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THE ENERGY OF A DYONIC DILATON BLACK HOLE

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We calculate the energy distribution of a dyonic dilaton black hole by using the Tolman's energy-momentum pseudotensor. All the calculations are performed with approximation in quasi-Cartesian coordinates. The energy distribution of the dyonic dilaton black hole depends on the mass M, the electric charge Q_e, the magnetic charge Q_m and asymptotic value of the dilaton Φ_0. We get the same result as obtained by Y-Ching Yang, Ching-Tzung Yeh, Rue-Ron Hsu and Chin-Rong Lee by using the Einstein's prescription.

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1 Introduction

The energy-momentum localization has been a problematic issue since the outset of the theory of relativity. A large number of definitions of the gravitational energy have been given since now. Some of them are coordinate independent and other are coordinate-dependent. An adequate coordinate-independent prescription for energy-momentum localization for all the type of space-times has not given yet in General Relativity.

We remark that it is possible to evaluate the energy distribution and momentum by using various energy-momentum pseudotensors.

The physical interpretation of these nontensorial energy-momentum complexes have been questioned by a number of physicists, including Weyl, Pauli and Eddington.

There prevails suspicion that different energy-momentum pseudotensors could give different energy distributions in a given space-time. Virbhadra and his collaborators have considered many space-times and have shown that several energy-momentum pseudotensors give the same and acceptable result for a given space-time.

Many authors obtained dilaton black hole solutions and studied theirs properties [1]-[4]. Garfinkle, Horowitz and Strominger (GHS) [5] obtained a form of static spherical symmetric charged dilaton black hole solutions which exhibit several different properties compared to the Reissner-Nordström (RN) black holes. In their theory the gravity is coupled to the electromagnetic and dilaton fields and can be described by the four-dimensional effective string action.

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Chamorro and Virbhadra [6] obtained in the Einstein's prescription the energy of a charged dilaton black hole based on the GHS [5] solutions. They found that the energy distribution which has the expression $E(r) = M - \frac{Q^2}{2r}(1-\beta^2)$, depends on the mass M, electric charge Q_e and the coupling parameter β between the dilaton and the Maxwell fields.

For the value $\beta = 0$ they obtained the energy distribution in the Reissner-Nordström (RN) field. Also, only for $\beta = 1$ the energy is confined to its interior, and for all other values of β the energy is shared by the interior and exterior of the black holes. The total energy of the charged dilaton black hole is independent of β and is given by the mass parameter of the black hole.

With increasing the radial distance, $E(r)$ increases for $\beta = 0$ (RN metric) as well for $\beta < 1$, decreases for $\beta > 1$, and remains constant for $\beta = 1$.

S. S. Xulu [7] get the same energy distribution as Chamorro and Virbhadra [6] by using the Tolman's prescription. The energy distribution that is given by $E(r) = M - \frac{Q^2}{2r}(1-\beta^2)$ can be interpreted as the "effective gravitational mass" that a neutral test particle "feels" in the GHS space-time. Also, the "effective gravitational mass" becomes negative at radial distances less than $\frac{Q^2}{2M}(1-\beta^2)$.

Virbhadra and Parikh [8] investigated, in the Einstein's prescription [9], the energy of a static spherical symmetric charged dilaton black hole for $\beta = 1$ and found that the entire energy is confined to its interior with no energy shared by the exterior of the black hole. This result is similar to the case of the Schwarzschild black hole and unlike the RN black hole.

Cheng, Lin and Hsu (CLH) [10] using the standard spherical coordinate system which is more suitable for describing the structure of the charged dilaton black hole, obtained the more general solutions which are the dyonic dilaton black hole solutions. The GHS solutions can be obtained from the CLH solution as a special case when electric or magnetic charges are switched off.

I-Ching Yang, Ching-Tzung Yeh, Rue-Ron Hsu and Chin-Rong Lee [11] employing the Einstein's energy-momentum pseudotensor obtained that the energy distribution of a dyonic dilaton black hole depends on the mass M, electric charge Q_e, magnetic charge Q_m and asymptotic value of the dilaton Φ_0.

In this paper we compute the energy distribution of a dyonic dilaton black hole by using the Tolman's prescription [12] . We obtain the same result as obtained by I. Ching-Yang, Ching-Tzung Yeh, Rue-Ron Hsu and Chin-Rong Lee [11]. We also make a discussion of the results. We use the geometrized units $(G = 1, c = 1)$ and follow the convention that the Latin indices run from 0 to 3.

2 The energy distribution in the Tolman's prescription

The static, spherical symmetric dyonic dilaton black hole solutions are in terms of the standard spherical coordinate [10]

$$ds^2 = \Delta^2 dt^2 - \frac{\sigma^2}{\Delta^2} dr^2 - r^2 d\theta^2 - r^2 \sin\theta^2 d\varphi^2 \tag{1}$$

where

$$\sigma^2 = \frac{r^2}{r^2 + \lambda^2}$$

$$
\begin{aligned}
&\Delta^2 = 1 - \frac{2M}{r^2}\sqrt{r^2+\lambda^2} + \frac{\beta^2}{r^2} \\
&\lambda = \frac{1}{2M}(Q_e^2 e^{2\Phi_0} - Q_m^2 e^{-2\Phi_0}) \\
&\beta = Q_e^2 e^{2\Phi_0} + Q_m^2 e^{-2\Phi_0} \\
&e^{2\Phi} = e^{-2\Phi_0}(1 - \frac{2\lambda}{\sqrt{r^2+\lambda^2}+\lambda})
\end{aligned}
\tag{2}
$$

The only non-zero components of the electromagnetic field tensor are

$$F_{01} = \frac{Q_e}{r^2}e^{2\Phi} \tag{3}$$

and, respectively

$$F_{23} = \frac{Q_m}{r^2} \tag{4}$$

The properties of the dyonic dilaton black holes are characterized by the mass M, electric charge Q_e, magnetic charge Q_m and asymptotic value of the dilaton Φ_0. Their structures are similar to that of the RN [10] black holes.

The Tolman's energy-momentum pseudotensor [12] is given by

$$\Upsilon_i^k = \frac{1}{8\pi}U_i^{kl}{}_{,l} \tag{5}$$

where Υ_0^0 and Υ_α^0 are the energy and momentum components. We have

$$U_i^{kl} = \sqrt{-g}(-g^{pk}V_{ip}^l + \frac{1}{2}g_i^k V_{pm}^l) \tag{6}$$

with

$$V_{jk}^i = -\Gamma_{jk}^i + \frac{1}{2}g_j^i V_{mk}^m + \frac{1}{2}g_k^i V_{mj}^m. \tag{7}$$

The energy-momentum pseudotensor Υ_i^k also satisfies the local conservation laws

$$\frac{\partial \Upsilon_i^k}{\partial x^k} = 0. \tag{8}$$

The Tolman's energy-momentum pseudotensor gives the correct result if the calculations are carried out in quasi-Cartesian coordinates.

We transform the line element (1) to quasi-Cartesian coordinates t, x, y, z according to

$$
\begin{aligned}
u &= t - r \\
x &= r\sin\theta\cos\varphi \\
y &= r\sin\theta\sin\varphi \\
z &= r\cos\theta
\end{aligned}
\tag{9}
$$

and

$$r = (x^2 + y^2 + z^2)^{\frac{1}{2}} \tag{10}$$

The line element (1) becomes

$$ds^2 = \Delta^2 dt^2 - (dx^2 + dy^2 + dz^2) - \frac{\sigma^2/\Delta^2 - 1}{r^2}(xdx + ydy + zdz)^2 \tag{11}$$

The only required components of U_i^{kl} in the calculation of the energy are the following

$$\begin{aligned} U_0^{01} &= \frac{x\Pi}{r^2} \\ U_0^{02} &= \frac{y\Pi}{r^2} \\ U_0^{03} &= \frac{z\Pi}{r^2} \end{aligned} \tag{12}$$

In the relations (12) we denote by Π

$$\Pi = \sigma(1 - \frac{\Delta^2}{\sigma^2}) \tag{13}$$

The components of the pseudotensor U_i^{kl} are calculated with the program Maple GR Tensor II Release 1.50.

The energy and momentum in the Tolman's prescription are given by

$$P_i = \Upsilon_i^0 dx^1 dx^2 dx^3. \tag{14}$$

Using the Gauss's theorem we obtain

$$P_i = \frac{1}{8\pi} U_i^{0\alpha} n_\alpha dS \tag{15}$$

where $n_\alpha = (x/r, y/r, z/r)$ are the components of a normal vector over an infinitesimal surface element $dS = r^2 \sin\theta d\theta d\varphi$.

Using (2), (12), (14) and applying the Gauss's theorem we evaluate the integral over the surface of a sphere with radius r

$$E(r) = \frac{1}{8\pi} \oint \frac{\sigma}{r}(1 - \frac{\Delta^2}{\sigma^2}) r^2 \sin\theta d\theta d\varphi \tag{16}$$

We find that the energy within a sphere with radius r is given

$$E(r) = M + \frac{M\lambda^2}{r^2} - \frac{1}{2\sqrt{r^2 + \lambda^2}}[\frac{\beta\lambda^2}{r^2} + \lambda^2 + \beta]. \tag{17}$$

The energy distribution depends on the mass M, electric charge Q_e, magnetic charge Q_m and asymptotic value of the dilaton Φ_0.

The energy is shared both by the interior and by the exterior of the black hole.

3 Discussion

The subject of the localization of energy continues to be an open one. Bondi [13] sustained that a nonlocalizable form of energy is not admissible in relativity.

Other authors consider that because the energy-momentum pseudotensor are not tensorial objects and give results which are coordinate dependent they are not adequate for describing the gravitational field.

A favorable argument for using these objects to calculate the energy distribution is that some of them or, even, all of them [1]-[4] , [6]-[8], [11] give the same result for a given space-time if the calculations are carried out in the quasi-Cartesian coordinates.

The results obtained by Chamorro and Virbhadra [6], Xulu [7], Virbhadra and Parikh [8] and I-Ching Yang, Ching-Tzung Yeh, Rue-Ron Hsu and Chin-Rong Lee [11] lend support to the idea that the several energy-momentum pseudotensors can give the same result for a given space-time.

We obtain the same result as obtained by Y-Ching Yang, Ching-Tzung Yeh, Rue-Ron Hsu and Chin-Rong Lee [11]. This is an encouraging result and, also, it is one more proof that the Einstein's and Tolman's energy-momentum pseudotensors can give the same result for a static spherical symmetric solution.

The energy distribution depends on the mass M, electric charge Q_e, magnetic charge Q_m and asymptotic value of the dilaton Φ_0.

The result is different from the results obtained by Virbhadra and Parikh because we have a different choice of coordinate representations and the gravitational energy is a component of a cuadrivector and it will change at a changing of the coordinates.

The energy is shared both by the interior and by the exterior of the black hole.

The point $r = 0$ is a singularity for the energy distribution $E(r)$.

We obtain the same expression for the energy distribution as in the case of Schwarzschild black holes as Q_e, Q_m and Φ_0 vanish.

For $Q_e = 0$ or $Q_m = 0$ we find the case of the pure electric or pure magnetic charged black hole and the energy distribution is always positive except at the singular point $r = 0$.

If the dilaton field was suppressed $\Phi_0 = 0$, $\lambda = 0$ (or $Q_e = Q_m$) the dilaton gravity will reduce to the Einstein-Maxwell theory and the dilaton dyonic black hole solutions will become to be Reissner-Nordström solutions.

In the case of the ADM mass we obtain the same result as the result of Virbhadra

$$M_{ADM} = E(r)_{r\to\infty} = M. \tag{18}$$

From (18) we deduce that the ADM mass does not depend on the coordinate representation of the black hole.

Also, the concept of a black hole lends support to the idea that the gravitational energy is localizable.

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