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# DECOHERENCE DUE TO STATISTICALLY DISTRIBUTED JUMP-LIKE EVENTS*

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We investigate an interacting quantum system which is additionally subjected to jump-like events occurring at time instants that are distributed according to a given statistics. Assuming that the latter can be descibed by a stationary renewal process, we consider a Poissonian and a regular distribution as well as a super-Poissonian one. To apply our method we study a two-level system being resonantly driven by a classical field and undergoing jump-like phase decoherence (e.g. caused by quantum-nondemolition measurements of the level population). We obtain analytical results for the steady state and for the quantum Zeno dynamics that illustrate the influence of the statistics. It turns out that a Poissonian distribution of the dephasing events is still half as effective as a regular one in increasing the lifetime of the initial state.

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## 1 Introduction

We study the problem as to how the statistical distribution in time of certain quasi-instanteous, i.e. jump-like changes of the density operator of a quantum system affects the evolution of this system. The motivation arises from the renewed attention [1] devoted to the theoretical treatment and possible experimental realization of the quantum Zeno effect [2] which has also been proposed as a means for error prevention in quantum computing [3]. The quantum Zeno effect rests on the suppression of the unitary Hamiltonian evolution of a quantum system due to intermittent measurements in rapid succession. When, in the limit of continuous measurements, the evolution is completely inhibited, the system is said to be frozen in its initial state. According to standard quantum measurement theory each measurement results in an instantaneous reduction of the wave function, or in a projection of the state vector, respectively, onto an eigenstate corresponding to the particular detected eigenvalue of the observable being measured. Non-selective measurements destroy the quantum-mechanical phase relations between different eigenstates and reduce the state of the system to a statistical mixture where the non-diagonal elements of

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the corresponding density matrix vanish. Therefore all jump-like dephasing events, i.e. all interaction processes that provide for a rapid quantum-mechanical phase destruction in a short time interval but leave the diagonal elements of the density matrix unchanged, cause the same dynamical effect on the evolution of the system like genuine quantum-nondemolition measurements. The first experimental manifestation of the quantum Zeno effect was the inhibition of radiation-induced atomic transitions between two energy levels in a three-level atom due to perturbations caused by periodically repeated short radiation pulses suitable for detecting the energy state [4].

In the standard treatment the quantum Zeno dynamics is studied for measurements being periodically repeated, i.e. regularly distributed in time, which corresponds to the experimental situation investigated in [4]. However, one may also think of measurements or jump-like dephasing events, respectively, which are not regularly distributed but obey a different statistics. For instance, as we will point out, a Poissonian distribution is implicitly present in all situations where the action of the dephasing events on the system can be described by a Markovian master equation for its density operator. We mention that such a master-equation description has been applied in a recent proposal [5] for observing the quantum Zeno effect in Bragg scattering of atomic waves at a standing wave laser field by adding random fluctuations to the spatial phase of the laser fields in order to mimic the effect of a non-selective measurement process which suppresses the coherent oscillations of the population between the two scattered atomic beams. In this contribution we first supply the basic theory necessary to treat the influence of the statistics of jump-like events. In the second part we study a resonantly driven two-level system subjected to statistically distributed jump-like dephasing events and investigate the stationary state as well as the quantum Zeno dynamics.

## 2 General theory

Let us consider a quantum system the evolution of which is interrupted by identical jump-like events occurring at statistically distributed time instants $t_1, t_2, \ldots, t_k, \ldots$. Each of these events is represented by a superoperator $M$ describing sudden jump-like changes of the density operator $\rho$ of the system according to

$$\rho(t_k + \Delta t) = M\rho(t_k), \tag{1}$$

where $\Delta t$ is supposed to be short in comparison to all time scales relevant for the evolution of $\rho$. We assume that in the time intervals between the jumps the evolution of the system is a Markovian one. This means that in the most general case it can be described by the equation

$$\dot{\rho} = \frac{1}{i\hbar}[H, \rho] + L\rho \equiv \mathcal{L}\rho \tag{2}$$

which is composed of a Hamiltonian part and an additional contribution resulting from the coupling of the system to external reservoirs and being represented by a Lindblad operator $L$ accounting for irreversible dynamics. The superoperator $\mathcal{L}$ formally combines these two contributions and allows the Markovian evolution in the absence of jump-like events to be written as $\rho(t) = \mathrm{e}^{\mathcal{L}t}\rho(0)$. To take into account the effects of both the random instantaneous changes given by Eq. (1) and the smooth Markovian evolution according to Eq. (2), we proceed along the lines developed in [6], with the only difference that a Hamiltonian part of the evolution is included. We describe the distribution of the random jump-like events by a stationary stochastic point process which is completely characterized by the whole set of exclusive probability densities $Q_{k+2}(t_0, t_1, \ldots t_{k+1})$ for the occurrence of exactly $k+2$ $(k = 0, 1 \ldots)$ jump-like events at the time instants $t_0, t_1, \ldots t_{k+1}$. The evolution equation for the density operator $\rho$ takes the form

$$\rho(t) = \int_t^\infty dt_1 \int_{-\infty}^0 dt_0 Q_2(t_0, t_1) e^{\mathcal{L}t} \rho(0) + \sum_{k=1}^{\infty} \int_t^\infty dt_{k+1} \int_0^t dt_k \int_0^{t_k} dt_{k-1} \ldots$$

$$\ldots \int_0^{t_2} dt_1 \int_{-\infty}^0 dt_0 Q_{k+2} \left(t_0, t_1, \ldots t_{k+1}\right) e^{\mathcal{L}(t-t_k)} M e^{\mathcal{L}(t_k - t_{k-1})} \ldots M e^{\mathcal{L}t_1} \rho(0) \,. \quad (3)$$

It can be shown that the resulting dynamics is non-Markovian except for the case of a Poissonian distribution of the jump-like events [6]. In the following we specify to the so-called renewal processes for which the functions $Q_{k+2}\left(t_0, t_1, \ldots t_{k+1}\right)$ factorize in the form

$$Q_{k+2}(t_0, t_1 \ldots t_{k+1}) = r \prod_{i=1}^{k+1} f(t_i - t_{i-1}), \tag{4}$$

where the rate $r$, i.e. the number of events per unit time, is given by $r = \left[\int_0^\infty t f(t) dt\right]^{-1}$. In this case the statistics of the jump-like events is uniquely defined by the function $f(t)$ which has the meaning of the waiting-time distribution between consecutive events.

A *regular* distribution of the jump-like events is characterized by the waiting-time distribution $f(t) = \delta(t - T)$ with the time interval $T$ separating consecutive events, the rate being $r = T^{-1}$. In analogy to [7,8] the evolution equation of the density operator is found to be

$$\rho(t) = \frac{1}{T} \left[ \int_0^{xT} dt' \mathrm{e}^{\mathcal{L}(xT - t')} M (\mathrm{e}^{\mathcal{L}T} M)^{\left[\frac{t}{T}\right]} \mathrm{e}^{\mathcal{L}t'} + \int_{xT}^{T} dt' \mathrm{e}^{\mathcal{L}(xT - t')} (\mathrm{e}^{\mathcal{L}T} M)^{\left[\frac{t}{T}\right]} \mathrm{e}^{\mathcal{L}t'} \right] \rho(0) \tag{5}$$

with $x = t/T - [t/T]$, where $[\ldots]$ stands for the integer part. Eq. (5) refers to the whole quantum mechanical ensemble where the initial time $t = 0$ has not been specified with respect to its separation in time from the next jump-like event. We mention that due to the periodicity it can be exactly proven that in the stationary state the ensemble average is equal to the time average [8].

When the jump-like events are statistically independent, their statistics is a *Poissonian* one described by Eq. (4) with the waiting-time distribution $f(t) = r\mathrm{e}^{-rt}$. Using the procedure outlined in [6] we find that in the Poissonian case Eq. (3) is equivalent to the Markovian master equation

$$\dot{\rho} = r(M - 1)\rho + \mathcal{L}\rho. \tag{6}$$

Hence for a Poissonian statistics the effect of the jump-like events can be considered separately from the uninterrupted evolution between the jumps. This is not true for any other statistics since for any deviation from a Poissonian distribution there are correlations between the events which induce a memory effect resulting in a non-Markovian evolution of the system. In the frame of the renewal-process assumption (4) we finally can also model a *super-Poissonian* distribution of events exhibiting bunching. For this purpose we assume their waiting time to be distributed according to the specific Ansatz [6] $f(t) = \lambda_1\lambda_2(e^{-\lambda_1 t} + \alpha e^{-\lambda_2 t})/(\lambda_2 + \alpha\lambda_1)$, where $\alpha, \lambda_1$ and $\lambda_2$ are real positive parameters.

## 3 Phase decoherence in a driven two-level system

In the following we apply Eqs. (3) - (6) to investigate jump-like phase decoherence in a two-level system with ground state $|1\rangle$ and excited state $|2\rangle$ that is driven by a resonant classical field with a Rabi frequeny $\Omega$ which is chosen to be real for simplicity. To account for complete dephasing in a single jump-like event, we introduce a superoperator $M$ which destroys the nondiagonal density-matrix elements, i.e. we require that

$$M\rho = \rho_{11}|1\rangle\langle 1| + \rho_{22}|2\rangle\langle 2|. \tag{7}$$

Eq. (7) can be interpreted as the change of the density operator of the system due to a non-selective quantum-nondemolition measurement of its energy state. When the evolution of the system in the time intervals betweeen the dephasing events is written in the interaction picture, we obtain from Eq. (2) the expression [9]

$$\mathcal{L}\rho = -i\frac{\Omega}{2}[(\sigma_+ + \sigma_-), \rho] - \frac{\gamma}{2}(\sigma_+\sigma_-\rho - 2\sigma_-\rho\sigma_+ + \rho\sigma_+\sigma_-). \tag{8}$$

Here $\gamma$ is the spontaneous decay rate of the upper level and we have introduced the spin-flip operators $\sigma_+ = |2\rangle\langle 1|$ and $\sigma_- = |1\rangle\langle 2|$.

First we consider the stationary density operator $\overline{\rho} = \lim_{t\to\infty}\rho(t)$. In particular, we are interested in the steady state of the population inversion $w = \rho_{22} - \rho_{11}$. Making use of the methods developed in [6] we can derive a general expression which is valid for an arbitrary waiting-time distribution and reads

$$\overline{w} = \overline{\rho}_{22} - \overline{\rho}_{11} = -\frac{1}{2\Omega^2 + \gamma^2}\left[\gamma^2 + \frac{4r\Omega^2}{\gamma}\,\frac{(16\Omega^2 - \gamma^2)F^2 + \gamma^2 G^2}{(2\Omega^2 + \gamma^2)(F + G)}\right], \tag{9}$$

where the abbreviations

$$F = \frac{\gamma}{4\sqrt{\Omega^2 - \gamma^2/16}}\int_0^\infty dt\, f(t)\,\mathrm{e}^{-\frac{3}{4}\gamma t}\sin\left(t\sqrt{\Omega^2 - \gamma^2/16}\right) \tag{10}$$

and

$$G = 1 - \int_0^\infty dt\, f(t)\,\mathrm{e}^{-\frac{3}{4}\gamma t}\cos\left(t\sqrt{\Omega^2 - \gamma^2/16}\right) \tag{11}$$

have been used. Inserting the waiting-time distribution $f(t) = \delta(t - T)$ which describes a regular distribution of events and taking into account that $T = r^{-1}$ we obtain from Eqs. (9) - (11) an approximate analytical result for $\overline{w}^{reg}$ which is valid when $r \gg \gamma$. On the other hand, for a Poissonian waiting-time distribution $f(t) = r\mathrm{e}^{-rt}$ Eqs. (9) - (11) can be transformed to yield an exact result for $\overline{w}^{poiss}$. Comparing the expressions

$$\overline{w}^{poiss} = -\frac{\gamma(r+\frac{\gamma}{2})}{\gamma(r+\frac{\gamma}{2})+\Omega^2} \qquad \text{and} \qquad \overline{w}^{reg} \approx -\frac{2\gamma r}{2\gamma r+\Omega^2} \tag{12}$$

we find that for $r \gg \gamma$ but $\gamma \neq 0$ a regular distribution of phase-destroying events is just twice as effective in reducing the steady-state population inversion as a Poissonian distribution with the same rate. Conversely, numerical results show that a super-Poissonian distribution of events exhibiting bunching is less effective than a Poissonian distribution. Finally we remark that for $\gamma = 0$, i.e. without level damping, the stationary state $\overline{w} = 0$ is reached for any waiting-time distribution $f(t)$ between the phase-destroying events. This expresses the fact that due to repeated interruption of the coherent Hamiltonian evolution all information about the initial state is lost and after a sufficiently long time the ideal two-level-system system will be found with equal probability in the upper and the lower level.

Now we turn to the quantum Zeno dynamics. Supposing that $\gamma = 0$ we investigate the evolution in time of a resonantly driven ideal two-level system under the influence of repeated quantum-nondemolition measurements of its energy state or of repeated phase-destroying events, respectively, which are described by Eq. (7). For simplicity we assume that at the initial time $t = 0$ the system is in its excited state, the initial population inversion therefore being $w(0) = 1$. For a Poissonian distribution of the measuring events Eq. (6) yields the result

$$w^{poiss} = \mathrm{e}^{-\frac{r}{2}t}\left(\cos\left(t\sqrt{\Omega^2-\frac{r^2}{4}}\right)+\frac{r}{2}\frac{\sin\left(t\sqrt{\Omega^2-\frac{r^2}{4}}\right)}{\sqrt{\Omega^2-\frac{r^2}{4}}}\right), \tag{13}$$

where Eq. (8) with $\gamma = 0$ and Eq. (7) have been used. The time dependence of $w$ exhibits an oscillatory behaviour as long as $r < 2\Omega$. If we consider a regular distribution of the measuring events we find from Eq. (5) that for $t \geq T$

$$\begin{aligned} w^{reg} = \frac{[\cos(\Omega T)]^{[\frac{t}{T}]-1}}{2}\Bigg[&\cos(x\Omega T)\left(\cos(\Omega T)\frac{\sin(\Omega T)}{\Omega T}\right) \\ &+(x-1)\sin(x\Omega T)\sin(\Omega T)\Bigg] \end{aligned} \tag{14}$$

with $x = t/T - [t/T]$. We emphasize again that in Eq. (14) the time instant of the occurrence of the first measuring event has not been fixed with respect to the initial time $t = 0$. Doing this we would instead arrive at a stroboscopic result, e.g. $w^{strob}(kT) = \cos^k(\Omega T)$ with $k = 1, 2, \ldots$.

By investigating the population inversion at a constant time for more and more frequent phase destruction we recover the quantum Zeno effect when $r \to \infty$ or $T = r^{-1} \to 0$, respectively. In particular, if we expand Eqs. (13) and (14) for $r \gg \Omega$ we find

$$w^{poiss} \approx \exp\left(-\frac{\Omega^2}{r}t\right) \qquad \text{and} \qquad w^{reg} \approx \exp\left(-\frac{\Omega^2}{2r}t\right). \tag{15}$$

We note that the difference between Eq. (14) and a stroboscopic result can be neglected in the limit $\Omega T \ll 1$. Obviously, $\lim_{r\to\infty} w = 1$ which expresses just the quantum Zeno effect, i.e.

the freezing of the two-level system in its initial state. From Eq. (15) we conclude that a mean lifetime [10] $\tau$ of the system in its initial state may be introduced which is given by $\tau^{poiss} = r/\Omega^2$ and $\tau^{reg} = 2r/\Omega^2$, i.e. a Poissonian distribution of phase-destroying events is half as effective as a regular one in suppressing the Hamiltonian evolution of the quantum system. The enlarged effectivity of a regular distribution can be qualitatively explained in physical terms. Starting from zero after a previous phase-destroying event, a certain time interval is needed until the nondiagonal density-matrix element takes on its maximum absolute value in the cycle of field-induced Rabi oscillations. Therefore a regular distribution of events could enhance the overall effect of phase destruction by excluding events that follow immediately one after the other thus being ineffective.

To summarize, we have investigated the influence of statistics on decoherence and quantum Zeno dynamics. For a resonantly driven two-level system it turned out that a Poissonian statistics of dephasing events, which can be treated very easily with the help of a Markovian master equation, causes an effect which is still half as large as the effect of a regular distribution with the same rate.

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**References**

[1] V. Frerichs, A. Schenzle: *Phys. Rev. A* **44** (1991) 1962; W. L. Power, P. L. Knight: *Phys. Rev. A* **53** (1996) 1052; M. Kitano: *Phys. Rev. A* **56** (1997) 1138; L. S. Schulman: *Phys. Rev. A* **57** (1998) 1509; B. Nagels, L. J. F. Hermans, P. L. Chapovsky: *Phys. Rev. Lett* **79** (1997) 3097

[2] B. Misra, E. C. G. Sudarshan: *J. Math. Phys. (N. Y.)* **18** (1976) 756

[3] L.-M. Duan, G.-C. Guo: *Phys. Rev. A* **57** (1998) 2399

[4] W. M. Itano, D. J. Heinzen, J. J. Bollinger, D. J. Wineland: *Phys. Rev. A* **41** (1990) 2295

[5] S. Dyrting, M. J. Gagen: *Phys. Rev. A* **56** (1997) 1453

[6] U. Herzog: *Phys. Rev. A* **52** (1995) 602; U. Herzog: *acta physica slovaca* **45** (1995) 367

[7] J. D. Cresser: *Phys. Rev. A* **46** (1992) 5913

[8] U. Herzog, J. A. Bergou: *Phys. Rev. A* **54** (1996) 5334; U. Herzog, J. A. Bergou: *acta physica slovaca* **46** (1996) 399

[9] D. F. Walls, G. J. Milburn: *Quantum Optics* (Springer-Verlag Berlin, 1994)

[10] V. B. Braginsky, F. Ya. Khalili: *Quantum Measurement* (Cambridge University Press, 1992)