TWO-LEVEL ATOM IN A STRONG FIELD AND/OR A TAILORED RESERVOIR*

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We present a generalized master equation, in an operator form, for a two-level atom driven by a strong classical field and damped into a "tailored" reservoir with non-flat density of modes. The master equation is derived under the Born and Markov approximations. To derive the master equation the dressing transformation on the atomic operators is performed first and next the dressed operators are coupled to the reservoir and the corresponding damping rates are calculated. The modifications introduced by a strong field and/or by the reservoir with non-flat density of modes lead to non-standard terms in the master equation, some of which are reminiscent of terms known for squeezed vacuum reservoirs.

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1 Introduction

When an exited atom is interacting with a reservoir of electromagnetic field modes it emits irreversibly in the spontaneous emission process its energy into the reservoir. Usually, it is supposed that the damping rate, at which the atom looses its energy by radiating photons into the reservoir, is an inherent property of the atom and can be expressed as the Einstein A coefficient for spontaneous emission. This is, however, only true when the atom is radiating to the ordinary vacuum and is excited by not too strong external field, which does not affect the level structure of the atom. In fact, when the atom is placed inside a cavity its damping rates depend on the cavity mode structure [1–3]. The modifications are important when the density of modes of the reservoir essentially depends on frequency and can be ignored for when the density of modes is flat.

When the driving field is very strong the energy structure of the atom can be changed in such a way that also damping rates will depend on the intensity of the driving field [4, 5]. The influence of the mode structure on the atomic properties was analyzed by Lewenstein *et al.* [3, 6] within a non-Markovian approach.

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In this paper we present an operator form of the master equation for a two-level atom driven by a strong laser field and damped into a "tailored" reservoir. The equation has been derived under the Born and Markov approximations, but nonetheless it takes into account the dependence of the atomic relaxation rates on the strength of the driving field as well as on the density of modes of the reservoir with a non-flat mode structure. It is a generalization of earlier results. A transparent and very simple form of the master equation allows, for example, an easy identification of new squeezing-like terms, which are reminiscent of the real squeezing terms in the master equation for an atom in a squeezed vacuum. We discuss some consequences of such terms here.

2 Master equation

We analyze a model of a two-level atom driven by a strong monochromatic laser field of frequency ω_L with the Rabi frequency Ω and detuned by $\Delta = \omega_L - \omega_A$ from the atomic transition frequency ω_A. The atom is placed in a reservoir with a density of modes structure that depends in an essential way on frequency. The traditional approach to include damping into the evolution of an atom driven by a strong laser field assumes that the damping rates do not depend on the strength of the field and are the same as for spontaneous emission. An alternative approach takes into account the interaction of the strong field with the atom first to derive the "dressed states" of the atom, and next takes the coupling of the dressed atomic states to the reservoir in order to calculate the damping rates [7]. The order in which the two couplings are invoked is important and leads to different results [8]. In this paper, we first perform the dressing transformation on the atomic operators and only after that we couple the dressed atomic operators to the reservoir. In this way we obtain a quite simple and transparent, operator form of the master equation.

The Hamiltonian of the system has the form

$$H = H_A + H_R + H_L + H_I\,, \tag{1}$$

where

$$H_A = \frac{1}{2}\hbar\omega_A\,\sigma_z = -\frac{1}{2}\hbar\Delta\,\sigma_z + \frac{1}{2}\hbar\,\omega_L\,\sigma_z \tag{2}$$

is the Hamiltonian of the atom, which we split into two terms for further convenience,

$$H_R = \hbar\int_0^\infty \omega\, b^+(\omega)\, b(\omega)\,\mathrm{d}\omega\,, \tag{3}$$

is the Hamiltonian of the reservoir field,

$$H_L = \frac{1}{2}\hbar\Omega\left[\,\sigma_+\exp(-i\omega_L t - i\varphi) + \sigma_-\exp(i\omega_L t + i\varphi)\,\right] \tag{4}$$

is the interaction between the atom and the classical laser field, and

$$H_I = i\hbar\int_0^\infty K(\omega)\left[\,\sigma_+\, b(\omega) - b^+(\omega)\,\sigma_-\,\right]\,\mathrm{d}\omega \tag{5}$$

describes the interactions of the atom and the reservoir. Operators $b(\omega)$ and $b^+(\omega)$ are the annihilation and creation operators of the reservoir field, respectively, satisfying the bosonic commutation relation, $\sigma_\pm$ are the atomic raising and lowering operators, σ_z is the atomic inversion operator, and $K(\omega)$ is the coupling between the atom and the reservoir which is related to the natural atomic linewidth γ (FWHM) through the relation

$$K^2(\omega) = \frac{\gamma}{2\pi}\left(\frac{\omega}{\omega_A}\right)^3 \eta(\omega)\,. \tag{6}$$

In further calculations the phase of the driving field ϕ will be absorbed in the atomic operators, $\sigma_- \exp(i\phi) \to \sigma_-$, $\sigma_+ \exp(-i\phi) \to \sigma_+$. As a first step we transform the Hamiltonian into the frame rotating with the laser frequency ω_L, which gives us

$$H_0 = -\frac{1}{2}\hbar\Delta\sigma_z + \frac{1}{2}\hbar\Omega(\sigma_+ + \sigma_-)\,, \tag{7}$$

$$\begin{aligned} H_I^r(t) &= i\hbar\int_0^\infty K(\omega)\big\{\sigma_+ b(\omega)\exp[i\phi + i(\omega_L - \omega)t] \\ &\quad - b^+(\omega)\sigma_- \exp[-i\phi - i(\omega_L - \omega)t]\big\}\,\mathrm{d}\omega \end{aligned} \tag{8}$$

The second step is the unitary dressing transformation performed with the Hamiltonian (7)

$$\sigma_\pm(t) = \exp\left[-\frac{i}{\hbar}H_0 t\right]\sigma_\pm \exp\left[\frac{i}{\hbar}H_0 t\right]\,, \tag{9}$$

which leads to the following time-dependent atomic raising and lowering operators

$$\sigma_\pm(t) = \frac{1}{2}\left[\mp(1\pm\tilde{\Delta})\tilde{\sigma}_- \exp(-i\Omega' t) \pm (1\mp\tilde{\Delta})\tilde{\sigma}_+ \exp(i\Omega' t) + \tilde{\Omega}\tilde{\sigma}_z\right]\,, \tag{10}$$

where

$$\begin{aligned} \tilde{\sigma}_- &= \frac{1}{2}\left[(1-\tilde{\Delta})\sigma_- - (1+\tilde{\Delta})\sigma_+ - \tilde{\Omega}\sigma_z\right]\,, \\ \tilde{\sigma}_+ &= \frac{1}{2}\left[-(1+\tilde{\Delta})\sigma_- + (1-\tilde{\Delta})\sigma_+ - \tilde{\Omega}\sigma_z\right]\,, \\ \tilde{\sigma}_z &= \tilde{\Omega}(\sigma_- + \sigma_+) - \tilde{\Delta}\sigma_z\,, \end{aligned} \tag{11}$$

and

$$\Omega' = \sqrt{\Omega^2 + \Delta^2}\,, \quad \tilde{\Omega} = \frac{\Omega}{\Omega'}\,, \quad \tilde{\Delta} = \frac{\Delta}{\Omega'}\,. \tag{12}$$

After the dressing transformation (9) the interaction Hamiltonian (5) takes the form

$$\begin{aligned} H_I(t) &= i\hbar\int_0^\infty K(\omega)\big\{\sigma_+(t)b(\omega)\exp[i\phi + i(\omega_L - \omega)t] \\ &\quad - b^+(\omega)\sigma_-(t)\exp[-i\phi - i(\omega_L - \omega)]\big\}\mathrm{d}\omega\,, \end{aligned} \tag{13}$$

where $\sigma_+(t)$ and $\sigma_-(t)$ are given by (10).

Using standard methods [9], we derive the master equation, in the Born approximation, for the reduced atomic density matrix

$$\frac{\partial\rho^{(d)}}{\partial t} = -\frac{1}{\hbar^2}\int_0^t \mathrm{Tr}_R\left\{[H_I(t), [H_I(t-\tau), \rho_R(0)\rho^{(d)}(t-\tau)]]\right\}\mathrm{d}\tau\,, \tag{14}$$

where the superscript (d) stands for the dressed picture, $\rho_R(0)$ is the density operator for the field reservoir. Tr_R is the trace over the reservoir states and the Hamiltonian $H_I(t)$ is given by (13). At this stage we make the Markov approximation [9] by replacing $\rho^{(d)}(t-\tau)$ in (14) by $\rho^{(d)}(t)$, substitute the Hamiltonian (13) and take the trace over the reservoir variables, assuming that

$$\begin{aligned} \mathrm{Tr}_R\{b(\omega)b^+(\omega)\rho_R(0)\} &= [N(\omega)+1]\delta(\omega-\omega')\,, \\ \mathrm{Tr}_R\{b^+(\omega)b(\omega)\rho_R(0)\} &= N(\omega)\delta(\omega-\omega')\,, \end{aligned} \tag{15}$$

where $N(\omega)$ is the mean number of photons at frequency ω. In the Markov approximation we extend the upper limit of the integral over τ to infinity and perform necessary integrations using the formula

$$\int_0^\infty \exp(\pm i\epsilon\tau)\,\mathrm{d}\tau = \pi\delta(\epsilon) \pm i\,\mathcal{P}\frac{1}{\epsilon}\,, \tag{16}$$

where $\mathcal{P}$ means the Cauchy principal value.

After performing all the calculations and transforming back from the dressed picture to the original density operator, in the frame rotating with the laser frequency, we finally arrive at the following master equation

$$\begin{aligned} \frac{\partial}{\partial t}\rho &= \frac{i}{2}\Delta^{'}\,[\sigma_z,\rho] - \frac{i}{2}\Omega\,[\sigma_+ + \sigma_-,\rho] \\ &+ \frac{1}{2}N\,(2\sigma_+\rho\sigma_- - \sigma_-\sigma_+\rho - \rho\sigma_-\sigma_+) \\ &+ \frac{1}{2}\,(N+a)\,(2\sigma_-\rho\sigma_+ - \sigma_+\sigma_-\rho - \rho\sigma_+\sigma_-) \\ &- M\sigma_+\rho\sigma_+ - M^\star\sigma_-\rho\sigma_- \\ &+ \frac{1}{2}L\,[\sigma_+,\rho\sigma_z] - \frac{1}{2}L^\star\,[\sigma_-,\sigma_z\rho] \\ &+ \frac{1}{2}\,(L+b)\,[\sigma_-,\rho\sigma_z] - \frac{1}{2}\,(L+b)^\star\,[\sigma_+,\sigma_z\rho]\ , \end{aligned} \tag{17}$$

where

$$\begin{aligned} \Delta^{'} &= \Delta + \Delta_p\,, \\ \Delta_p &= \frac{\gamma}{8}\Big[(1+\tilde\Delta)^2(1+2N_-)b_- + (1-\tilde\Delta)^2(1+2N_+)b_+ \\ &\quad +2(1-\tilde\Delta^2)(1+2N_0)b_0\Big]\ , \\ N &= \frac{\gamma}{4}\Big[(1+\tilde\Delta)^2N_-a_- + (1-\tilde\Delta)^2N_+a_+ + 2(1-\tilde\Delta^2)N_0a_0\Big]\ , \\ a &= \frac{\gamma}{4}\Big[(1+\tilde\Delta)^2a_- + (1-\tilde\Delta)^2a_+ + 2(1-\tilde\Delta^2)a_0\Big]\ , \\ M &= \frac{\gamma}{8}(1-\tilde\Delta^2)\,[(1+2N_-)(a_- - ib_-) + (1+2N_+)(a_+ - ib_+) \\ &\quad - 2(1+2N_0)(a_0 - ib_0)]\ , \\ L &= \frac{\gamma}{4}\tilde\Omega\,\Big[(1+\tilde\Delta)N_-(a_- + ib_-) - (1-\tilde\Delta)N_+(a_+ + ib_+) \end{aligned} \tag{18}$$

$$
\begin{aligned}
& - 2\tilde{\Delta} N_0 (a_0 + i b_0) \Big] \, , \\
b &= \frac{\gamma}{4} \tilde{\Omega} \Big[(1+\tilde{\Delta})(a_- + i b_-) - (1-\tilde{\Delta})(a_+ + i b_+) - 2\tilde{\Delta}(a_0 + i b_0) \Big] \, ,
\end{aligned}
$$

where $\eta(\omega)$ describes the deviation of the reservoir density of modes from the vacuum density of modes (for vacuum $\eta(\omega) = 1$). The other quantities are defined by

$$
\begin{aligned}
N_0 &= N(\omega_L) \, , \quad N_\pm = N(\omega_L \pm \Omega^{'}) \, , \\
a_0 &= \left(\frac{\omega_L}{\omega_A} \right)^3 \eta(\omega_L) \, , \quad a_\pm = \left(\frac{\omega_L \pm \Omega^{'}}{\omega_A} \right)^3 \eta(\omega_L \pm \Omega^{'}) \, , \\
b_0 &= -\frac{1}{\gamma} \mathcal{P} \int_0^\infty \frac{K^2(\omega)}{\omega_L - \omega} \, \mathrm{d}\omega \, , \quad b_\pm = -\frac{1}{\gamma} \mathcal{P} \int_0^\infty \frac{K^2(\omega)}{\omega_L - \omega \pm \Omega'} \, \mathrm{d}\omega \, ,
\end{aligned}
\tag{19}
$$

where $N(\omega)$ is the mean number of photons at frequency ω. Lamb shifts are included via the redefinition of the atomic transition frequency. We have also included in (18) the shifts coming from the principal value terms. They contribute essentially to the atomic evolution in the cavity situation when the density of modes $\eta(\omega)$ has a non-trivial ω dependence.

We model the reservoir density of modes by a dimensionless Lorentzian $\eta(\omega)$ centered at frequency ω_c with the width γ_c

$$
\eta(\omega) = \frac{\gamma_c^2}{(\omega - \omega_c)^2 + \gamma_c^2} \, . \tag{20}
$$

For very broad reservoir, ($\gamma_c \to \infty$), the Lorentzian (20) becomes constant ($\eta(\omega) \to 1$), and our results reproduce the results for ordinary vacuum.

For the reservoir described by the Lorentzian (20), parameters b_0 and $b_\pm$ take the following form

$$
\begin{aligned}
b_0 &= -\frac{1}{2} \left(\frac{\omega_L}{\omega_A} \right)^3 \frac{\delta_c \gamma_c}{\delta_c^2 + \gamma_c^2} \, , \\
b_\pm &= -\frac{1}{2} \left(\frac{\omega_L \pm \Omega^{'}}{\omega_A} \right)^3 \frac{(\delta_c \pm \Omega^{'})\gamma_c}{(\delta_c \pm \Omega')^2 + \gamma_c^2} \, .
\end{aligned}
\tag{21}
$$

It is clear from (21) that the shifts coming from the principal value terms give nonzero contributions for moderately intense laser fields and reservoirs with finite bandwidth. The most interesting situations appear when the peak of the Lorentzian is centered at the laser frequency ($\delta_c = 0$) or the Rabi sidebands ($\delta_c = \pm\Omega^{'}$).

From the form of the master equation (17) one can identify the new terms coming from the interaction of the atom with a very strong laser field, seen via the ω^3 terms in $a_\pm$, and/or from the interaction with the reservoir, seen via the density of modes $\eta(\omega_L \pm \Omega^{'})$ at the dressed atom transitions and via the principal value terms b_0 and $b_\pm$. Our approach is a generalization, on the one hand, of the Bloch equations introduced by Kocharovskaya *et al.* [4] and, on the other hand, the Bloch equations introduced by Keitel *et al.* [8] for the cavity situation in the secular approximation.

For weak driving fields and thermal reservoirs, for which $a_\pm = a_0 = 1$ and $N_\pm = N_0$, our master equation (17) has the well known standard form, but for stronger fields and/or tailored reservoirs it is easy, from the transparent form of the equation (17), to identify the new terms proportional to M, L and b. Terms proportional to M are reminiscent of the terms appearing in interactions of an atom with squeezed reservoirs [10, 11]. These terms, however, are not the real squeezing terms, because, despite the fact that they depend of the phase of the driving field, they do not give the phase dependence of the fluorescence or absorption spectra, which is characteristic for the squeezed reservoirs.

3 Generalized Bloch equations

The generalized master equation (17) leads immediately to the generalized Bloch equations describing the evolution of the expectation values of the atomic operators. The Bloch equations can be written in the matrix form as

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix}\langle\sigma_-(t)\rangle\\ \langle\sigma_+(t)\rangle\\ \langle\sigma_z(t)\rangle\end{pmatrix} = \mathbf{A}\begin{pmatrix}\langle\sigma_-(t)\rangle\\ \langle\sigma_+(t)\rangle\\ \langle\sigma_z(t)\rangle\end{pmatrix} + \frac{1}{2}\begin{pmatrix}b_r - i\Lambda_i\\ b_r + i\Lambda_i\\ -2a\end{pmatrix}, \tag{22}$$

where the matrix $\mathbf{A}$ has the form

$$\mathbf{A} = \begin{pmatrix} i\Delta' - \Gamma & -M & \frac{i}{2}\Omega\\ -M^\star & -i\Delta' - \Gamma & -\frac{i}{2}\Omega\\ i(\Omega + b_i) + \Lambda_r & -i(\Omega + b_i) + \Lambda_r & -2\Gamma \end{pmatrix}. \tag{23}$$

To make the notation shorter we denote the real part of the complex number Q by Q_r and the imaginary part by Q_i, and we have used the substitutions

$$\Gamma = \frac{1}{2}(a + 2N)\,, \quad \Lambda = b + 2L\,. \tag{24}$$

On introducing the Hermitian operators representing the two quadrature components of the atomic dipole moment

$$\sigma_x = \frac{1}{2}(\sigma_- + \sigma_+)\,, \quad \sigma_y = \frac{1}{2i}(\sigma_- - \sigma_+)\,, \tag{25}$$

the Bloch equations can be rewritten as

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix}\langle\sigma_x(t)\rangle\\ \langle\sigma_y(t)\rangle\\ \langle\sigma_z(t)\rangle\end{pmatrix} = \mathbf{B}\begin{pmatrix}\langle\sigma_x(t)\rangle\\ \langle\sigma_y(t)\rangle\\ \langle\sigma_z(t)\rangle\end{pmatrix} + \frac{1}{2}\begin{pmatrix}b_r\\ -\Lambda_i\\ -2a\end{pmatrix} \tag{26}$$

with the matrix $\mathbf{B}$ given by

$$\mathbf{B} = \begin{pmatrix} -\Gamma - M_r & -\Delta' + M_i & 0\\ \Delta' + M_i & -\Gamma + M_r & \frac{1}{2}\Omega\\ \Lambda_r & -2(\Omega + b_i) & -2\Gamma \end{pmatrix}. \tag{27}$$

From the matrix $\mathbf{B}$ it is easily seen that two components of the atomic dipole moment, $\langle\sigma_x\rangle$ and $\langle\sigma_y\rangle$ decay with different rates when M_r is different from zero. This effect is well known

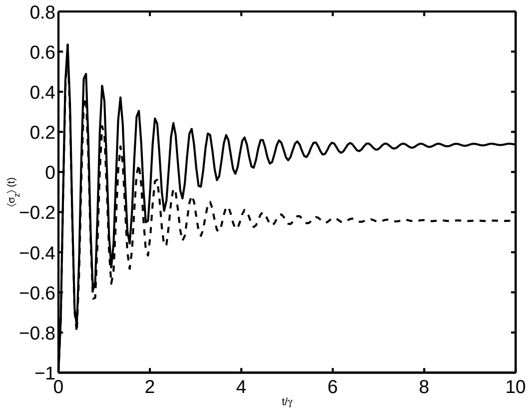

Fig. 1. Time dependence of the atomic inversion $\langle\sigma_z\rangle(t)$ for $\Omega/\gamma = 15\ \omega_c = \omega_L - \Omega^{'}$, $\Delta = -0.4\Omega$, and $\gamma_c/\gamma = 10$ (solid line), $\gamma_c/\gamma = 10000$ (dashed line).

for squeezed reservoirs [14], but here it is associated with the modification of the damping rates in very strong fields and/or with the non-flat density of modes of the reservoir. A new feature of the Bloch equations (26) is the presence of the shifts b_0 and $b_\pm$, given by (21), which do not appear in other Markovian approaches, but they do appear in the non-Markovian approach [6]. On neglecting the shifts, the Bloch equations (22) are equivalent of the Bloch equations obtained by Kocharovskaya *et al.* [4].

The steady state solutions of the Bloch equations (26) can be easily obtained for general case. In particular, for very strong laser fields and equal number of photons, at each frequency, $N(\omega) = N_0 = N_\pm$, the steady state solutions for the atomic operators take the approximate form

$$
\begin{aligned}
\langle\sigma_x\rangle_{ss} &= \frac{\tilde{\Omega}}{2(1+2N_0)}\,\frac{(1+\tilde{\Delta})^2\,a_- - (1-\tilde{\Delta})^2\,a_+}{(1+\tilde{\Delta})^2\,a_- + (1-\tilde{\Delta})^2\,a_+}\,, \\
\langle\sigma_y\rangle_{ss} &= -\frac{\tilde{\Omega}}{4\Omega'}\,\frac{2(1-\tilde{\Delta}^2)\,a_-\,a_+ + [(1+\tilde{\Delta})^2\,a_- + (1-\tilde{\Delta})^2\,a_+]\,a_0}{(1+\tilde{\Delta})^2\,a_- + (1-\tilde{\Delta})^2\,a_+}\,, \\
\langle\sigma_z\rangle_{ss} &= -\frac{\tilde{\Delta}}{1+2N_0}\,\frac{(1+\tilde{\Delta})^2\,a_- - (1-\tilde{\Delta})^2\,a_+}{(1+\tilde{\Delta})^2\,a_- + (1-\tilde{\Delta})^2\,a_+}\,.
\end{aligned}
\tag{28}
$$

From solutions (28), it is seen that for thermal reservoir and very strong laser fields the steady state inversion between the atomic states for the specific range of the laser field detuning can be realized, the effect reported in [5] and attributed to $((\omega \pm \Omega^{'})/\omega_A)^3$ factors. Moreover, a nonzero solution for $\langle\sigma_x\rangle_{ss}$ component in the resonant case can be found. It is also obvious that placing the atom inside a cavity, where there is a peak in the density of modes at some characteristic frequency, may increase the values of $\langle\sigma_x\rangle_{ss}$ and the steady state atomic inversion [6, 12].

For not too strong laser fields the exact solutions have to be used. In this laser field intensity regime it is possible to observe changes of the atomic behavior coming from the nonzero principal

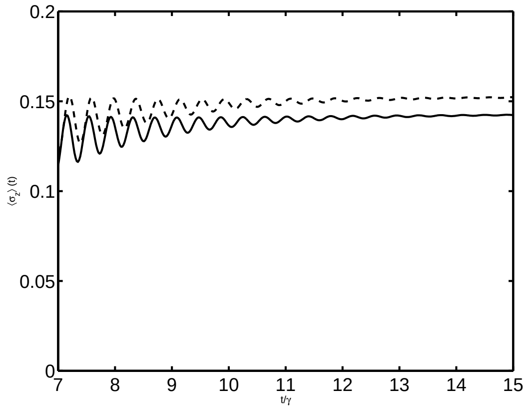

Fig. 2. Time dependence of the atomic inversion $\langle\sigma_z\rangle(t)$ for $\Omega/\gamma = 15$ $\omega_c = \omega_L - \Omega^{'}$, $\gamma_c/\gamma = 10$, and $\Delta = -0.4\Omega$. Exact solution (solid line), no shifts (dashed line).

value terms in the case of structured reservoirs. In Fig. 1 we have plotted the time dependence of the mean value of the atomic inversion $\langle\sigma_z\rangle(t)$. The figure shows the differences, for the moderate laser field intensity $\Omega/\gamma = 15$, between the broadband reservoir (dashed line) and the reservoir with the mode structure being a Lorentzian centered at frequency $\omega_c = \omega_L - \Omega^{'}$ (solid line). The laser field is detuned by $\Delta = -0.4\Omega$ from the atomic resonance, and we can see that there are significant differences in behavior of $\langle\sigma_z\rangle(t)$ for the two cases. When the atom is placed inside a tailored reservoir with narrow bandwidth the Rabi oscillations of the atomic population inversion have larger amplitude than in the case of broadband reservoir and their decay time is longer. In the long time limit there is a considerable amount of population inversion, $\langle\sigma_z\rangle_{ss} \approx 0.15$, in the case of narrow bandwidth reservoir, while there is no inversion between atomic states for broadband reservoir. In Fig. 2 we have illustrated the role of the shifts coming from the principal value terms in the atomic evolution by plotting the long time behavior of the atomic inversion: the exact solution (solid line) and the solution with the shifts equal to zero (dashed line). It is clear that the exact solution gives slightly lower atomic inversion in the long time limit, for the parameter values taken in the figure, and whole the trajectory is shifted by the shift terms. The possibility of creating the steady state atomic inversion by tuning the cavity was discussed in the non-Markovian approach by Lewenstein and Mossberg [6]. We can see that also the much simpler Markovian approach presented here, leads to similar effects.

4 Resonance fluorescence

We define the fluorescence spectrum into the structured reservoir modes as a rate at which the mean number of photons $b^{+}(\omega)b(\omega)$ of the reservoir mode at frequency ω changes in time for the steady state conditions. It is given by

$$\mathcal{F}(\omega) = \lim_{t\to\infty} \frac{d}{dt}\langle b^{+}(\omega,t)b(\omega,t)\rangle = \lim_{t\to\infty}\langle \frac{d}{dt}b^{+}(\omega,t)b(\omega,t) + b^{+}(\omega,t)\frac{d}{dt}b(\omega,t)\rangle \quad (29)$$

Using the Heisenberg equations of motion for the bosonic reservoir operators, we obtain the following formula for the fluorescence spectrum emitted into the cavity modes

$$\mathcal{F}(\omega) = 2K^2(\omega)\,\mathrm{Re}\int_0^{\infty} d\tau \langle\sigma_+(0)\sigma_-(\tau)\rangle e^{i(\omega-\omega_L)\tau}\,. \tag{30}$$

Formula (30) differs from the standard definition of the resonance fluorescence spectrum, as the Fourier transform of the atomic correlation function, by the frequency dependent factor $K^2(\omega)$, which is important here. The standard definition assumes that the atomic rate is constant. The equations of motion for the atomic correlation function appearing in (30) can be obtained from the generalized Bloch equations (22) with the use of the quantum regression theorem [13]. Taking the Laplace transform of the evolution equations for the atomic correlation functions with the appropriate initial conditions, we finally arrive at the following expression for the Laplace transform of the correlation function $\langle\sigma_+(0)\sigma_-(\tau)\rangle$, which enters the definition of the resonance fluorescence spectrum

$$\begin{aligned} F(z) &= \frac{1}{2zd(z)}\Bigg\{\frac{z}{2}(1+\langle\sigma_z\rangle_{ss})\left[2(z+2\Gamma)(z+\Gamma+i\Delta') + \Omega(\Omega+b_i+i\Lambda_r)\right] \\ &+ \langle\sigma_+\rangle_{ss}\Big[-i\left[\Omega(z+a)+\Lambda_i(z+2\Gamma)\right](z+\Gamma+M+i\Delta') \\ &+ b_r\left[\Omega(\Omega+b_i+i\Lambda_r)+(z+2\Gamma)(z+\Gamma-M+i\Delta')\right]\Big]\Bigg\}\,. \end{aligned} \tag{31}$$

The incoherent part of the spectrum can be calculated from

$$\mathcal{F}_{inc}(\omega) = \frac{\gamma}{\pi}\left(\frac{\omega}{\omega_A}\right)^3 \eta(\omega)\,\mathrm{Re}\left\{\left[F(z) - \frac{1}{z}\lim_{z\to 0} zF(z)\right]_{z=-i(\omega-\omega_L)}\right\}, \tag{32}$$

where we have used the expression (6) for the frequency dependent coupling constant $K(\omega)$. We would like to emphasize that the presence of this factor is necessary when one wish to derive the fluorescence spectrum into the structured reservoir modes. When the fluorescent light is emitted to the structureless background modes the traditional definition is applicable and $K(\omega)$ can be omitted. This factor is crucial for "tailored" reservoirs and/or very strong laser fields. However, the expressions (31) and (32) are quite general and they are applicable for both strong and weak driving fields and all reservoirs with sufficiently broad linewidth, which is much broader than the atomic linewidth to justify the Markovian approximation used to derive the master equation. Of course, for very strong driving fields, in the secular limit, the results can be simplified considerably.

To illustrate our results, we have plotted in Fig. 3 and Fig. 4 the fluorescence spectra for moderately strong laser fields, for which the principal value terms (shifts) and the density of modes of the reservoir play an important role. We can observe that the structured reservoir with non-flat density of modes leads to narrowing of the central line and simultaneously broadening of the sidebands when the cavity is tuned to the central frequency (Fig. 3). This is the effect reminiscent of the analogous effect observed for the squeezed vacuum reservoir [14], and it is related to a possibility of getting negative values of M_r by tuning the Lorentzian representing the reservoir density of modes to the central line.

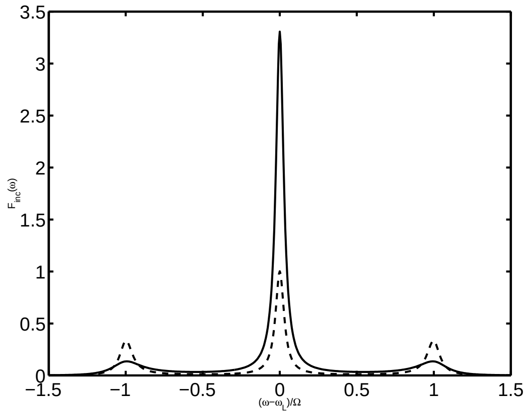

Fig. 3. The incoherent part of the fluorescence spectrum $\mathcal{F}_{inc}(\omega)$ for $\Delta = 0$, $\Omega/\gamma = 15$, $\omega_c = \omega_L$, $\gamma_c/\gamma = 10$ (solid line), and $\gamma_c/\gamma = 10000$ (dashed line).

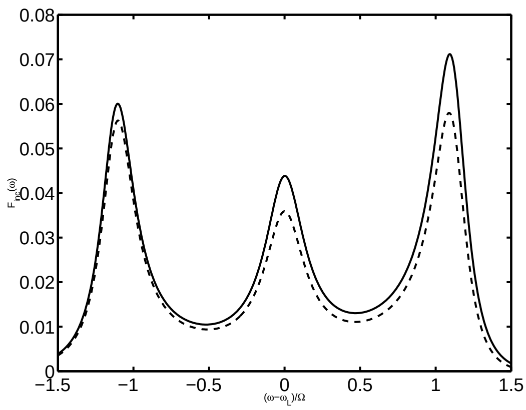

Fig. 4. The incoherent part of the fluorescence spectrum $\mathcal{F}_{inc}(\omega)$ for $\Delta/\gamma = 5$, $\Omega/\gamma = 15$, $\omega_c = \omega_L + \Omega'$, $\gamma_c/\gamma = 10$. Exact solution (solid line), no shifts (dashed line).

We can also observe that including the principal value terms (shifts) in the derivation of the fluorescence spectrum, *i.e.* using our solutions for "tailored" reservoirs can lead to the observable asymmetry in the fluorescence emitted into the cavity modes. This effect is clearly seen from Fig. 4, where we compare the spectra with and without the shift terms. For very strong driving fields, in the secular limit, the shift terms become negligible, and the resonance fluorescence spectrum becomes symmetric in accordance with the earlier results [8].

5 Summary

In this paper we have presented the generalized master equation for the reduced density matrix within the Born and Markov approximations for a two-level atom driven by a strong laser field and damped by a "tailored" reservoir. The dressing transformation on the atomic operators has been performed first and after that the coupling between the dressed atom and the reservoir has been included. The generalized master equation is valid for a wide range of laser intensities (for very strong laser fields it includes the dependence of the damping rates on the laser intensity) and different types of Markovian reservoirs. It includes the shifts coming from the nonzero principal value terms. We have identified some new terms in the master equation that are reminiscent of the well known terms occurring when atom is interacting with the squeezed vacuum. Such terms give similar effects as if the atom were damped by a squeezed vacuum (narrowing of the central line of the Mollow triplet, the different damping rates for the two quadrature components of the atomic dipole) although, in fact, they are not real squeezing terms because they do not give, for example, the phase dependence of the resonance fluorescence spectra.

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