acta physica slovaca vol. 50 No. 2, 199 – 211 **April 2000**

DECOHERENCE AND MILBURN DYNAMICS OF OPEN SYSTEMS: THE CASE OF THE LINEAR COUPLER

Marek Šašura
Institute of Physics, Slovak Academy of Sciences,
Dúbravská cesta 9, Bratislava 842 28, Slovakia

Received 20 October1999, in final form 28 January 2000, accepted 2 February 2000

Within the framework of the Milburn model of the intrinsic decoherence [G.J.Milburn, *Phys.Rev.* A44, 5401 (1991)] which is based on the assumption of the existence of the fundamental time step we study the dynamics of the linear coupler. We show that the evolution governed by the Milburn equation can be significantly modified compared to the standard quantum mechanics. Dicsussion about closed systems is also included.

PACS: 03.65.Bz, 42.50.-p

1 Introduction

Inspite of the success of quantum theory in accounting with striking accuracy for a vast variety of physical phenomena there are still some unsolved epistemological and conceptual questions concerning quantum theory. The feature of quantum physics that distinguishes it from classical one is the existence of the linear superposition of physical states. Apparently, the superposition principle doesn't work on the macroscopic level though nothing in the present formulation of quantum theory indicates this. Thus, the question is why we don't observe *quantum coherences*[1] on the macroscopic level. There exist two basic approaches to explain this problem.

The first one says it's difficult to prepare a closed system, i.e. each system is embedded in a large system with many degrees of freedom and a constant temperature called *reservoir*. Quantum coherences of a system interacting with the reservoir spread over its degrees of freedom what we effectively observe as a process of the *decoherence*. It's important to add, generally speaking, quantum coherences decay much faster compared to the energy dissipation of the system.

The second approach to the problem seeks to modify the Schrödinger equation in such a way that the coherences of a closed system are automatically destroyed as the system evolves – we call this effect *intrinsic decoherence*. Most known models come from Ghirardi, Rimini, Weber [1] and Milburn [2]. In this paper we analyze the Milburn model. He proposed a modification of the Schrödinger equation based on the assumption that on sufficiently short time steps a closed system doesn't evolve continuously under unitary evolution but rather in a stochastic sequence of unitary phase changes. Further, Milburn effectively introduces a minimum time step in the

[1] Off-diagonal elements of the density matrix in the eigenstate basis of the total Hamiltonian of a closed system.

universe θ_0. The inverse of this time step is equal to the mean frequency of unitary steps, i.e. $\gamma = \frac{1}{\theta_0}$. The smaller the value of γ the bigger the deviation from unitary evolution is. Further, Milburn derived the relation which we call the *Milburn equation*

$$\frac{d}{dt}\hat{\rho}(t) = \gamma\left[\exp\left(-\frac{i\hat{H}}{\hbar\gamma}\right)\hat{\rho}(t)\exp\left(+\frac{i\hat{H}}{\hbar\gamma}\right) - \hat{\rho}(t)\right], \tag{1}$$

where $\hat{\rho}(t)$ denotes the density operator of the system, $\hat{H}$ is its Hamiltonian and $\hbar$ is the Planck constant. We call γ the *parameter of the intrinsic decoherence*. Expanding (1) to the Taylor series to the first order in $\frac{1}{\gamma}$ we obtain the following equation

$$\frac{d}{dt}\hat{\rho}(t) = -\frac{i}{\hbar}[\hat{H},\hat{\rho}(t)] - \frac{1}{2\hbar^2\gamma}[\hat{H},[\hat{H},\hat{\rho}(t)]]\,. \tag{2}$$

The standard quantum mechanics is governed by the *von Neumann equation*

$$\frac{d}{dt}\hat{\rho}(t) = -\frac{i}{\hbar}[\hat{H},\hat{\rho}(t)]\,. \tag{3}$$

If $\gamma \to \infty$ the relation (2) becomes the equation (3), thus for $\gamma \to \infty$ the Milburn equation reduces to the standard von Neumann equation describing the Schrödinger evolution. The description of the evolution for $\gamma \to 0$ is discussed in detail in the following section.

Milburn in his paper discussed in detail a number of testable consequences of his model. Nevertheless, he studied only closed systems. In this paper we analyze an open system, i.e. a system interacting with its enviroment. We took a mode of the electromagnetic (EM) field as the open system coupled with another mode of the EM field as its enviroment.

This paper is organized as follows: In Sec.II we present the solution of the Milburn equation for the total Hamiltonian $\hat{H}$ of a composite system, the linear coupler is definied in Sec.III. Standard and Milburn dynamics of the system under consideration is described in Sec.IV and V, respectively. In Sec.VI we introduce Wigner functions of states of EM field modes. The paper is concluded with a discussion.

2 General solution of the Milburn equation for composite systems

Let us consider a composite system governed by the Hamiltonian

$$\hat{H} = \hat{H}_S + \hat{H}_E + \hat{H}_I\,, \tag{4}$$

where $\hat{H}_S$ denotes the Hamiltonian of the open system, $\hat{H}_E$ is for its enviroment and $\hat{H}_I$ is the interaction part of the total Hamiltonian $\hat{H}$.

Let us denote

$$\begin{aligned} \hat{H}|\Phi_\mu\rangle &= E_\mu|\Phi_\mu\rangle\,, && (5)\\ (\hat{H}_S+\hat{H}_E)|\varphi_i\rangle &= \epsilon_i|\varphi_i\rangle\,, && (6)\\ |\Phi_\mu\rangle &= \sum_i c_i^\mu|\varphi_i\rangle && (7)\end{aligned}$$

and also the density operator

$$\hat{\rho}(t) = \sum_{\mu,\nu} \rho_{\mu\nu}(t)|\Phi_\mu\rangle\langle\Phi_\nu| = \sum_{i,j} \tilde{\rho}_{ij}(t)|\varphi_i\rangle\langle\varphi_j| \,. \tag{8}$$

We can rewrite (8) with the use of (7)

$$\rho_{\mu\nu}(t) = \sum_{i,j} \tilde{\rho}_{ij}(t)(c_i^\mu)^*(c_j^\nu)\,, \tag{9}$$

$$\tilde{\rho}_{ij}(t) = \sum_{\mu,\nu} \rho_{\mu\nu}(t)(c_i^\mu)(c_j^\nu)^* \,. \tag{10}$$

The solution of the Milburn equation (1) in the eigenstate basis of the total Hamiltonian (5) then reads

$$\rho_{\mu\nu}(t) = \rho_{\mu\nu}(0)\exp\left(\gamma t\left\{\exp\left[\frac{-i}{\hbar\gamma}(E_\mu - E_\nu)\right] - 1\right\}\right)\,, \tag{11}$$

where the matrix elements $\rho_{\mu\nu}(0)$ describe the initial state $(t = 0)$ of the composite system. Diagonal elements are not affected, i.e. $\rho_{\mu\mu}(t) = \rho_{\mu\mu}(0)$. Off diagonal elements $\rho_{\mu\nu}(t)$ are reduced gradually to zero with the factor $\exp\left\{\gamma t\left[\cos\left(\frac{E_\mu - E_\nu}{\hbar\gamma}\right) - 1\right]\right\}$ because $\forall x \in \mathcal{R};\, |\cos(x)| \leq 1$. If $\gamma \to 0$ the exponential factor in (11) reduces to unity[2] and we get $\rho_{\mu\nu}(t) = \rho_{\mu\nu}(0)$. It means the system effectively does not evolve, we say that the system *freezes*. According to Milburn for $\gamma \to 0$ the minimum time step $\theta_0 \to \infty$ and the system can't *make* even a single evolution step.

Substituting (11) into (10) and using (9) for $t = 0$ we get the solution of the Milburn equation in the basis of the free Hamiltonian (6)

$$\begin{aligned}\tilde{\rho}_{ij}(t) &= \sum_{\mu,\nu} \rho_{\mu\nu}(0)\exp\left(\gamma t\left\{\exp\left[\frac{-i}{\hbar\gamma}(E_\mu - E_\nu)\right] - 1\right\}\right)(c_i^\mu)(c_j^\nu)^* \qquad (12)\\ &= \sum_{\mu,\nu}\left[\sum_{I,J}\tilde{\rho}_{IJ}(0)(c_I^\mu)^*(c_J^\nu)\right]\exp\left(\gamma t\left\{\exp\left[\frac{-i}{\hbar\gamma}(E_\mu - E_\nu)\right] - 1\right\}\right)(c_i^\mu)(c_j^\nu)^* \,.\end{aligned}$$

By performing the trace over the enviroment we obtain the density operator of the open system under consideration, i.e. $\hat{\rho}_S(t) = \mathrm{Tr}_E\hat{\rho}_{SE}(t)$.

3 Simple model of the open system: Linear coupler

Let us assume a simple model of a *linear coupler* - an effective device which allows a linear EM interaction between two modes of the EM field. We assume one of the modes to play a role of the open system (mode A) while the other to be in the role of its nondissipative enviroment (mode B). The linear coupler is described by the Hamiltonian [3]

$$\hat{H} = \hbar\omega\hat{a}^\dagger\hat{a} + \hbar\omega\hat{b}^\dagger\hat{b} + \hbar\lambda(\hat{a}^\dagger\hat{b} + \hat{a}\hat{b}^\dagger)\,, \tag{13}$$

[2] $\lim_{\gamma\to 0}\gamma\cos(\frac{1}{\gamma}) = 0,\ \lim_{\gamma\to 0}\gamma\sin(\frac{1}{\gamma}) = 0\,.$

where $\hat{a}, \hat{a}^\dagger$ and $\hat{b}, \hat{b}^\dagger$ denotes annihilation and creation field operators for the modes A and B, respectively. Further, we have assumed $\omega_A = \omega_B = \omega$ and we don't take into account absolute terms $\frac{1}{2}\hbar\omega$ in the Hamiltonians of single modes. λ is the coupling constant. Introducing the dressed modes

$$\hat{c} = \frac{\hat{a} + \hat{b}}{\sqrt{2}} \,, \qquad \hat{d} = \frac{\hat{a} - \hat{b}}{\sqrt{2}} \,, \tag{14}$$

the Hamiltonian (13) becomes

$$\hat{H} = \hbar(\omega + \lambda)\hat{c}^\dagger \hat{c} + \hbar(\omega - \lambda)\hat{d}^\dagger \hat{d} \,, \tag{15}$$

which can be diagonalized as follows

$$\hat{H}|\phi_j^{(N)}\rangle = E_j^{(N)}|\phi_j^{(N)}\rangle \,, \quad j = 0, \ldots, N \,, \tag{16}$$

$$|\phi_j^{(N)}\rangle = \left(\frac{1}{\sqrt{2}}\right)^N \frac{1}{\sqrt{(N-j)!\,j!}} \sum_{k=0}^{N-j} \sum_{l=0}^{j} (-1)^{j-l} \binom{N-j}{k} \binom{j}{l} \times \sqrt{(k+l)!}\sqrt{(N-k-l)!}\,|k+l, N-k-l\rangle \,, \tag{17}$$

$$E_j^{(N)} = \hbar\omega N + \hbar\lambda(N - 2j) \,. \tag{18}$$

Relations (16)-(18) express the eigensystem of the Hamiltonian (13) in the Hilbert subspace (with N excitations) generated by the composite Fock states $|N,0\rangle$, $|N-1,1\rangle$, $\ldots$, $|1,N-1\rangle$, $|0,N\rangle$, where we denote $|m,n\rangle \equiv |m\rangle_A|n\rangle_B$. $|m\rangle$ denotes the Fock state with m photons.

In what follows we will consider the mode A initially prepared in the coherent state

$$|\alpha\rangle_A = \exp(\alpha\hat{a}^\dagger - \alpha^*\hat{a})|0\rangle_A = \sum_{n=0}^{\infty} c_n|n\rangle_A = \sum_{n=0}^{\infty} \exp(-\frac{|\alpha|^2}{2})\frac{\alpha^n}{\sqrt{n!}}|n\rangle_A \tag{19}$$

and the mode B in the vacuum state $|0\rangle_B$.

4 Standard dynamics

4.1 Closed system

Let us firstly consider a single mode of the EM field (closed system) governed by the Hamiltonian $\hat{H} = \hbar\omega\hat{a}^\dagger\hat{a}$ and initially prepared in the coherent state $|\alpha\rangle$. The unitary evolution of such a system is described by the wave function

$$|\psi(t)\rangle = \hat{U}(t)|\psi(0)\rangle = \exp\left(-\frac{i\hat{H}t}{\hbar}\right)|\alpha\rangle = |\alpha\exp(-i\omega t)\rangle \,. \tag{20}$$

The state of the system changes from $|\alpha\rangle$ into $|\alpha\exp(i\omega t)\rangle$. The mean photon population in the mode stays constant, $\bar{n} = \langle\alpha|\hat{n}|\alpha\rangle = \langle\alpha|\hat{a}^\dagger\hat{a}|\alpha\rangle = |\alpha|^2$ where $\hat{a}|\alpha\rangle = \alpha|\alpha\rangle$ and $\langle\alpha|\hat{a}^\dagger = \langle\alpha|\alpha^*$.

4.2 Open system

The situation is different when we consider the linear coupler, i.e. two coupled modes of the EM field where one of them (mode A) plays a role of the open system and the other is the enviroment (mode B). We assume the mode A to be iniatially in the coherent state $|\alpha\rangle$ (in what follows we assume for simplicity that α is real), the mode B in the vacuum. The wave function of the coupled modes in the standard evolution then reads [4], [5]

$$|\Psi(t)\rangle = \exp\left(-\frac{i\hat{H}t}{\hbar}\right)|\alpha\rangle_A|0\rangle_B = |\alpha(t)\rangle_A|\beta(t)\rangle_B\,, \tag{21}$$

where

$$\alpha(t) = \alpha\cos(\lambda t)\exp(-i\omega t)\,, \tag{22}$$
$$\beta(t) = -i\alpha\sin(\lambda t)\exp(-i\omega t) \tag{23}$$

and the mean photon population in the modes reads

$$\bar{n}_A(t) = \alpha^2\cos^2(\lambda t)\,, \tag{24}$$
$$\bar{n}_B(t) = \alpha^2\sin^2(\lambda t)\,. \tag{25}$$

When the mode A (open system) is coupled with another mode B ($\omega = 1$) and governed by the standard Schrödinger dynamics the state of the system (22) reconstructs with the period $\frac{2\pi}{\lambda}$. There is no loss of the phase information. Coherences are present. The energy exchanges periodically between the system (mode A) and its enviroment (mode B). The mean photon population (24) reconstructs with the period $\frac{\pi}{\lambda}$. This is also one of reasons why we chose the linear coupler to test the Milburn dynamics. Because modes reconstructs after a certain time there is no decoherence through the interaction between modes.

5 Milburn dynamics

5.1 Closed system

Firstly we again consider a single mode of the EM field (closed system) governed by the Milburn equation (1) prepared initially in the coherent state $|\alpha\rangle$. We find out that diagonal elements of the density matrix are not affected by the evolution, i.e. all integrals of the motion stay constant. For example, the mean photon population in the mode is constant. However, off--diagonal elements (quantum coherences) are influenced in the Milburn dynamics and decay at the rate of the intrinsic decoherence parameter γ what we effectively observe as the intrinsic decoherence of the system. See relation (11) and the discussion below.

5.2 Open system

If we take into account the mode A (open system) coupled with the mode B the Milburn dynamics is more complex because not only off-diagonal elements but also diagonal elements of the density matrix are influenced due to the Milburn assumption of the fundamental time step. It implies

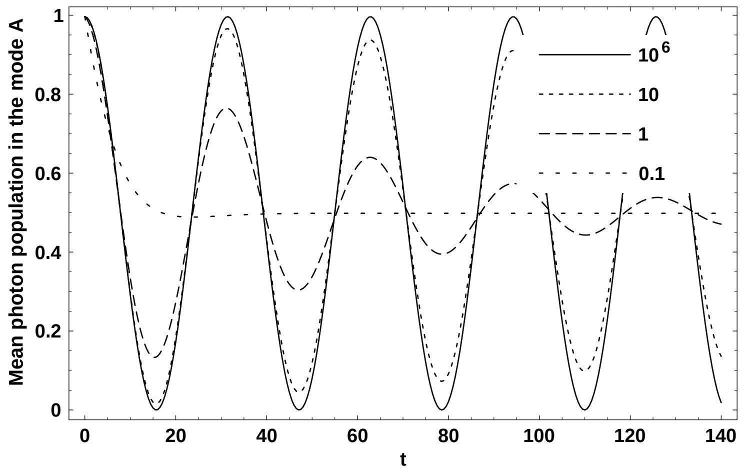


Fig. 1. The mean photon population in the mode A as a function of the time plotted for 4 values of the parameter γ. The mode A initially prepared in the coherent state with the real amplitude $\alpha = 1$, the mode B in the vacuum. The coupling constant is $\lambda = 0.1$. If we consider the real physical coupling $\lambda_{real} = 7$MHz then $t = 140$ corresponds to $t_{real} = 2 * 10^{-6}$s, $\gamma = 10^6$ to $\gamma_{real} = 7 * 10^{13}$Hz, $\gamma = 10$ to $\gamma_{real} = 7 * 10^8$Hz, $\gamma = 1$ to $\gamma_{real} = 7 * 10^7$Hz and $\gamma = 0.1$ to $\gamma_{real} = 7 * 10^6$Hz.

that all observables associated with the photon distribution – which directly depend on diagonal elements – are strongly affected, too.

Further in this section we will present the numerical solution $(\omega = 1)$ of the Milburn equation for the mode A coupled with the mode B. Though there exists the exact analytical subscription for the eigenvectors (17) and eigenvalues (18) of the linear coupler, from the computation point of view it is more advantageous to use the method of the numerical diagonalization of the Hamiltonian (13) rather than to perform infinite multiple summations in the analytical solution. Tracing over the mode B we obtained the density operator of the mode A

$$\hat{\rho}_A(t) = \mathrm{Tr}_B\, \hat{\rho}_{AB}(t)\,. \tag{26}$$

The coherent state is the infinite superposition of the Fock states so we had to choose the cut-off dimension for the Hilbert space under consideration. For the amplitude of the coherent state $\alpha = 1$ we took the first 7 states $|\alpha = 1\rangle = \sum_{n=0}^{6} c_n |n\rangle$, $\langle\alpha|\alpha\rangle = \sum_{n=0}^{6} |c_n|^2 \doteq 0.9999$, i.e. the Hilbert space is spanned by 28 orthonormal vectors $|0,0\rangle$, $|1,0\rangle$, $|0,1\rangle$, $|2,0\rangle$, $|1,1\rangle$, $|0,2\rangle$, $|6,0\rangle$, $\ldots$,$|0,6\rangle$. For $\alpha = 2$ we took 13 states so the Hilbert space dimension was 91 and the norm 0.9997.

The temporal depence of the mean population in the mode A is plotted in Fig. 1 for 4 different values of the parameter γ. The mean photon population oscillates with the same period for different values of γ. For the coupling constant $\lambda = 0.1$ we observe no difference between the standard and Milburn dynamics already for $\gamma = 10^6$. If we consider a strong interaction we also have to rescale the value of γ under consideration. The mean photon popullation in Fig. 1. does not depend on the mode frequency ω.

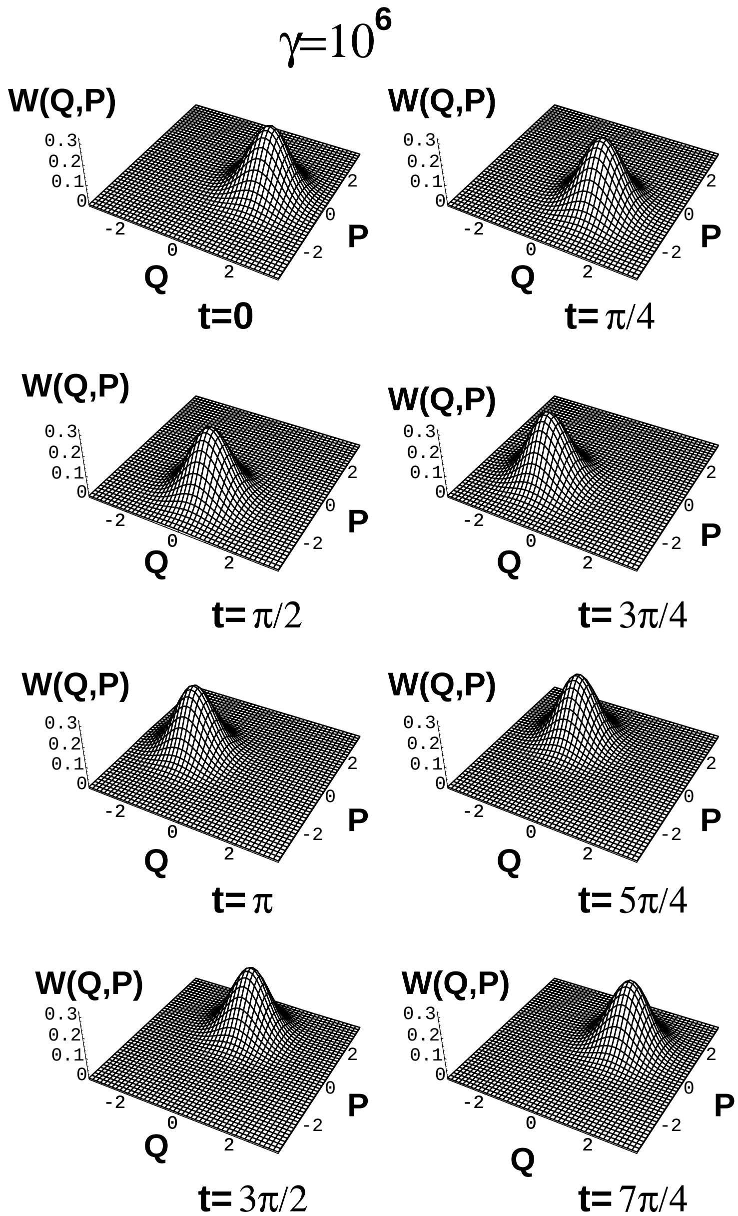


Fig. 2. Wigner functions of a single mode of the EM field ($\omega = 1$) in 8 time instants for the standard dynamics ($\gamma = 10^6$). The mode initially prepared in the coherent state with the amplitude $\alpha = 1$. If we now consider the real optical frequency $\omega_{real} = \pi * 10^{15}$ Hz (600 nm) then $\gamma = 10^6$ corresponds to $\gamma_{real} = \pi * 10^{21}$ Hz, time is scaled by the relation $t_{real} = \frac{t}{\pi} * 10^{-15}$s and $q_{real} = Q\sqrt{\frac{\hbar}{\omega \varepsilon_0}}$, $p_{real} = P\sqrt{\hbar \omega \varepsilon_0}$.

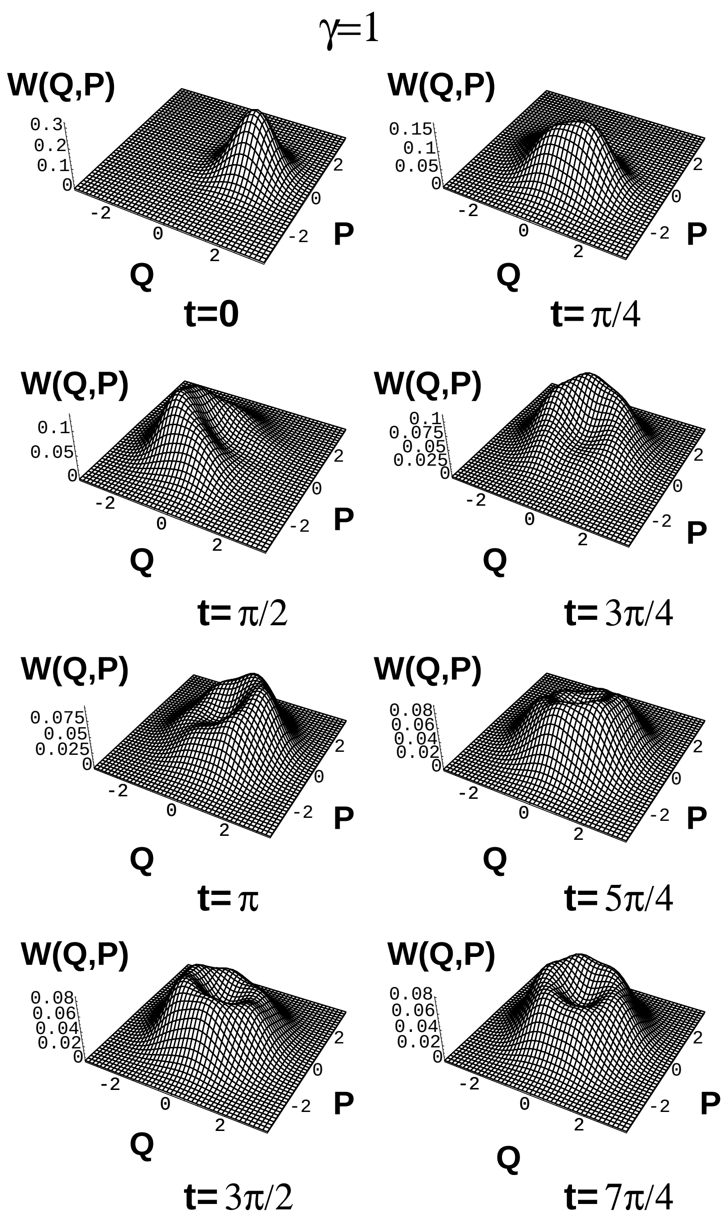
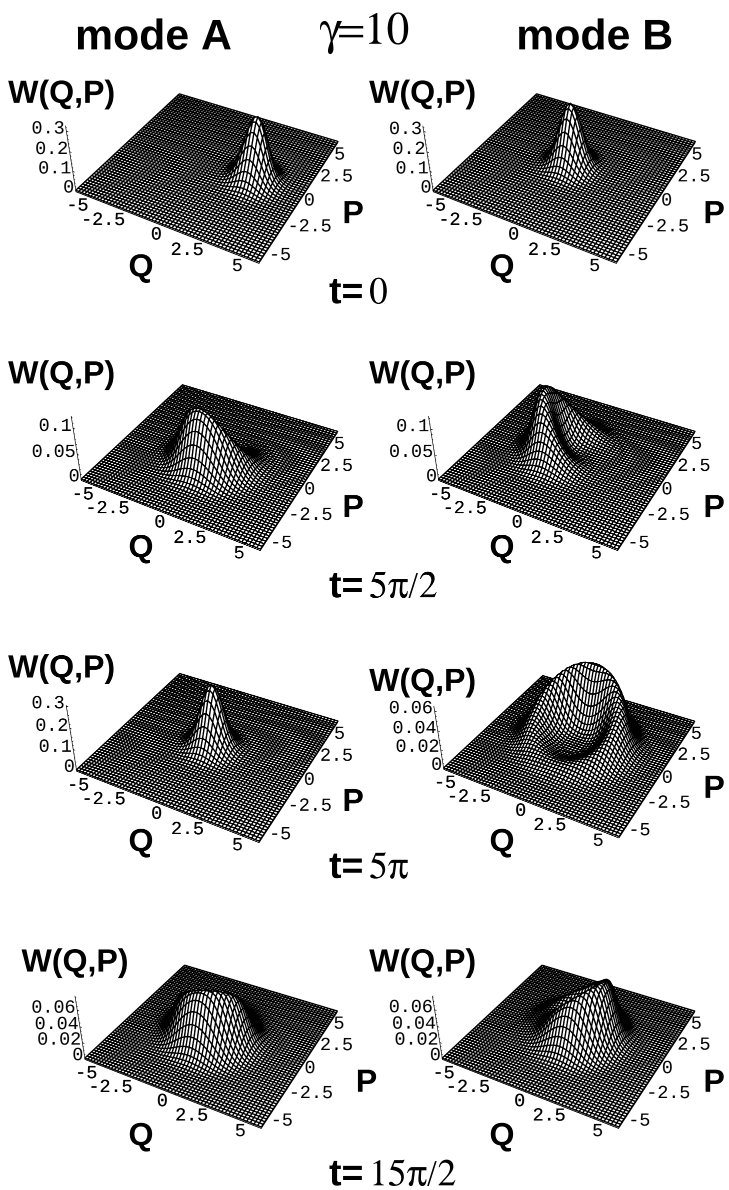


Fig. 3. Wigner functions of a single mode of the EM field ($\omega = 1$) in 8 time instants for the Milburn dynamics ($\gamma = 1$). The mode initially prepared in the coherent state with the amplitude $\alpha = 1$. If we now consider the real optical frequency $\omega_{real} = \pi * 10^{15}$Hz (600 nm) then $\gamma = 1$ corresponds to $\gamma_{real} = \pi * 10^{15}$Hz, time is scaled by the relation $t_{real} = \frac{t}{\pi} * 10^{-15}$s and $q_{real} = Q\sqrt{\frac{\hbar}{\omega\varepsilon_0}}$, $p_{real} = P\sqrt{\hbar\omega\varepsilon_0}$.


mode A
γ=10
mode B
W(Q,P)
Q
P
t= 0
t= 5π/2
t= 5π
t= 15π/2

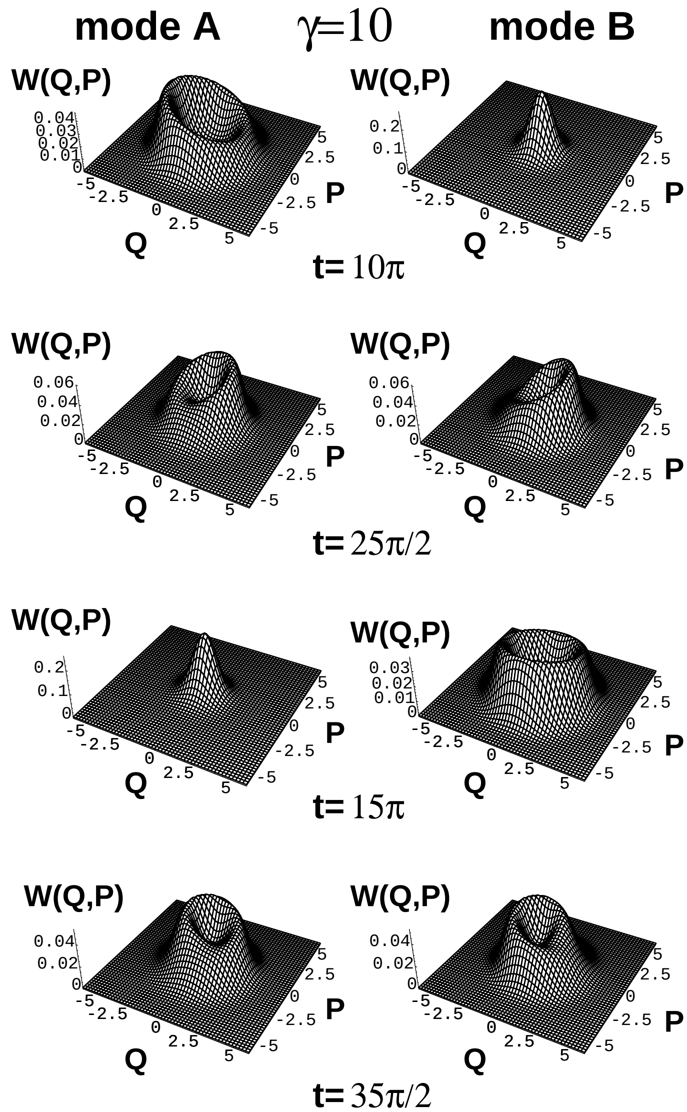


Fig. 4. Wigner functions of the modes A and B in 8 time instants for the Milburn dynamics ($\gamma = 10$). The mode A initially prepared in the coherent state with the amplitude $\alpha = 2$, the mode B in the vacuum. The coupling constant is $\lambda = 0.1$.

6 Wigner functions

The decoherence means a loss of the phase information. Quantum mechanics is usually formulated in the Hilbert space but it is sometimes appropriate to reformulate it within the phase space formalism. The Wigner function carries all the information about a quantum system decribed by the density operator $\hat{\rho}(t)$ and is defined in the phase space as follows

$$W(q,p,t) = \frac{1}{2\pi\hbar}\int\limits_{-\infty}^{\infty} d\eta \exp\left(\frac{ip\eta}{\hbar}\right) \langle q - \frac{\eta}{2}|\, \hat{\rho}(t)\, |q + \frac{\eta}{2}\rangle\,. \tag{27}$$

The Hilbert space under our consideration is generated by the Fock states, thus let us write the density operator in this basis

$$\hat{\rho}(t) = \sum_{m,n} A_{mn}(t)\, |m\rangle\langle n|\,. \tag{28}$$

Substituting (28) into (27) we can write

$$W(q,p,t) = \sum_{m,n} A_{mn}(t) W_{mn}(q,p)\,, \tag{29}$$

where

$$W_{mn}(q,p) = \frac{1}{2\pi\hbar}\int\limits_{-\infty}^{\infty} d\eta \exp\left(\frac{ip\eta}{\hbar}\right) \psi_m(q - \frac{\eta}{2})\, \psi_n^*(q + \frac{\eta}{2})\,, \tag{30}$$

$$\psi_m(x) = \frac{1}{\sqrt{2^m m!}}\left(\frac{\zeta}{\pi}\right)^{\frac{1}{4}} H_m(x\sqrt{\zeta}) \exp\left(\frac{-\zeta x^2}{2}\right)\,. \tag{31}$$

$\psi_m(x)$ denotes the wave function of the quantum oscillator (mode of the EM field) in q-representation. $H_m(x)$ is the Hermitte polynomial of the m-th order, $\zeta = \frac{\omega\varepsilon_0}{\hbar}$, where ω is the EM mode field frequency and ε_0 is the dielectric constant.

After substituting (31) into (30) and integration we get

$$\begin{aligned} W_{mn}(q,p) = {} & \frac{(-1)^m}{\pi\hbar}\sqrt{\frac{2^n}{2^m}}\sqrt{\frac{m!}{n!}}\left(\sqrt{\zeta}q + \frac{ip}{\hbar\sqrt{\zeta}}\right)^{n-ma} \\ & \times \exp\left[-\left(\frac{p^2}{\zeta\hbar^2} + \zeta q^2\right)\right] L_m^{n-m}\left[2\left(\frac{p^2}{\zeta\hbar^2} + \zeta q^2\right)\right]\,, \end{aligned} \tag{32}$$

where $L_m^{n-m}(x)$ denotes the generalized polynomial of the m-th order. For the numerical calculations we used the values $\omega = 1, \hbar = 1, \lambda = 0.1$ and $\zeta = 1$.

At the beginning of Sec.4 and 5 we discussed the standard and Milburn dynamics of the single mode of the EM field. For the standard evolution, we found out that the system evolves from the state $|\alpha\rangle$ into $|\alpha \exp(-i\omega t)\rangle$, i.e. the mode stays in the coherent state. When we considered the Milburn dynamics of the single mode we found out that the mode was losing just its phase information but diagonal elements of the density matrix stayed untouched (11).

In Fig. 2 we plotted Wigner functions of the time evolution of the single mode of the EM field iniatially prepared in the coherent state $|\alpha = 1\rangle$ for $\gamma = 10^6$ what essentially corresponds to the standard dynamics. We see that the system reconstructs with the period 2π. There is no decoherence.

In Fig. 3 we plotted almost the same situation but for $\gamma = 1$ (Milburn dynamics). We see the rotating effect of the Gaussian peak[3] but as the function rotates around the origin of the phase space the system loses gradually its phase information which is mostly pronounced for $t = \frac{7\pi}{4}$ when the Wigner function is almost symetrical around the origin of the phase space. We observe the intrinsic decoherence - the loss of the phase information as the closed system evolves but diagonal elements of the density matrix of the system are not affected.

Further, let us assume the linear coupler initially prepared in the state where the mode A is in the coherent state $|\alpha\rangle$ with the amplitude $\alpha = 2$ and the mode B in the vacuum. The interaction between the modes is then performed by four photons (24,25).

If we consider the standard dynamics both modes stay in the coherent states (21) and the initial state of the whole system recovers after $t = 20\pi$ because there is no deceherence through the interaction between the open system (mode A) with its enviroment (mode B). Thus, the Wigner functions of the modes are then Gaussian functions differently centered as the amplitudes of the coherent states change (22,23).

If we take into account the Milburn dynamics ($\gamma = 10$) we see that the dynamics is different (Fig. 4). The Wigner function of the open system (mode A) loses gradually its Gaussian shape and the system is not able to reconstruct its iniatial state anymore. Although there is no decoherence caused by the interaction between modes, diagonal elements of the density matrix of mode A are strongly affected and boths modes get for $t \to \infty$ to the quasithermal stationary state with two photons in each mode.

In this paper we describe the Milburn dynamics of the open system and we propose a simple idea how to verify experimentally its validity. We chose the values of the frequency ω and coupling λ in almost same order ($\omega = 1, \lambda = 0.1$) because it provided us the possibility to present the effect of the phase rising and coupling on the same time scale, see relations (22) and (23). If we tried to plot Fig. 4 with realistic optical values ($\omega = \pi * 10^{15}$Hz, $\lambda = 7$Mhz) then we would have to consider the time scale in microseconds (see Fig. 1) where the rotating effect of the coherent state would be pushed by the coupling which is relevant on this time scale.

7 Discussion

Milburn proposed a solution to answer the question why we don't observe the linear superpostion of physical states on the macroscopic level, i.e. why the system loses quantum coherences (off-diagonal elements of the density matrix in the eigenstate basis). In this paper we tested the Milburn dynamics on the open system (mode of the EM field) that doesn't lose quantum coherences through the interaction with its nondissipative enviroment (another mode of the EM field). We showed that this dynamics gives different results from what we know about the linear coupler governed by the Schrödinger equation with the same initial conditions. We demonstrated the difference between these two dynamics on the temporal dependce of the mean photon distribution and by plotting the Wigner functions of the states of the system under consideration.

In this paper we suggested the idea rather than a specific experimental test how to verify experimentally the validity of the Milburn equation by measuring the mean photon distribution (or some other observables associated with this distribution) in the linear coupler. The measurement on the open system interacting with its enviroment could give us the real chance to measure *disturbances* caused by the Milburn equation comparing to the standard dynamics on the realistic laboratory time scale by the suitable setting of the coupling constant between the system and its enviroment.

Acknowledgement This work has been supported by the Slovak Grant Agency VEGA Grant n. 2/7173/20.

[3]The Wigner function of the coherent state is the Gaussian function with the moved centre.

References

[1] G.C. Ghirardi, A. Rimini, T. Weber: *Phys. Rev.* **D34** (1986) 470
[2] G.J. Milburn: *Phys. Rev.* **A44** (1990) 5401
[3] W.K. Lai, V.Bužek, P.L. Knight: *Phys. Rev.* **A43** (1991) 6323
[4] G. Drobný, I. Jex, V. Bužek: *Acta Physica Slovaca* **44** (1994) 155
[5] M. Šašura: *Diploma Thesis*, MFF UK, Bratislava (1999)
[6] H. Moya-Cessa, V. Bužek, P.L. Knight: *Phys. Rev.* **A48** (1993) 3900
[7] V. Bužek, M. Konôpka: *Phys. Rev.* i**A58** (1998) 1735