least within the lowest order approximation given by formula (4). The remarkable exception turns out to be a de Sitter space-time. In this case, indeed, $C(\eta) = a\eta^{-2}$ and the result of the embedding (4) turns out to be nothing but a fine-tuning of the de Sitter parameter a. Hence, the underlying de Sitter symmetry is not lost and standard results of QFT in de Sitter space-time are reproduced up to the substitution $a \to a + A_{max}^{-2}$. Among the others, this means that there is still no particle production [1].

Above ideas and results as well as the relevance of the procedure outlined above, equation (2), as a simple model to account for essential features displayed by real systems in the Quantum Gravity regime can be certainly debated. It is clear, for instance, that solutions over a greater range of validity would be advisable. As one moves to shorter and shorter wavelengths the higher order corrections induced by A_{max} may be far from negligible and in fact can have a crucial influence on all the dynamical aspects. Nevertheless, in this letter we have been guided by the consideration that, in order to attack the fascinating problem of quantization of gravity and its consequences, all techniques and ideas should be carefully considered. Even if it may not appear fashionable, we believe that taking into account quantum fluctuations of gravitational fields by means of the embedding procedure (2) may be useful in giving some insight into a better understanding of the subject. We thus hope that our naive observations may suggest one to perform additional studies.

References

[1] N.D. Birrel, P.C.W. Davies: *Quantum Field Theory in curved space*, Cambridge University Press, Cambridge,1982

[2] E.R. Caianiello: *La Rivista del Nuovo Cimento* **15** (1992) 4; and references therein

[3] H.E. Brandt: *Chaos, Solitons and Fractals* **10** (1999) 267; and references therein

[4] G. Lambiase *et al.*: *Nuovo Cim. B* **112** (1997) 1003; and references therein

[5] E.R. Caianiello *et al.*: *Nuovo Cim. B* **105** (1990) 259

[6] S. Capozziello *et al.*: *Phys. Lett. A* **268** (2000) 247

[7] V.V. Nesterenko *et al.*: *Phys. Rev. D* **60** (1999) 065001

[8] E.R. Caianiello *et al.*: *Int. J. Theor. Phys.* **29** (1990) 131

[9] D.H. Lyth, D. Roberts: *Phys. Rev. D* **57** (1998) 7120

[10] I. Costa *et al.*: *Class. Quant. Grav.* **6** (1989) 1983

[11] M. Toller: *Phys. Lett. B* **256** (1991) 215; and references therein