acta physica slovaca vol. 49 No. 5, 805 – 809 **October 1999**

Thermodynamic Equilibrium of Pyrgons and Radiation

C. Wolf
Dept. of Physics, Massachusetts College of Liberal Arts, North Adams, MA 01247, USA

Received 7 April 1998, revised 26 August 1998, accepted 27 August 1998

By considering the pyrgons generated by the Planck scale excitations of massive scalar field to be equilibrium with radiation we calculate the favored equilibrium condition of pyrgons and radiation.

PACS: 05.20

1 Introduction

The possible existence of higher dimensions in both elementary particle theory and cosmology has been both a "curiosity" and useful construct in understanding in a natural way how cosmological evolution was catalyzed and how the standard model emerged as a low energy fragment of a superstring theory in higher dimensions [1-4]. The present belief is that an initially higher dimensional cosmology evolved so that the three space expanded and the extra compact space compactified [5]. With regard to particle theory an $E_8 \times E_8'$ super Yang - Mills field theory is coupled to the heterotic super-string in ten dimensions compactified through Calabi-Yau compactification to yield an $E_8' \times E_6$ G.U.T. group coupled to $N = 1$ supergravity in the gravitational sector [6]. The initial hope of all Kaluza-Klein theory was that coordinate symmetries in the higher dimensions appear as gauge symmetries in the compactified four dimensional world [7]. If indeed compactification is a real physical process, fields in the higher dimensional space can be expanded as periodic functions of the extra space time compactifiedi coordinates, the original proposal that an additional compactified space will generate gravitational excitations of mass proportional to the inverse of the copactification scale was due to Kolb and Slansky [8]. These excitations called pyrgons survive in the early universe if the temperature is of the order of the inverse compactification scale (in appropriate units). In Ref. 8 it is demonstrated that as the universe cools, pyrgons annihilate with antipyrgons into photons. It is also possible that highly massive pyrgons can decay into lower mass pyrgons, but never just into zero modes of the compactified theory [9]. Since these excitations have energy of the order of the Planck scale they are seldom considered because of the enormous energy necessary to excite them. If however the temperature is high enough these excitations can be in thermal equilibrium with radiation. In what follows we consider the thermodynamic equilibrium of radiation (propagating in $3+1$ space time) interacting with the gravitational excitations of an almost massless scalar field (pyrgons).

By calculating the thermal energy and entropy of a gas of (pyrgons) according to the canonical ensemble and then maximizing the entropy of a mixture of radiation and pyrgons we find a unique equilibrium point which is a stable maximum. From a physical point of view such a mixture of radiation and pyrgons might exist in the very early universe prior to the formation of black holes by "instanton effects" [10] and may provide us with the initial state from which inflationary cosmology evolved [11]. Even in the present universe, if very high temperature regions exist, such an equilibrium configuration might survive for a time and upon dispersing provide us with an ambient source of γ ray bursts. Whatever the possible ramifications are of such an equilibrium configuration, the fact that these two distinct forms of matter can coexist in thermal equilibrium if the temperature is high enough suggests that excitations of the higher dimensional world may give rise to real physical effects even though the size of the extra-compactified dimensions are unobservably small.

2 The equilibrium of radiation and excitations in the fifth dimension

We begin by considering a five dimensional Minkowski space with metric

$$g_{\mu\nu} = (\,1, \quad -1, \quad -1, \quad -1, \quad -1\,) \quad (x^\mu = x^0, x^1, x^2, x^3, x^5)$$

The fifth dimension we consider to be compactified and of the order of the Planck length with

$$x^5 = l_p \Theta \qquad (0 \leq \Theta \leq 2\pi)$$

For the relativistic relation between energy, momentum, mass and angular momentum in the fifth dimension we have [12]

$$E^2 = c^2 P^2 + m^2 c^4 + \frac{c^2 L_5^2}{l_p^2}. \tag{1}$$

Here l_p is Planck length, L_5 is angular momentum in fifth dimensions, m is the mass of particle, p is three momentum in three space. Eq. (1) could also be derived from the field equation for a real scalar field in five dimensions which reads

$$\Box\Phi + \frac{m^2 c^2}{\hbar^2}\Phi = 0 \tag{2}$$

with

$$\Box\Phi = \frac{1}{\sqrt{g}} \frac{\partial}{\partial x^\alpha} \left(\Phi_{,\beta} g^{\beta\alpha} \sqrt{g} \right).$$

When we expand

$$\Phi = \frac{1}{\sqrt{2\pi}} \sum_{n=-\infty}^{\infty} \Phi_n(x^0, x^1, x^2, x^3) \exp \frac{inx^5}{l_p}$$

and impose the condition

$$\Phi_n^* = \Phi_{-n}$$

we find the following four dimensional lagrangian after integrating the five dimensional lagrangian over the compact coordinates x^5

$$\mathcal{L}_4 = \frac{\partial_\mu \Phi_0 \partial^\mu \Phi_0}{2} - \frac{m^2 c^2}{2\hbar^2}\Phi_0 \Phi_{0'} + \sum_{n=1}^{\infty}\left(\partial_\mu \Phi_n \partial^\mu \Phi_n^* - \frac{m_n^2 c^2}{\hbar^2}\Phi_n \Phi_n^*\right) \tag{3}$$

here

$$\mathcal{L}_5 = \frac{\partial^N \Phi \partial_N \Phi}{2} - \frac{m^2 c^2}{2\hbar^2}\Phi\Phi_{'},$$

$$\mathcal{L}_4 = \frac{1}{l_p}\int \mathcal{L}_5 \mathrm{d}x^5$$

where $m_n^2 = m^2 + n^2 M_p^2$, $M_p = \sqrt{\frac{\hbar c}{G}}$ is Planck Mass. If we impose $L_5 = n\hbar$ $(n = 0, 1, \ldots)$ in Eq. (1), we have

$$E^2 = c^2 P^2 + m_n^2 c^4 \qquad (m_n^2 = m^2 + n^2 M_p^2)$$

which is identical to the formula for m_n from Eq. (3). We thus take the energy of an excitation in the fifth dimension as

$$E^2 = m^2 c^2 + n^2 M_p^2 c^4 \qquad (P = 0).$$

If $M_p >> m$ or $m = 0$ we have $E = nM_p c^2$. To calculate the thermal energy of a pyrgon we first calculate the canonical partition function given by [13]

$$Z = \int_1^\infty \exp - \frac{nM_p c^2}{\tau}\mathrm{d}n \tag{4}$$

(Note that $n = 0$ is excluded if $m = 0$, also $\tau = kT$ where k is Boltzmann constant and T is absolute temperature). Integrating Eq. (4) we have

$$Z = \frac{\tau}{M_p c^2}\exp - \frac{M_p c^2}{\tau},$$

for the average thermal energy we have

$$\langle \epsilon \rangle = \tau^2 \frac{\mathrm{d}}{\mathrm{d}\tau}\log_e Z = M_p c^2 + \tau. \tag{5}$$

For the entropy and energy of N pyrgons we have

$$\begin{aligned} S &= Nk\left[\tfrac{\langle \epsilon \rangle}{\tau} + \log_e Z\right] = Nk\left[1 + \tfrac{M_p c^2}{\tau} + \log_e \tfrac{\tau}{M_p c^2} - \tfrac{M_p c^2}{\tau}\right] \\ &= Nk\left[1 + \log_e \tfrac{\tau}{m_p c^2}\right] \end{aligned} \tag{6}$$

and

$$U = N(M_p c^2 + \tau).$$

We now consider the above pyrgons to interact with radiation propagating purely in the $3 + 1$ dimensional space-tme. For the total entropy and energy of pyrgons plus radiation we have

$$S = Nk\Big[1 + \log_e \frac{\tau}{M_p c^2}\Big] + \frac{4}{3} aVT^3 \tag{7}$$

$$U = N(M_p c^2 + \tau) + aVT^4 \tag{8}$$

where V is three spatial volume and a constant related to Stefan Boltzmann constant. If we solve Eq. (8) for N and substitute into Eq. (7) we obtain

$$S = k\Big(1 + \log_e \frac{kT}{M_p c^2}\Big) \frac{U - aVT^4}{M_p c^2 + kT} + \frac{4}{3} aVT^3 \tag{9}$$

with $U =$ constant. To find the equilibrium configuration we take the derivative of Eq. (9) and set it equal to 0. This gives

$$1 + \log_e x = 1 + \frac{1}{x} \qquad \text{where} \quad x = \frac{kT}{M_p c^2} \tag{10}$$

An approximate solution of this equation is $x \approx 1.8$. To test for a maximum we calculate the second derivative of Eq. (9) at $M_p c^2 = \frac{kT}{1.8}$:

$$\frac{\mathrm{d}^2 S}{\mathrm{d}T^2} = -\frac{U}{T^3}(2y^2 + y) + 4aVT \frac{1 - y + 2y^2}{y} < 0 \tag{11}$$

with $y = x/(1 + x)$. We have approximated $U - aVT^4 \approx U$ which is valid if V is small.

Thus, Eq. (11) gives a maximum of the entropy and a stable equilibrium configuration of radiation interacting with pyrgons at $kT = 1.8 M_p c^2$. There was no approximation made in calculating (10) and we note that the equilibrium temperature is independent of the volume. To calculate the number of pyrgons at equilibrium we have from Eq. (8)

$$N = \frac{U - aVT^4}{M_p c^2 + \tau} \approx \frac{U}{2.8 M_p c^2}$$

since $U >> aVT^4$ for V small.

3 Conclusion

The above calculation has demonstrated that a stable equilibrium configuration exists for pyrgons interacting with radiation where we have assumed that the radiation does not propagate along the compact dimension. Such a calculation also suggests that in the early universe where temperatures are high there is a limiting higher temperature not imposed by quantum gravity but rather by the thermodynamic equilibrium condition. Adding more energy to a small spatial volume will not raise the temperature but will only increase the number of modes (pyrgons) of the scalar field. Thus one more feature of higher dimensional physics appears that imposes limits on thermodynamics quantities without resorting to quantum gravitational effects.

Acknowledgement I would like to thank the physics departments at Williams College and Harvard University for the use of their facilities.

References

[1] O. Klein: *Z. Phys.* **21** (1926) 895
[2] A.D. Sakharov: *Sov. Phys. JETP (Eng. translation)* **60** (1984) 214
[3] J.W. Moffat: *Can. J. Phys* **64** (1986) 651
[4] M.B. Green, J. Schwarz: *Phys. Lett.* **B 149** (1984) 117
[5] M. Gleiser, S. Rajpoot, T.G. Taylor: *Ann. Phys. (NY)* **160** (1985) 299
[6] J.L. Rosner: *comments on Nuclear and Particle Physics* **15** (1986) 195
[7] O. Klein: *Nature* **118** (1926) 516
[8] E.W. Kolb, R. Slansky: *Phys. Lett. B* **135** (1984) 378
[9] E.W. Kolb: in *Proc. of John Hopkins Workshop on Current Problems in Particle Theory 8*, Baltimore, June 20–22 1984, eds. G. Domokos, S.K. Domokos (World Scientific, Singapore) 1984
[10] P.F. Gonzales-Diaz: *Il Nuovo Cimento B* **106** (1991) 335
[11] A. Guth: *Phys. Rev. D* **23** (1981) 347
[12] E.A. Matute: *Class. and Quantum Gravity* **14** (1997) 2771
[13] C. Kittel: *Thermal Physics* J. Willey & Sons, NY 1969