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# DECOHERENCE AND DISSIPATION OF ATOMIC SCHRÖDINGER CATS[*]

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We investigate the time development of a superposition of macroscopically distinct quantum states (Schrödinger cats) in an ensemble of two-level atoms. The system is interacting with a thermal environment of a macroscopic number of photon modes. The final equilibrium state of the atomic subsystem is diagonal in the energy eigenstates, and is determined by the Boltzmann distribution. The time scale of decoherence is, however, generally much shorter than that of dissipation. The initial fast regime of the time evolution associated with the decoherence is directed towards a classical state which is different from the thermal equilibrium. For general initial conditions the distance between the actual state of the system and this classical state is decreasing fast, suggesting an appropriate measure of decoherence.

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## 1 Introduction

In the present paper we investigate the effect of decoherence [1, 2] on a system of Rydberg atoms in a resonant cavity. As it is well-known, these atoms show collective behavior. If the product state of the atoms was initially invariant with respect permutations of the atoms i.e. it is a superposition of the totally symmetric Dicke states [3], dipole interaction with a resonant field mode would not destroy this symmetry. Therefore we may restrict our investigation for the $N$ atom system to the totally symmetric $N+1$ dimensional subspace of the whole $2^N$ dimensional Hilbert-space. The time evolution of the atomic subsystem is governed by the superradiant master equation [4, 5, 6]. We present results of numerical computations of the solution of the master equation, with the initial state of the system being in a superposition of two macroscopically distinct atomic coherent states [7]. In analogy with the optical case [8], we shall refer to these states as atomic Schrödinger cats [9, 10].

Although decoherence and energy dissipation are both related to the coupling of the system with its environment, their time scales differ by orders of magnitude. Based on this fact, we

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show how one can make a clear distinction between the decoherence and the flow of the energy between the atomic subsystem and the environment.

The classical statistical mixture resulting from decoherence is generally associated with well-localized quantum states. Furthermore, for certain initial conditions decoherence is shown to drive the system very close to the mixture of the initial coherent states.

We shall introduce two plausible definitions for the characteristic time of decoherence and conclude that these times have the same order of magnitude. For certain special initial conditions our results are in consent with the slower decoherence of these special cat states [10, 11].

## 2 Description of the model

Let us consider an ensemble of two level Rydberg atoms in a resonant cavity which has low quality mirrors. In this case the thermal equilibrium of the electromagnetic field in the cavity is not appreciably modified by the atomic emission, because the emitted photons are absorbed soon by the mirrors [6]. From a slightly different point of view, this means that the thermal field modes play the role of an environment containing a large degrees of freedom. This static reservoir continuously interacts with the atomic subsystem influencing its dynamics until eventually thermal equilibrium is reached. Besides the energy transfer between the reservoir and the atoms, the continuous ”monitoring” [12] of the atomic subsystem results in the total loss of the coherence of its quantum superpositions. So the physical system described above also serves as a specific model of decoherence.

Supposing a low-Q cavity one is allowed to write a master equation for the reduced density matrix of the atomic subsystem $\rho$ alone [4, 5, 6]. Recalling that the totally symmetric $N+1$ dimensional atomic subspace is isomorphous to the angular momentum eigensubspace labeled by $j = N/2$, the interaction picture master equation reads [5, 6]:

$$\begin{aligned} \hbar^2 \frac{\mathrm{d}\rho(\mathrm{t})}{\mathrm{dt}} &= -\frac{\gamma}{2}\left(\langle n\rangle + 1\right)\left(J_+J_-\rho(t) + \rho(t)J_+J_- - 2J_-\rho(t)J_+\right) \\ &\quad -\frac{\gamma}{2}\langle n\rangle\left(J_-J_+\rho(t) + \rho(t)J_-J_+ - 2J_+\rho(t)J_-\right). \end{aligned} \tag{1}$$

Here $\langle n\rangle$ is the mean number of photons in the environment and $\gamma$ denotes the damping rate.

The presence of quantum coherence is probably the most striking when the quantum system is in a superposition of two macroscopically distinct states. This classically unexpected (and unobserved) phenomenon can be modeled by the quantum superposition of two atomic coherent states [7]. A single atomic coherent state can be expanded in terms of the $|j,m\rangle$ Dicke states as follows:

$$|\theta,\phi\rangle_j = \sum_{m=-j}^{j} \sqrt{\begin{pmatrix} 2j \\ j+m \end{pmatrix}} \sin^{j+m}\frac{\theta}{2}\, \cos^{j-m}\frac{\theta}{2}\, e^{-i(j+m)\phi}|j,m\rangle. \tag{2}$$

Here the $\theta$ and $\phi$ parameters represents polar and azimuthal angles, in the sense that $|\theta,\phi\rangle_j$ can be regarded as the quantum counterpart of a $\hbar\sqrt{j(j+1)}$ magnitude classical angular momentum pointing in the direction defined by $\theta$ and $\phi$. Hereafter we shall concentrate mainly on the solution of Eq. (1), provided the atomic subsystem is initially in an atomic Schrödinger cat state [9, 10]:

$$|\Psi_{12}\rangle = \frac{|\theta_1, \phi_1\rangle_j + |\theta_2, \phi_2\rangle_j}{\sqrt{2(1 + \mathrm{Re}\ {}_j\langle\theta_1, \phi_1|\theta_2, \phi_2\rangle_j)}}. \tag{3}$$

It can be shown that the classical mixture of energy eigenstates representing thermal equilibrium with the environment is a stationary solution of Eq. (1). The corresponding stationary density matrix is clearly diagonal in the energy eigenstate basis. The population of the $|j, m\rangle$ states are defined by the Boltzmann factors related to the energy of $\hbar\omega_a m$, where $\omega_a$ denotes the transition frequency between two atomic levels.

However, as it will be seen in the next section, there is also a different process, which is extremely fast compared to the dissipation of the atomic subsystem's energy. By the aid of an appropriate measure of decoherence we shall be able to distinguish between the energy transfer and the process of decoherence.

## 3 Measures of decoherence

In order to be able to determine the characteristic time of decoherence for different initial density matrices it is necessary to characterize the process of decoherence quantitatively. We require the decoherence measure (also called “decoherence norm”, [11]) $\mathcal{M}$ to be general, i. e. in the case of an arbitrary density matrix ,$\rho$, the real number $\mathcal{M}(\rho)$ has to answer the question to what extent quantum coherence is present in $\rho$. For some special initial density matrices the effect of decoherence on the matrix elements is clear. The decoherence measure has to reproduce the expectations based on these special cases. It also should be independent of the particular choice of basis in terms of which $\rho$ is expanded.

We shall deal with two possible decoherence measures here. First let us consider the following definition: $\mathcal{M}^1(\rho) = \mathrm{Tr}(\rho^2)$, which is clearly invariant with respect to the change of basis. Note that $\mathcal{M}^1$ was used to describe decoherence also in [13]. If $\rho$ represents a pure state, $\mathcal{M}^1(\rho) = 1$; on the other hand, for $\rho = I \times 1/(N+1)$, where $I$ represents the $N+1$ dimensional identity matrix, $\mathcal{M}^1(\rho) = 1/(N+1)$, which is negligible if the number of atoms is large.

It is worth noting that the linear entropy $S$ of the atomic subsystem can be simply expressed in terms of the norm $\mathcal{M}^1$:

$$S = \mathrm{Tr}(\rho - \rho^2) = 1 - \mathrm{Tr}(\rho^2) = 1 - \mathcal{M}^1(\rho). \tag{4}$$

As the first application of $\mathcal{M}^1$, let us consider Fig. 1 which shows that the decoherence is a much faster process than dissipation: $\mathcal{M}^1\ (\rho(t))$ changes rapidly while the dissipated energy is just a fraction of the whole amount of energy to be transferred. This implies that the beginning of the time evolution is dominated by decoherence but later the energy dissipation becomes more important. In the next section we shall use this fact to define the characteristic time of decoherence.

For the second measure of decoherence to be considered we shall drop the requirement of generality by concentrating on a narrower class of initial density matrices. In return we gain physical insight into the decoherence of the particular systems described by these matrices.

Touching upon the general theory of decoherence we recall that the interaction with a large number of degrees of freedom naturally selects the so-called pointer basis [12]. The effect of

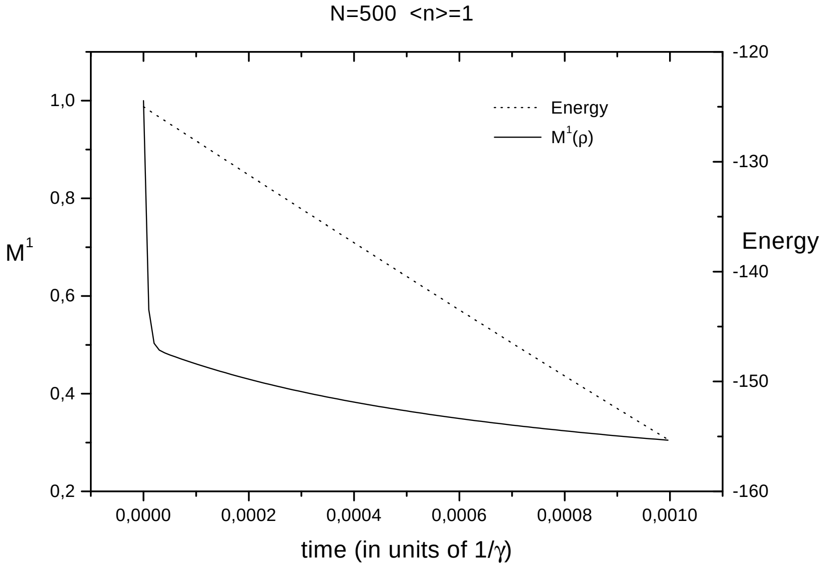

Fig. 1: The energy of the atomic subsystem (in units of $\hbar\omega_a$) and the trace of $\rho^2$ as a function of time. The parameters of the initial Schrödinger cat state (3) were $\theta_1 = 0.5\pi, \phi_1 = 0$; $\theta_2 = 0, \phi_2 = 0$. The number of atoms is $N = 500$ and the average number of photons is $\langle n \rangle = 1$.

decoherence is claimed to favor the constituent states of the pointer basis in the sense that provided there are no other processes, the system ends up in a classical statistical mixture of these states. However, in our system, apart from the initial period of the time evolution, the dissipation of the energy is the dominant process. (Although all density matrices finally arrive in thermal equilibrium with the environment, the energy eigenstates clearly can not always be considered as the constituents of the pointer basis.) On the contrary, for special cat states (3) (see Fig. 4 of the next section) it is possible to determine two pointer states, namely the coherent states from which the initial cat state is composed. At this point it is instructive to introduce the classical density matrix $\rho_{cl}(\theta_1, \phi_1, \theta_2, \phi_2)$ which corresponds to the classical statistical mixture of two coherent states:

$$\rho_{cl}(\theta_1, \phi_1, \theta_2, \phi_2) = c\left(|\theta_1, \phi_1\rangle_{jj}\langle\theta_1, \phi_1| + |\theta_2, \phi_2\rangle_{jj}\langle\theta_2, \phi_2|\right), \tag{5}$$

where the factor $c$ provides normalization. Now the scheme of the decoherence can be written in the following manner:

$$|\Psi_{12}\rangle\langle\Psi_{12}| \rightarrow \rho_{cl}(\theta_1, \phi_1, \theta_2, \phi_2). \tag{6}$$

Therefore it is plausible to define a second measure of decoherence, $\mathcal{M}^2(\rho)$ as the distance (in

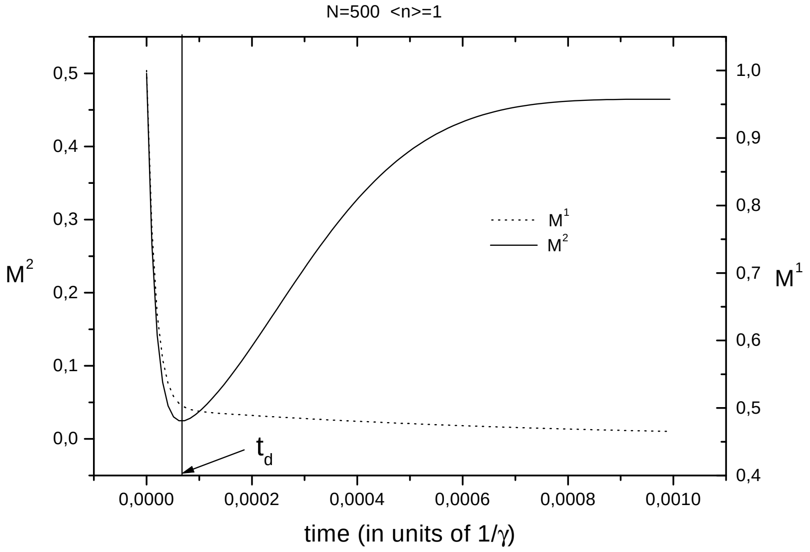

Fig. 2: The comparison of the two decoherence norms. (Initially: $\theta_1 = 0.5\pi, \phi_1 = 0$; $\theta_2 = 0, \phi_2 = 0$.) The number of atoms is $N = 500$ and the average number of photons is $\langle n\rangle = 1$, $t_d \approx 6 \times 10^{-5}/\gamma$

the sense of $\mathcal{M}^1$) between the current density matrix and $\rho_{cl}$:

$$\mathcal{M}^2\left(\rho(t)\right) = \mathrm{Tr}\left[\left(\rho(t) - \rho_{cl}\right)^2\right]. \tag{7}$$

Having defined two measures of decoherence, we may ask whether they really characterize the same property of the physical process. We can perform the comparison in the regions of the atomic system's phase-space (see the marked areas in Fig. 4) where the application of both $\mathcal{M}^1$ and $\mathcal{M}^2$ is relevant. Fig. 2 shows that the characteristic times based on the two measures are approximately the same. In addition even the initial slope of the two curves are very close to each other.

So we can conclude that the measure $\mathcal{M}^2(\rho)$, which has clear physical meaning, supports the applicability of the general measure $\mathcal{M}^1(\rho)$ on the whole phase-space.

## 4 Characteristic time of the decoherence and the pointer basis

In this section we shall define the characteristic time of the decoherence and examine the dependence of this decoherence time on the initial conditions.

Returning to Fig. 2 we can see that the slope of the $\mathcal{M}^1\left(\rho(t)\right)$ curve changes rapidly at a well-defined time, denoted by $t_d$ in the figure. This implies that the dominant underlying physical process changes at this time. Similarly the graph of $\mathcal{M}^2\left(\rho(t)\right)$ exhibits a sharp and

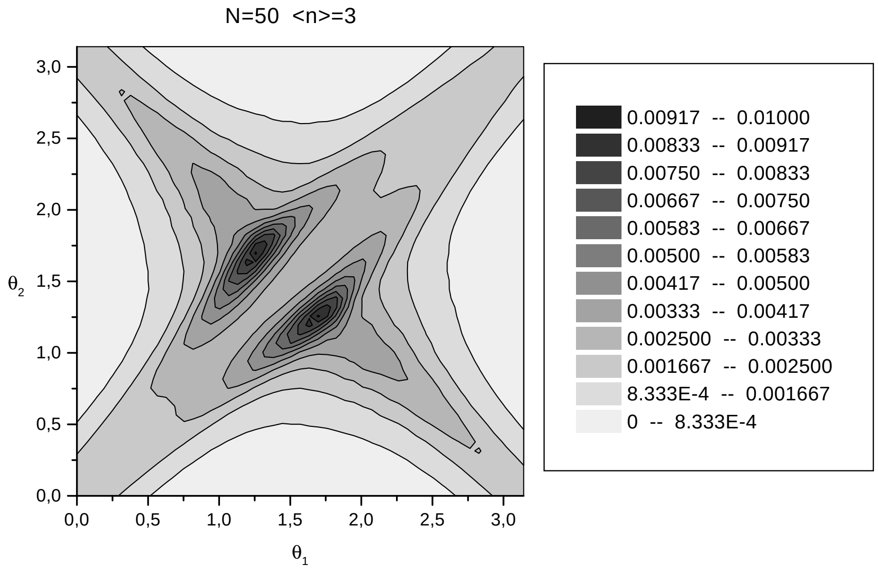

Fig. 3: The dependence of the characteristic time of the decoherence on the initial $\theta_1$, $\theta_2$ parameters of the Schrödinger cat (3) states. (Note that $\phi_1 = \phi_2 = 0$.) The number of atoms is $N = 50$ and the average number of photons is $\langle n \rangle = 3$.

very deep minimum at about $t_d$. The rapid decrease of the $\mathcal{M}^1(\rho(t))$ curve at a certain time is a general feature of this norm, provided the number of atoms is large enough and the temperature is not too low.

From these facts it follows that in general it is reasonable to define the characteristic time of the decoherence as the instant when the time derivative of the function $\mathcal{M}^1(\rho(t))$ appreciably decreases.

Now we can turn to the investigation of the dependence of the characteristic time on the initial conditions. Roughly speaking we shall answer the question “ How long does decoherence take provided the system was initially in the superposition of two particular atomic coherent states?” Fig. 3 shows the contour plot of the decoherence time versus the $\theta_1$ and $\theta_2$ parameters of the initial atomic Schrödinger cat states (3). For the sake of convenience $\phi_1 = \phi_2 = 0$ were chosen. As we can see, the effect of decoherence is remarkably slower when $\theta_1 \approx \theta_2$ which was expected taking into account that in this case the overlap of the two initial coherent states is not negligible, so these states can not be considered as “macroscopically distinct” any more. The fact that cat states which were initially symmetric with respect to the $\theta = \pi/2$ plane (i. e. $\theta_1 \approx \pi - \theta_2$ ) also decohere slower, is more surprising [10] but it is in accordance with the analytical estimations of Braun et. al. [11].

Besides the characteristic time of the decoherence the question of the pointer basis is also of great importance. By definition, the reduced density matrix of the atomic subsystem is diagonal

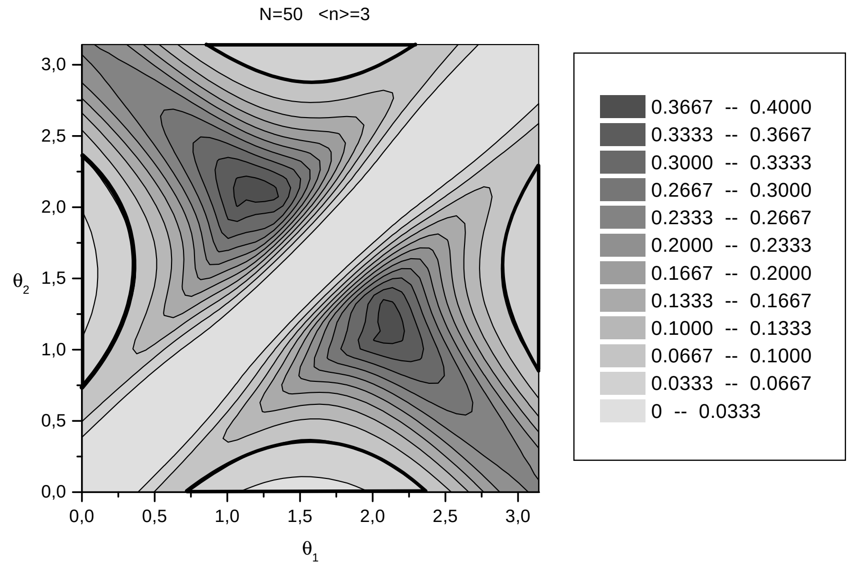

Fig. 4: The minimal distance between $\rho$ and $\rho_{cl}$ during the time evolution. The initial state of the system was an atomic Schrödinger cat (3) labeled by $\theta_1, \phi_1 = 0$; $\theta_2, \phi_2 = 0$. The number of atoms is $N = 50$ and the average number of photons is $\langle n \rangle = 3$.

in the pointer basis at $t_d$. This allows us to compute the numerical expansion coefficients of the pointer states. However, the physical interpretation of these vectors is a rather involved procedure. Nevertheless, we have obtained that the initial coherent states themselves are the pointer states to a very good approximation, provided the process of decoherence is fast. The thick solid lines in Fig. 4 enclose the initial $\theta_1$, $\theta_2$ parameter ranges, for which the minimal value of $\mathcal{M}^2\left(\rho(t)\right)$ that is the minimal distance between the current density matrix and the corresponding $\rho_{cl}$ is practically negligible. So if the parameters of the two initial coherent states fall in the marked area, the scheme (6) is clearly valid.

The time evolution can be suggestively mapped onto the phase-space even when the decoherence is slow. The corresponding spherical Wigner functions reflect the dynamics of the atomic subsystem. Using these functions we shall visualize the decoherence and dissipation of the slowly decohering atomic Schrödinger cat states in a subsequent publication.

## 5 Summary

We have considered the time evolution of atomic Schrödinger cat states during the interaction with an environment of photon modes. We have solved numerically the corresponding master equation of the atomic subsystem in the case of different initial conditions and number of atoms.

We have investigated two possible measures of decoherence and determined the range of their applicability. We have demonstrated that both measures provide the same characteristic time of decoherence.

Based on the more general measure $\mathcal{M}^1$ we verified the slower decoherence of the Schrödinger cat states which were initially symmetric with respect to the $\theta = \pi/2$ plane.

We have shown how one can distinguish between the dissipation of the atomic subsystem's energy and the process of decoherence. Considering a special class of the initial atomic Schrödinger cat states we have found that the initial coherent states are approximate pointer states.

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