SQUEEZED-VACUUM ASSISTED QUANTUM TELEPORTATION*

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We show that the fidelity of teleportation of continuous quantum variables can be improved by conditional photon-number measurement of the entangled state. Further, we propose a teleportation scheme based on photon counting on the output fields of a squeezer that combines the mode whose quantum state is desired to be teleported and one mode of the two-mode squeezed vacuum playing the role of the entangled state.

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1 Introduction

In quantum teleportation, an unknown state of a system is destroyed and created on another, distant system. The method was first suggested in [1] and realized in [2, 3] for discrete variables, namely photonic (polarization) qubits. Subsequently, the concept has been extended to continuous variables [4,5], and then realized experimentally to teleport a coherent state by means of parametrically entangled (squeezed) optical beams and quadrature-component measurements [6].

The basic requirement of quantum teleportation is that the two parties share an entangled state with each other. In continuous-variable teleportation of quantum states of optical field modes, a two-mode squeezed vacuum is suited for playing the role of the entangled state. The quadrature components $\hat{q}_k$ and $\hat{p}_k$ ($[\hat{q}_k, \hat{p}_k]=i$, $k=1,2$) are correlated and anti-correlated, respectively, such that $\Delta(\hat{q}_1 - \hat{q}_2)\,\Delta(\hat{p}_1 + \hat{p}_2) < 1$. For large squeezing, the correlations approach the original Einstein-Podolsky-Rosen (EPR) correlations [7] (for EPR correlations in optical fields, see, e.g., [8, 9]).

Here we want to discuss two aspects of quantum teleportation based on a two-mode squeezed vacuum as entangled state. First, we suggest a way for improving the teleportation fidelity by

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conditional measurement (CM) of the numbers of photons reflected by low-reflectance beam splitters the two entangled modes are fed into. Such an operation can increase the strength of squeezing of the two-mode squeezed vacuum and the amount of entanglement as well. As a result, the input state can be teleported with increased fidelity. Second, we propose a conditional teleportation scheme, where Alice transmits through a squeezer the mode whose state is desired to be teleported and one of the modes of the two-mode squeezed vacuum, measures the photon numbers of the outgoing modes, and communicates the result to Bob. For some measurement results, Bob can then accomplish the teleportation by appropriate transforming his part of the entangled quantum state.

2 Teleportation improvement by photon subtraction via conditional measurement

The prospects of realizing teleportation of continuous quantum variable are limited by the available strength of squeezing of the entangled two-mode state. Perfect teleportation requires an infinitely squeezed vacuum, which is, of course, not available; the fidelity of the teleported state decreases with decreasing squeezing. Our objective here is to increase the fidelity by improving the entanglement of the shared state using conditional photon-number measurement. It has been shown that when a single-mode squeezed vacuum is transmitted through a beam splitter and a photon-number measurement is performed on the reflected beam, then a Schrödinger-cat-like state is generated [10]. Even though photons have been subtracted, the mean number of photons remaining in the transmitted state has increased. The newly generated states have often larger squeezing than the input squeezed vacuum.

Here we demonstrate that, by transmitting each mode of the two-mode squeezed vacuum through a low-reflectance beam splitter and detecting photons in the reflected beams, the transmitted modes are prepared in an entangled state which differs from the original one due to the photons subtracted by the measurement. Teleportation is performed if the two detectors simultaneously register photons. We show that such a CM can increase the degree of entanglement of the transmitted modes, as well as the fidelity of the teleported state.

2.1 Basic formulas

We first consider the standard scheme of teleportation of continuous quantum variables (Fig. 1 without the beam splitters BS_1 and BS_2). The entangled state shared between Alice and Bob is supposed to be a two-mode squeezed vacuum, which, in the Fock basis, can be written as

$$|\psi_E\rangle = \sqrt{1-q^2}\sum_{n=0}^{\infty} q^n |n\rangle_1 |n\rangle_2 \,, \tag{1}$$

where the indices 1 and 2 refer to the two modes, and $q \in (0;1)$. The first mode is mixed on the 50%-50% beam splitter BS_0 in Fig. 1 with the input mode prepared in a state $\hat{\varrho}_{\mathrm{in}}$ Alice wishes to teleport. Homodyne detection is performed on the two output modes of the beam splitter BS_0 in order to measure the conjugate quadrature components $\hat{x}_0$ and $\hat{p}_1$. By sending classical information, Alice communicates the measured values X_0 and P_1 to Bob, who uses the value $\sqrt{2}(X_0+iP_1)$ as a displacement parameter for shifting the quantum state of the second mode of the entangled state.

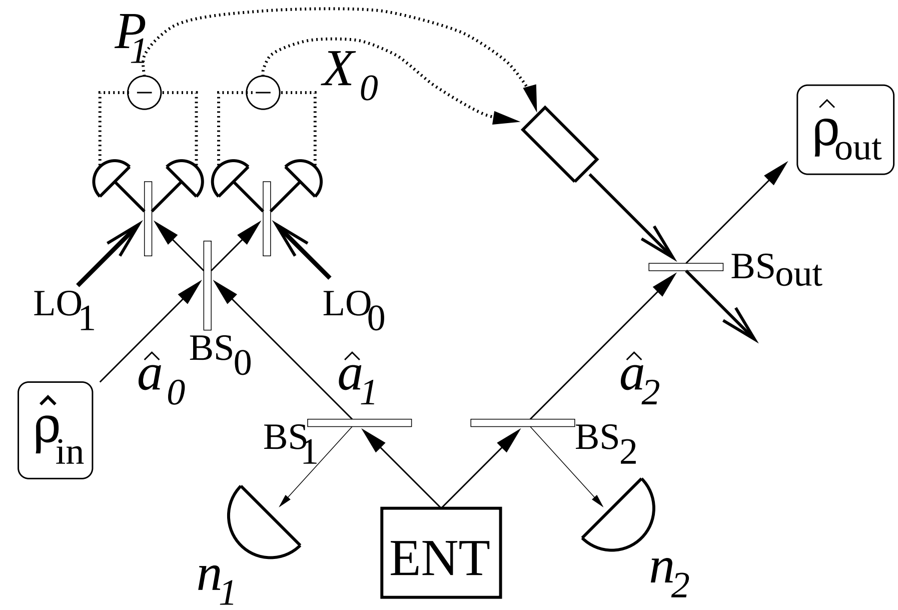

Fig. 1. Teleportation scheme: An input state $\hat{\varrho}_{\rm in}$ is destroyed by measurement and it appears, with certain fidelity, at a distant mode as $\hat{\varrho}_{\rm out}$. The essential means is the entangled state created as a two-mode squeezed vacuum in the box ENT. The degree of entanglement is improved by CM of the numbers of photons n_1 and n_2 reflected at the beam-splitters BS_1 and BS_2. After mixing the input mode prepared in the state $\hat{\varrho}_{\rm in}$ with the mode 1 of the entangled state, the quadrature components $\hat{x}_0$ and $\hat{p}_1$ are measured and their values X_0 and P_1 are communicated by Alice to Bob via classical channels (dotted lines). Using these values as displacement parameters for shifting the quantum state of the mode 2 on the beam splitter $\mathrm{BS}_{\rm out}$, Bob creates the output state $\hat{\varrho}_{\rm out}$ which imitates $\hat{\varrho}_{\rm in}$.

For the sake of simplicity, we restrict our attention to pure states and use the $\hat{x}_j$ quadrature-component basis [$\hat{x}_j = 2^{-1/2}(\hat{a}_j + \hat{a}_j^\dagger)$, $\hat{p}_j = -2^{-1/2} i(\hat{a}_j - \hat{a}_j^\dagger)$, $j = 0,1,2$]. Let the input-state wave function be $\psi_{\rm in}(x_0)$ and the entangled state have the wave function $\psi_E(x_1,x_2)$, so that the initial overall wave function is

$$\psi_I(x_0,x_1,x_2) = \psi_{\rm in}(x_0)\psi_E(x_1,x_2). \tag{2}$$

Assuming the beam-splitter BS_0 mixes the quadratures as

$$\hat{x}_0 \to 2^{-1/2}(\hat{x}_1 + \hat{x}_0), \quad \hat{x}_1 \to 2^{-1/2}(\hat{x}_1 - \hat{x}_0), \tag{3}$$

the transformed wave function is

$$\psi_{II}(x_0,x_1,x_2) = \psi_{\rm in}\left(\frac{x_1+x_0}{\sqrt{2}}\right)\psi_E\left(\frac{x_1-x_0}{\sqrt{2}},x_2\right). \tag{4}$$

Upon measuring the quadratures $\hat{x}_0$ and $\hat{p}_1$, in order to obtain the values X_0 and P_1, the (unnormalized) wave function of the mode 2 reads

$$\psi_{X_0,P_1}(x_2) = (2\pi)^{-1/2}\int dx_1\, e^{-iP_1x_1}\,\psi_{\rm in}\left(\frac{x_1+X_0}{\sqrt{2}}\right)\psi_E\left(\frac{x_1-X_0}{\sqrt{2}},x_2\right), \tag{5}$$

and the probability density of measuring the values X_0 and P_1 is given by

$$\mathcal{P}(X_0, P_1) = \int dx_2 |\psi_{X_0,P_1}(x_2)|^2. \tag{6}$$

Using the measured values X_0 and P_1 to realize a displacement transformation on the mode 2,

$$\hat{x}_2 \to \hat{x}_2 - \sqrt{2}X_0, \quad \hat{p}_2 \to \hat{p}_2 + \sqrt{2}P_1, \tag{7}$$

the resulting (unnormalized) wave function of the mode is found to be

$$\psi_{\rm tel}(x_2) = (2\pi)^{-1/2} \int dx_1 \Bigg[e^{iP_1(\sqrt{2}x_2 - x_1)} \times \psi_{\rm in}\left(\frac{x_1 + X_0}{\sqrt{2}}\right) \psi_E\left(\frac{x_1 - X_0}{\sqrt{2}}, x_2 - \sqrt{2}X_0\right)\Bigg]. \tag{8}$$

An infinitely squeezed two-mode vacuum, $q \to 1$ in Eq. (1), can be described, apart from normalization, by the Dirac delta function,

$$\psi_E(x_1, x_2) \to \delta(x_1 - x_2). \tag{9}$$

It can easily be checked that Eq. (8) then reduces to

$$\psi_{\rm tel}(x_2) \to \psi_{\rm in}(x_2), \tag{10}$$

i.e., the input quantum state is perfectly teleported. In the realistic case of finite squeezing, one evaluates the teleportation success by fidelity, given as the overlap of the input state with the output one. For a given measurement outcome the fidelity is given by $F(X_0, P_1) = |\langle\psi_{\rm in}|\psi_{\rm out}\rangle|^2$; the average fidelity is then

$$F = \int dX_0 \int dP_1 \, \mathcal{P}(X_0, P_1) F(X_0, P_1). \tag{11}$$

2.2 Conditionally entangled state

Let us now consider the full scheme in Fig. 1 in which each mode of the two-mode squeezed vacuum is transmitted through a low-reflectance beam splitter BS$_j$ ($j = 1, 2$) and the numbers of reflected photons n_j are detected. Each beam splitter BS$_j$ is described by a transformation matrix

$$T_j = \begin{pmatrix} t_j & r_j \\ -r_j & t_j \end{pmatrix} \tag{12}$$

with real transmittance t_j and real reflectance r_j, for simplicity. These matrices act on the operators of the input modes. After detecting n_j photons in the reflected modes, the Fock states $|k_j\rangle$ transform as ($n_j \le k_j$)

$$|k_j\rangle \to (-1)^{n_j} \sqrt{\begin{pmatrix} k_j \\ n_j \end{pmatrix}} |r_j|^{n_j} |t_j|^{k_j - n_j} |k_j - n_j\rangle \tag{13}$$

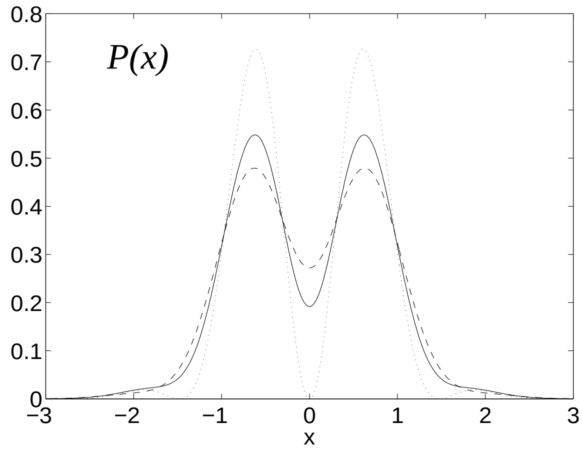

Fig. 2. Probability distribution of measuring the x quadrature in the teleported quantum state for input state $|\Psi\rangle_{\text{in}} \sim (|\alpha\rangle - |-\alpha\rangle)$, $\alpha = 1.5\,i$; dotted line - input state; dashed line - teleported state if the entangled state is a squeezed vacuum ($q = 0.8178$); solid line - teleported state if the entangled state is the photon-subtracted squeezed vacuum obtained by CM ($n_1 = n_2 = 1$; $r_1 = r_2 = 0.15$).

(for details and more general beam-splitter transformations, see [10, 12]).
The expansion coefficients $a_{k,l}^{(E)}$ of the entangled two-mode wave function,

$$a_{k,l}^{(E)} = \langle k, l | \psi_E \rangle \tag{14}$$

are then transformed into

$$a_{k,l}^{(E,\,\text{new})} = (-1)^{n_1 + n_2} \sqrt{\frac{(k+n_1)!(l+n_2)!}{k!\; l!\; n_1!\; n_2!}}\, |r_1|^{n_1} |r_2|^{n_2} |t_1|^k |t_2|^l a_{k+n_1, l+n_2}^{(E,\,\text{old})}. \tag{15}$$

Note that in this form the wave function is not normalized. The sum of the squares of moduli of the coefficients $a_{k,l}^{(E,\,\text{new})}$ gives the probability of the measurement results n_1 and n_2. When the original entangled state is the two-mode squeezed vacuum, Eq. (1), i.e.,

$$a_{k,l}^{(E,\,\text{old})} = \sqrt{1 - q^2}\; q^k \delta_{k,l}, \tag{16}$$

then the expansion coefficients (15) of the new state read

$$\begin{aligned} a_{k,l}^{(E,\,\text{new})} &= (-1)^{n_1 + n_2} \frac{\sqrt{1-q^2}\; (k+n_1)!}{\sqrt{k!\; (k+n_1-n_2)!\; n_1!\; n_2!}} \\ &\qquad \times |r_1|^{n_1} |r_2|^{n_2} |t_1|^k |t_2|^{k+n_1-n_2} q^{k+n_1} \delta_{k+n_1, l+n_2}\,. \end{aligned} \tag{17}$$

2.3 Discussion

Equation (17) reveals that the polynomial increase with k of the expansion coefficients $a_{k,l}^{(E,\,\text{new})}$ of the new entangled quantum state can, for small values of k, overcome the exponential decrease

$(q|t_1t_2|)^k$ and thus increase the mean number of photons. This is especially the case when $|t_j|$ is close to unity, i.e. large transmittance of the beam splitters. The price for that is, however, a decrease in the probability of detecting the photons. The increase of entanglement of the conditional state has been demonstrated in [13].

We have simulated numerically the teleportation based on the conditional entangled state (17) in place of the state (1) (for details, see [13]). In Fig. 2 we have demonstrated the achievable improvement of teleportation considering the interference fringes of the x-quadrature-component distribution of a Schrödinger-cat-like quantum state. In the example under study, the fringe visibility of the teleported state is 26.6% for the squeezed vacuum and 48.2% for the photon-subtracted squeezed vacuum obtained by conditional measurement. Whereas the input state shows perfect interference fringes, the teleported states have the fringes smeared and their visibility decreased in general, because of the imperfect entanglement.

The results show that conditional photon-number measurement can improve the squeezing properties of the entangled state and thus increase the fidelity of teleportation. As for any conditional scheme, the required entangled state is obtained with some probability, which can be very small. Note, however, that it does not mean that the scheme represents conditional teleportation: only the preparation of the entangled state is conditional. As soon as the entangled state is known to be prepared, Alice uses it for teleporting the desired state. Teleportation is then completed regardless of Alice's measurement results.

3 Conditional teleportation using optical squeezers and photon counting

Besides the correlations of quadrature components, it has been realized that there are photon-number and phase correlations in a two-mode squeezed vacuum which could also be used for a potential teleportation protocol [11]. In the scheme in [11] it is assumed that (single-event) measurements of the photon-number difference and the phase sum of the two modes on Alice's side are performed. The obtained information is then sent to Bob who has to transform the quantum state of his mode by appropriate phase and photon-number shifting, thus creating the teleported state. The scheme is conditional, as for some measured photon-number differences Alice's original state cannot be re-created by Bob.

Unfortunately, the scheme in [11] requires phase-sum measurements for which no methods have been known so far. In this section we suggest a viable modification of the scheme which is based on (single-event) photon-number measurements on the output of a parametric amplifier (squeezer). The scheme is also conditional, and it applies to certain classes of quantum states. Even though the scheme is not universal, it can produce (for the same degree of squeezing) for some states higher teleportation fidelities than the scheme based on quadrature-component measurements [5].

3.1 Teleportation scheme

Let us consider the scheme sketched in Fig. 3. The entangled state is a two-mode squeezed vacuum produced by the first parametric amplifier from the vacuum state, α being the squeezing parameter. One of the two output modes of the first parametric amplifier is then used as one of the input modes of the second parametric amplifier (squeezing parameter β), and the mode whose quantum state $|\psi_{\rm in}\rangle$ is desired to be teleported is the other input mode. Alice measures

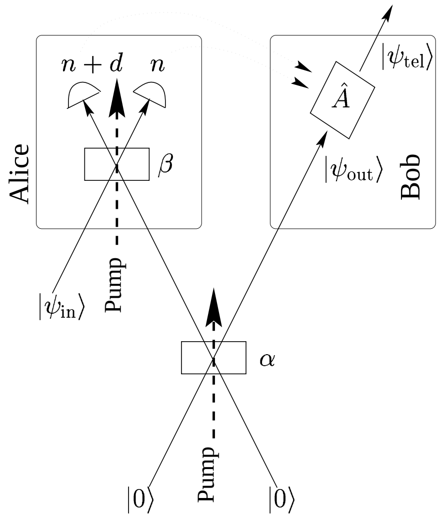

Fig. 3. Teleportation scheme based on photodetection in outputs of Alice's squeezer.

the photon numbers n and $n+d$ ($d\geq -n$) at the output of the second parametric amplifier and communicates the result to Bob. Owing to Alice's measurement, the state of the mode that was sent to Bob from the first parametric amplifier has been projected onto the state $|\psi_{\rm out}\rangle$. Bob now reproduces the input state by means of the transformation $\hat{A}|\psi_{\rm out}\rangle = |\psi_{\rm tel}\rangle$, where the operator $\hat{A}$ shifts the photon number according to the measured photon-number difference $|d|$.

The three modes are initially (i.e., before they enter any of the parametric amplifiers) prepared in the states $|\psi_{\rm in}\rangle$, $|0\rangle$, and $|0\rangle$, where the state $|\psi_{\rm in}\rangle$ that is desired to be teleported can be written in the Fock basis as

$$|\psi_{\rm in}\rangle = \sum_k |k\rangle\langle k|\psi_{\rm in}\rangle. \tag{18}$$

After passing the parametric amplifiers and detecting n and n' photons in the outgoing modes (modes 0 and 1) on Alice's side, the state of Bob's mode (mode 2) is

$$|\psi_{\rm out}\rangle_2 = P^{-\frac{1}{2}}\,{}_0\langle n|{}_1\langle n'|\hat{S}_{01}(\beta)\hat{S}_{12}(\alpha)|\psi_{\rm in}\rangle_0|0\rangle_1|0\rangle_2, \tag{19}$$

where P is the probability of that measurement event. The two-mode squeeze operator $\hat{S}_{kl}(\alpha)$ is given by

$$\hat{S}_{kl}(\alpha) = \exp\Big(\alpha^*\hat{a}_k\hat{a}_l - \alpha\hat{a}_k^\dagger\hat{a}_l^\dagger\Big), \tag{20}$$

with $\hat{a}_k$ ($\hat{a}_k^\dagger$) being the photon destruction (creation) operator of the kth mode. It can be written in the Fock basis as

$$\begin{aligned}{}_k\langle m|\,{}_l\langle m'|\hat{S}_{kl}(\alpha)|n\rangle_k|n'\rangle_l &= \delta_{m-m',n-n'}\, e^{i(m'-n')\varphi_\alpha}\\ &\quad\times(-1)^{n'}\sqrt{m!m'!n!n'!}\,\frac{(\sinh|\alpha|)^{n'}(\tanh|\alpha|)^{m'}}{(\cosh|\alpha|)^{n+1}}\end{aligned}$$

$$\times \sum_{j=\max\{0,n'-n\}}^{\min\{m',n'\}} \frac{\left(-\sinh^2|\alpha|\right)^{-j}}{j!(m'-j)!(n'-j)!(n-n'+j)!}\,, \tag{21}$$

where $\alpha = |\alpha|e^{i\varphi_\alpha}$. For the following it is useful to introduce the coefficients

$$\begin{aligned} S^m_{m'}(d;\alpha) &= {}_k\langle m+d|\,{}_l\langle m|\hat{S}_{kl}(\alpha)|m'+d\rangle_k|m'\rangle_l \\ &= {}_k\langle m|\,{}_l\langle m+d|\hat{S}_{kl}(\alpha)|m'\rangle_k|m'+d\rangle_l \\ &= e^{i(m-m')\varphi_\alpha}(-1)^{m'}\sqrt{m!m'!(m+d)!(m'+d)!} \\ &\quad\times \frac{(\tanh|\alpha|)^{m+m'}}{(\cosh|\alpha|)^{d+1}} \sum_{j=0}^{\min\{m,m'\}} \frac{\left(-\sinh^2|\alpha|\right)^{-j}}{j!(m-j)!(m'-j)!(d+j)!}\,. \end{aligned} \tag{22}$$

The properties of the conditional quantum state $|\psi_{\rm out}\rangle$, Eq. (19), in which the mode 2 is prepared after the detection of n and n' photons in the modes 0 and 1 respectively, are qualitatively different for different sign of the observed difference $d=n'-n$. In the case when $d<0$ is valid, then from Eq. (19) together with Eq. (22) it follows that ($|\psi_{\rm out}\rangle_2 \to |\psi_{\rm out}\rangle$)

$$\langle m|\psi_{\rm out}\rangle = P^{-\frac{1}{2}} S^{n-|d|}_m(|d|;\beta) S^m_0(0;\alpha)\langle m+|d||\psi_{\rm in}\rangle, \tag{23}$$

where the detection probability P is given by

$$P = \sum_m |S^{n-|d|}_m(|d|;\beta)|^2 |S^m_0(0;\alpha)|^2 |\langle m+|d||\psi_{\rm in}\rangle|^2. \tag{24}$$

In the second case when $d\geq 0$ is valid, we derive

$$\langle m|\psi_{\rm out}\rangle = \begin{cases} P^{-\frac{1}{2}} S^n_{m-d}(d;\beta) S^m_0(0;\alpha)\langle m-d|\psi_{\rm in}\rangle, & m \geq d, \\ 0, & m < d, \end{cases} \tag{25}$$

where

$$P = \sum_{m\geq d} |S^n_{m-d}(d;\beta)|^2 |S^m_0(0;\alpha)|^2 |\langle m-d|\psi_{\rm in}\rangle|^2. \tag{26}$$

From an inspection of Eqs. (23) and (25) we see that when the coefficients $S^{n-|d|}_m S^m_0$ and $S^n_{m-d} S^m_0$, respectively, change sufficiently slowly with m, then the state $|\psi_{\rm out}\rangle$, Eq. (19), imitates the state $|\psi_{\rm in}\rangle$, Eq. (18), but with a *shifted Fock-state expansion*, where the shift parameter is just given by the measured photon-number difference d. Obviously, if $d<0$ then the state $|\psi_{\rm out}\rangle$ does not contain any information about the Fock-state expansion coefficients c_k of the state $|\psi_{\rm in}\rangle$ for $k<|d|$. With regard to teleportation, this means that the method is conditional. Successful teleportation of a quantum state whose Fock-state expansion starts with the vacuum can only be achieved if the number of photons detected in the mode 1 is not smaller than the number of photons detected in the mode 0. This limitation is exactly of the same kind as in the scheme in [11]: the teleportation fidelity tends sharply to zero as the photon-number difference exceeds some (state-dependent) threshold value.

To complete the teleportation procedure, Bob transforms the state $|\psi_{\rm out}\rangle$ applying on it photon-number shifting. Thus, the teleported state is

$$|\psi_{\rm tel}\rangle = \begin{cases} \hat{E}^{\dagger|d|}|\psi_{\rm out}\rangle \text{ if } d<0, \\ \hat{E}^{d}|\psi_{\rm out}\rangle \quad \text{if } d>0, \end{cases} \tag{27}$$

where

$$\hat{E} = \sum_n |n\rangle\langle n+1|. \tag{28}$$

The teleportation fidelity is then given by

$$F = |\langle\psi_{\rm in}|\psi_{\rm tel}\rangle|^2. \tag{29}$$

For $d\neq 0$, the teleportation scheme requires a realization of the transformations $\hat{E}$ and $\hat{E}^\dagger$. Unfortunately, there has been no exact implementation of these transformations in quantum optics so far. Photon adding and subtracting are transformations that are very close to the required ones. They are based on conditional measurement and could be realized using presently available experimental techniques [14]. Their use of course reduces the efficiency of the scheme. Thus, the scheme may be presently confined to the case where $d=0$.

3.2 Discussion

From Eqs. (23) and (25) together with Eqs. (27) – (29), the main results can be summarized as follows. (i) Fock states can perfectly be teleported, i.e., the fidelity, Eq. (29), is equal to unity, which follows from the fact that parametric amplifiers conserve the photon-number difference. Therefore, high teleportation fidelities can also be expected for states with small photon number dispersion. For such states our method may be more suitable than the method in [5], where teleportation via measurement of conjugate quadrature components is realized. On the other hand, high teleportation fidelities are not expected for states with large mean photon number and large photon-number dispersion. In particular, for teleportation of highly excited coherent states or phase squeezed states the method in [5] may be more suitable. (ii) In comparison to the method in [11], our scheme does not require phase shifting of the output state $|\psi_{\rm out}\rangle$. The squeezing parameters α and β can be chosen such that the coefficients $S_m^{n-|d|}S_0^m$ and $S_{m-d}^nS_0^m$ in Eqs. (23) and (25), respectively, are real, so that the Fock-state expansion coefficients $\langle m|\psi_{\rm out}\rangle$ have the same phase as the coefficients $\langle m\pm|d||\psi_{\rm in}\rangle$. (iii) A high teleportation fidelity can be expected, provided that the values of the coefficients $S_m^{n-|d|}S_0^m$ and $S_{m-d}^nS_0^m$ vary sufficiently slowly with m in the relevant range of the Fock-state expansion of the input state $|\psi_{\rm in}\rangle$. On the other hand, in ranges where the coefficients change rapidly, reliable teleportation cannot be achieved.

Although our method is realizable in principle, there are several non-trivial experimental challenges. First, precise photodetection is needed, i.e., detectors are required that are able to distinguish between different photon numbers. This does not only concern Alice's measurement but also Bob's photon-number shifting, e.g., by means of photon adding and subtracting. Second, the photodetection should be sufficiently mode-selective, i.e, one must be able to distinguish whether an incident photon comes from the mode under study or from another part of the spectrum generated by the parametric amplifiers. A central problem in any scheme that exploits quantum coherence is that of decoherence due to unavoidable losses. The effect of decoherence may be reduced, if the squeezing strengths are reduced. However, using smaller squeezing decreases the available teleportation fidelity, so that one has to find an optimum regime for the teleportation of a given class of states, the losses in the scheme, and the required fidelity.

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