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ON DECOHERENCE IN NEUTRON INTERFEROMETRY*

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Consistency concerning decoherence in neutron interferometry is achieved by using stochastic differential equations. In interferometry inhomogeneities of the density and/or of the surface roughness of a phase shifter are of great influence to coherent beam superposition. The interferometric process is described by Wigner's quasi-probability which is a solution of the diffusion equation.

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1 Introduction

A quantum-mechanical process such as a beam superposition in a neutron interferometer is not an isolated process. The system interacts with its environment [1]. The results show that the off-diagonal terms of the density matrix which are related to the interference terms become rapidly dephased at a rate governed by the separation of the coherent states.

The influence of particle density inhomogeneities and of a phase shifter's surface roughness in an interferometer result in a coherence loss of the beams' superposition. In this paper this influence is investigated in detail. It is shown that the process can be adapted by the Wigner formalism, which can be interpreted by a diffusion process in ordinary space.

2 Stochastic differential equations

To gain some insight into the physics described by dephasing, the one-dimensional Fokker-Planck equation for the distribution function $P(x,t)$ is a useful starting point [2]:

$$\frac{\delta}{\delta t}P(x,t) = [-\frac{\delta}{\delta x}A(x) + \frac{1}{2}\frac{\delta^2}{\delta x^2}D(x)]P(x,t). \tag{1}$$

The motion of the mean is governed by $A(x)$. This term is called the *drift term*. $D(x)$ acts as a *source of fluctuations*. If $A = 0$ and D is constant (diffusion constant) the diffusion equation results:

$$\frac{\delta}{\delta t}P(x,t) = \frac{1}{2}D\frac{\delta^2}{\delta x^2}P(x,t). \tag{2}$$

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This equation's solution is well known:

$$P(x,t) = \int P_1(x,t,x_0,0)\, P(x_0,0)\, dx_0. \tag{3}$$

Here $P_1(x,t,x_0,0)$ is the conditional probability (Green's function)

$$P_1(x,t,x_0,0) = \frac{1}{\sqrt{2\pi Dt}} \exp[-\frac{(x-x_0)^2}{2Dt}] \tag{4}$$

which is a function of the space variable x and time t (Brownian Motion). In Eq. (3) $P(x_0,0)$ denotes the initial condition. Both functions P_1 and P are solutions of Eq. (2), naturally.

3 Application to interferometry

In the case of neutron interferometry the superposition (index s) of wavepackets are produced by means of a phase shifter in one of the arms of the interferometer:

$$\psi_s(x,\Delta_0) = \psi(x) + \psi(x+\Delta_0). \tag{5}$$

The phase shift Δ_0 can be expressed by

$$\Delta_0 = \frac{2\pi}{k_0^2} N b_c D_P = (1-n_0) D_P. \tag{6}$$

N is the particle density, b_c the coherent scattering length, D_P the thickness of the phase shifter, k_0 the mean wave number of the wave packet and n_0 the corresponding mean refractive index of the material of the phase shifter.

It can be shown that the related Wigner function W_s of superposition $[3,4]$

$$W_s(k,x,\Delta_0) = \frac{1}{2\pi} \int \psi_s^*(x+x'/2,\Delta_0)\psi_s(x-x'/2,\Delta_0)\, \exp[ikx'] dx' \tag{7}$$

can be expressed as [5]

$$W_s(k,x,\Delta_0) = W(k,x) + W(k,x+\Delta_0) + 2W(k,x+\Delta_0/2)\, \cos(k\Delta_0). \tag{8}$$

The Wigner function of a single wave packet is given by

$$W(k,x) = A(k)\, \exp[-2(\delta k)^2 x^2]\, ,\; A(k) = \frac{1}{\pi} \exp[-\frac{(k-k_0)^2}{2(\delta k)^2}]. \tag{9}$$

It is seen that the Wigner function of superposition Eq. (8) is a sum of three terms, the first two describing the two separated wavepackets plus an interference term (off-diagonal terms of the density operator) containing information about coherence properties.

Now if $W_s(k,x,\Delta_0)$ is used as an initial condition $P(x_0,0)$ as in Eq. (3) the following expression can be recorded:

$$\bar{W}_s(k,x,\bar{\Delta}_0) = \int P_1(\bar{\Delta}_0,t,\Delta_0,0)\, W_s(k,x,\Delta_0)\, d\Delta_0 \tag{10}$$

with the following conditional probability in this case:

$$P_1(\bar{\Delta}_0, t, \Delta_0, 0) = \frac{1}{\sqrt{2\pi Dt}} \exp[-\frac{(\Delta_0 - \bar{\Delta}_0)^2}{2Dt}]. \tag{11}$$

$\bar{W}_s$ is the mean value of W_s related to a mean phase shifting value $\bar{\Delta}_0$. Equation (10) can be calculated analytically using Eqs. (8) and (11) and one obtains:

$$\bar{W}_s(k, x, \bar{\Delta}_0) = W(k, x) + \frac{A(k)}{\sqrt{1+\epsilon}} \exp\left[-2(\delta k)^2 \frac{(x+\bar{\Delta}_0)^2}{1+\epsilon}\right] +$$
$$+2\frac{A(k)}{\sqrt{1+\frac{\epsilon}{4}}} \cos\left(k\frac{\frac{\epsilon x}{2} - \bar{\Delta}_0}{1+\frac{\epsilon}{4}}\right) \exp\left[-2(\delta k)^2 \frac{(x+\frac{\bar{\Delta}_0}{2})^2}{1+\frac{\epsilon}{4}} - \frac{k^2}{2(\delta k)^2}\frac{\frac{\epsilon}{4}}{1+\frac{\epsilon}{4}}\right]. \tag{12}$$

Here the parameter ϵ contains the diffusion coefficient D and the time t, which is the mean transmission time of the beam through the phase shifter:

$$\epsilon = 4(\delta k)^2 Dt. \tag{13}$$

The cosine - term (last term on the right hand side) of Eq. (12) is strongly affected by the decoherence parameter ϵ.

4 Discussion

If we consider Eq. (11) the exponent can be expressed by

$$\frac{(\frac{\Delta_0}{\bar{\Delta}_0} - 1)^2}{2\frac{Dt}{\bar{\Delta}_0^2}} = \frac{(\frac{N}{\bar{N}} - 1)^2}{2(\frac{\delta N}{\bar{N}})^2} \tag{14}$$

taking into account density fluctuations $\delta N/\bar{N}$, for example, according to Eq. (6). The denominators have to be equal and one reaches:

$$(Dt) = \bar{\Delta}_0^2 (\frac{\delta N}{\bar{N}})^2. \tag{15}$$

If the surface roughness of the phase shifter is additionally taken into account the parameter ϵ is $[6, 7]$

$$\epsilon = 4(\delta k)^2 \bar{\Delta}_0^2 [(\frac{\delta N}{\bar{N}})^2 + (\frac{\delta D_P}{\bar{D}_P})^2]. \tag{16}$$

Equations (10), (11), (2), (3) and (4) show that the Wigner function $\bar{W}_s$ of superposition meets the following differential equation of decoherence:

$$\frac{\delta}{\delta t}[\bar{W}_s(k, x, \bar{\Delta}_0)] = \frac{1}{2} D \frac{\delta^2}{\delta \bar{\Delta}_0^2} [\bar{W}_s(k, x, \bar{\Delta}_0)]. \tag{17}$$

This equation can immediately be proven by inserting Eqs. (12) and (13) directly in Eq. (17). Equation (17) is a diffusion equation of the Wigner function [8] and in this case *diffusion* appears as a process in ordinary space. The damping of oscillations in the Wigner function $\bar{W}_s$ of

Eq. (12) is a general consequence of the non-unitarity dynamics caused in this case by density inhomogeneities and surface roughness of a phase shifter in an interferometer.

It can be concluded that decoherence in neutron interferometry can be formally interpreted very simply by a *diffusion* process. But it must be emphasized that an interpretation of these dynamics as a result of random kicks (as in classical theory) would mistakenly intermingle classical and quantum concepts.

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