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A CONDITIONAL MEASUREMENT SCHEME FOR THE GENERATION OF MAXIMALLY ENTANGLED BIMODAL FIELD STATES*

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A scheme for generating maximally entangled states of bimodal high-Q cavity field is presented. Its implementation is based on a Jaynes-Cummings like one photon interaction mechanism between a single three level Rydberg atom in the lambda configuration and two modes of the radiation field. The practical feasibility of the project is discussed.

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The term entanglement, introduced many years ago by Schrödinger [1], describes the state of a system, composed by two or more subsystems, characterized by the fact that it cannot be expressed as a product of states of each component. In such a condition the system exhibits the astonishing property that the results of measurement on one subsystem cannot be specified independently of the parameters of the measurements on the other components. Moreover, as shown by Gisin [2], for any entangled state suitable observables can be found which lead to a violation of a variant of Bell's inequality. These puzzling behaviours, as well as others like the Schrödinger cat paradoxes and the quantum non locality, also stemming from entanglement, have stimulated a great deal of interest both of theoreticians and experimentalists. Apart their intrinsic theoretical interest, entangled states form also the basis of experiments in the realm of quantum information. In this context, a particularly interesting example is quantum state teleportation. Teleportation protocol allows an unknown state of a quantum system to be faithfully transmitted between two spatially separated parties. An essential step of this procedure is that sender and receiver are initially sharing a maximally entangled state. Moreover, from an applicative point of view, the possibility of generating in laboratory such states is of crucial importance in quantum cryptography and, in general, in all implementations of quantum computers [3].

Previous works have concentrated essentially on the generation of entangled states of atoms [4]. Recently, several kinds of entangled states of the electromagnetic field, such as, for example, entangled coherent states [5], multimode even and odd coherent states [6], entangled photon number states [7] and so on, have been discussed in literature. In the framework of cavity quantum electrodynamics J.A. Bergou [8] has proposed a theoretical scheme for the generation of entangled states of photons in spatially separated cavities.

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In this paper we propose a simple scheme, based on a conditional measurement approach, for generating a class of maximally entangled states of two modes of a single cavity. Our method exploits a Jaynes-Cummings-like one-photon interaction mechanism between a three level Rydberg atom in the lambda configuration and two modes of the radiation cavity field. We will show that, following the single atom procedure we are going to describe, it is possible to guide the cavity field toward bimodal field states having the maximally entangled form

$$|\phi_n\rangle = \frac{1}{\sqrt{2}}(|n, n-1\rangle + e^{i\phi}|n-1, n\rangle) \quad n = 1, 2, ... \tag{1}$$

The implementation of our scheme requires a single three levels Rydberg atom only, in the lambda configuration, and a bimodal high-Q microcavity ($Q \sim 10^{10}$). Let's indicate by ω_1 and ω_2 the frequencies of the two field modes and suppose that $\omega_1 \neq \omega_2$. The three relevant atomic circular Rydberg states are denoted by $|0\rangle$, $|1\rangle$ and $|2\rangle$.

We suppose that, indicating by E_j^A the energy level correspondent to the atomic state $|j\rangle$ $(j = 0, 1, 2)$, with $E_0^A < E_1^A < E_2^A$, the two resonance conditions $E_2^A - E_1^A \sim \omega_2$, $E_2^A - E_0^A \sim \omega_1$ $(\hbar = 1)$, are satisfied. Under these hypothesis the effective Hamiltonian model describing our system, in the rotating wave approximation, can be written down in the following form:

$$H = H_0 + H_I \tag{2}$$

with

$$H_0 = \sum_{i=1}^{2} \omega_i a_i^\dagger a_i + \sum_{j=0}^{2} E_j^A |j\rangle\langle j| \tag{3}$$

and

$$H_I = (g_1 a_1 |2\rangle\langle 0| + h.c.) + (g_2 a_2 |2\rangle\langle 1| + h.c.) \tag{4}$$

In Eqs. (3) and (4) $a_i(a_i^\dagger)$ $(i = 1, 2)$ is the annihilation (creation) operator relative to the $i-$th mode of the bimodal cavity field whereas the two coupling constants g_1 and g_2 measure the strengths of the energy exchanges between the Rydberg atom and the mode 1 and 2 respectively.

Let's now observe that the two operators N_1, N_2 defined as:

$$N_1 = a_1^\dagger a_1 - |0\rangle\langle 0| \tag{5}$$

$$N_2 = a_2^\dagger a_2 - |1\rangle\langle 1|. \tag{6}$$

are constants of motion. It is in fact easy to verify that:

$$[N_1, H] = [N_2, H] = 0. \tag{7}$$

From Eqs. (5) and (6) immediately follows that the operator $N \equiv N_1 + N_2 = a_1^\dagger a_1 + a_2^\dagger a_2 + |2\rangle\langle 2| - 1$, is also a constant of motion. Moreover it is easy to see that

$$[N_1, N_2] = 0. \tag{8}$$

This circumstance in particular means that if we prepare the atom-field system in the state $|\psi_j(0)\rangle = |n_1, n_2, j\rangle$ with $j = 0, 1, 2$, common eigenstate of N_1 and N_2 correspondent to the eigenvalues $(n_1 - \delta_{j0})$ and $(n_2 - \delta_{j1})$ respectively, we may claim with certainty that, at any time instant t, the state of the system can be written as superposition of a finite number of states all pertaining to the same initial eigenvalues of N_1 and N_2. This fact is of particular relevance being at the origin of the possibility of exactly solving the time evolution of the coupled system.

Let's consider, in fact, the initial condition $|\psi_0(0)\rangle = |n_1, n_2, 0\rangle$ and assume for simplicity $g_1 = g_2 = g$. The state of the system at a generic time t can be written as:

$$\begin{aligned} |\psi_0(t)\rangle &= \Big[C^{(0)}_{n_1,n_2}(t)|n_1, n_2, 0\rangle + C^{(0)}_{n_1-1,n_2+1}(t)|n_1 - 1, n_2 + 1, 1\rangle \\ &+ C^{(0)}_{n_1-1,n_2}(t)|n_1 - 1, n_2, 2\rangle\Big] e^{-i(E_0+n_1\omega_1+n_2\omega_2)t}. \end{aligned} \tag{9}$$

Substituting Eq. (9) into the time dependent Schrödinger equation relative to the physical system under scrutiny, we get a system of differential equations from which we straightforwardly deduce the following three unknown amplitudes appearing into Eq. (9):

$$C^{(0)}_{n_1,n_2} = \left(\frac{n_1}{n_1 + n_2 + 1}\cos(gt\sqrt{n_1 + n_2 + 1}) + \frac{n_2 + 1}{n_1 + n_2 + 1}\right), \tag{10}$$

$$C^{(0)}_{n_1-1,n_2+1} = \frac{\sqrt{n_1(n_2 + 1)}}{n_1 + n_2 + 1}\left(\cos(gt\sqrt{n_1 + n_2 + 1}) - 1\right), \tag{11}$$

$$C^{(0)}_{n_1-1,n_2} = -i\frac{\sqrt{n_1}}{\sqrt{n_1 + n_2 + 1}}\sin(gt\sqrt{n_1 + n_2 + 1}). \tag{12}$$

Analogously, choosing $|\psi_1(0)\rangle = |n_1, n_2, 1\rangle$, it is possible to convince oneself that the state $|\psi_1(t)\rangle$ can be written down as follows:

$$\begin{aligned} |\psi_1(t)\rangle &= \Big[C^{(1)}_{n_1,n_2}(t)|n_1, n_2, 1\rangle + C^{(1)}_{n_1+1,n_2-1}(t)|n_1 + 1, n_2 - 1, 0\rangle \\ &+ C^{(1)}_{n_1,n_2-1}(t)|n_1, n_2 - 1, 2\rangle\Big] e^{-i(E_0+n_1\omega_1+n_2\omega_2+\Delta\omega)t} \end{aligned} \tag{13}$$

with $\Delta\omega = \omega_1 - \omega_2$ and

$$C^{(1)}_{n_1,n_2} = \left(\frac{n_2}{n_1 + n_2 + 1}\cos(gt\sqrt{n_1 + n_2 + 1}) + \frac{n_1 + 1}{n_1 + n_2 + 1}\right), \tag{14}$$

$$C^{(1)}_{n_1+1,n_2-1} = \frac{\sqrt{n_2(n_1 + 1)}}{n_1 + n_2 + 1}\left(\cos(gt\sqrt{n_1 + n_2 + 1}) - 1\right), \tag{15}$$

$$C^{(1)}_{n_1,n_2-1} = -i\frac{\sqrt{n_2}}{\sqrt{n_1 + n_2 + 1}}\sin(gt\sqrt{n_1 + n_2 + 1}. \tag{16}$$

The building up of a target state $|\phi_n\rangle$ results from the success of a *single* event.

Let's suppose that a Rydberg atom, initially prepared in the state $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ by means an appropriate Ramsey zone, is injected into the cavity where the two field modes of interest are excited in a Fock state with the same number n of photons. The initial condition of the atom-field system is thus given by

$$|\psi(0)\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)|n, n\rangle. \tag{17}$$

Exploiting the results previously presented, it is immediate to demonstrate that at a generic time t the state of the system, within an irrelevant overall phase factor, assumes the form:

$$\begin{aligned} |\psi(t)\rangle &= [C^{(0)}_{n,n}(t)|n,n\rangle + e^{i\Delta\omega t}C^{(1)}_{n+1,n-1}(t)|n+1,n-1\rangle]|0\rangle + \\ &+ [C^{(0)}_{n-1,n+1}(t)|n-1,n+1\rangle + e^{i\Delta\omega t}C^{(1)}_{n,n}(t)|n,n\rangle]|1\rangle + \\ &+ [C^{(1)}_{n,n-1}(t)|n,n-1\rangle + e^{i\Delta\omega t}C^{(0)}_{n-1,n}(t)|n-1,n\rangle]|2\rangle \end{aligned} \tag{18}$$

where the probability amplitudes $C^{(j)}_{m,p}$, $(j = 0,1)$ and $(m, p = n-1, n, n+1)$, can be obtained from Eqs. (10) - (16) putting $n_1 = n_2 = n$.

Immediately after the atom has left the cavity its internal state is measured using the ionization technique. Looking at Eq. (18) and taking into account Eqs. (12) and (16), it is easy to convince oneself that, if the atomic state detector finds at $t = t_1$ the atom in its excited state $|2\rangle$, the radiation field is projected onto the normalized state

$$|\phi_n\rangle = \frac{1}{\sqrt{2}}(|n,n-1\rangle + e^{i\Delta\omega t_1}|n-1,n\rangle). \tag{19}$$

In other words our scheme provides a method for guiding the cavity field from the factorized state $|n,n\rangle$ toward the maximally entangled state expressed by Eq. (19) exploiting the interaction between the cavity and a single Rydberg atom only. A relevant consequence is that, in the context of our method, the low efficiency (∼50%) of the currently used experimental apparatus for detecting the atomic state turns out to be reasonable countered.

It is worth noting that the state into which the cavity collapses due to a successful measurement of the atomic internal state exhibits maximum entanglement regardless both of the initial equal population given to the two cavity modes and the duration of interaction t_1. In other words, whatever n and t_1 are, if the atom is measured in its excited state we may claim with certainty that the cavity field reduces to a maximally entangled state having the form (19).

Of course to judge the efficiency of our scheme and in particular to appreciate whether it is of experimental significance, we must evaluate the probability of detecting the atom in its excited state. Taking into account Eq. (18) it is not difficult to convince oneself that the probability of success P of our experimental scheme is given by:

$$P = \frac{n}{2n+1}\sin^2(gt_1\sqrt{2n+1}). \tag{20}$$

Thus, choosing the interaction time t_1 between the atom and the cavity field in such a way that the condition

$$gt_1 = \frac{\pi}{2}\frac{1}{\sqrt{2n+1}} \tag{21}$$

is satisfied, the probability of measuring the atom in the excited state $|2\rangle$ becomes

$$\frac{1}{3} \le P \equiv \frac{n}{2n+1} < \frac{1}{2} \tag{22}$$

thus providing values of experimental interest. The circumstance that our procedure is a single-atom scheme turns out once again of particular relevance since, in this case, the velocity of a single atom only has to be controlled. On the other hand, the accuracy of the velocity selectors today available may be estimated of the order of $1\% - 2\%$. Both these considerations allows us to guess that the probability of success of our scheme is practically immune from the fluctuations of the interaction time t_1.

It is important moreover to underline that the success of the procedure reported in this paper for generating maximally entangled bimodal cavity field having the form (19), is strictly related to the capacity of preparing the atom-field system in the state expressed by Eq. (17).

As far as the atomic initial state, it is well known that the Ramsey zone method, currently used in laboratory for mixing two atomic states, is very efficient [9]. On the other hand, it is possible to prepare the cavity field in an equally intensity bimodal Fock state $|n,n\rangle$, following, for example, an experimental scheme very recently presented in literature by one of the authors [10]. In Ref. [10] it is shown that, taking into consideration important technological limits of the apparatus currently used in laboratory, the probability of realizing the state $|n,n\rangle$, decreases with n maintaining however values of experimental interest in correspondence to $n \sim 10$.

It is important to observe, at this point, that, notwithstanding the high values of the quality factor Q of the resonators today available, it could be illegitimate to neglect the cavity losses when n is too large, in view of the fact that the photon damping time τ is such that $n\tau\omega = Q$.

These considerations suggest that from an experimental point of view it should be better to choose initial conditions such that the number of photons contained in both cavity modes at $t = 0$ does not exceed 10.

In conclusion we wish to point out that the states generated following the procedure discussed in this paper can be regarded as Bell like states thus providing an interesting starting point for testing fundamental features of quantum mechanics.

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