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# INTRINSIC DECOHERENCE IN NON-DEGENERATE TWO-QUANTA JAYNES-CUMMINGS MODEL

**A.-S. F. Obada**
*Department of Mathematics, Faculty of Science, Al-Azhar University, Nasr City, Cairo, Egypt*

**A. M. Abdel-Hafez**
*Department of Mathematics, Faculty of Science, El-Minia University, El-Minia, Egypt*

**H. A. Hessian**[1]
*Department of Mathematics, Faculty of Science, Assiut University, Assiut, Egypt*

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The non-degenerate two-quanta Jaynes-Cummings model governed by Milburn equation is studied. This models the decoherence of a quantum system as it evolves through intrinsic mechanisms beyond conventional quantum mechanics governed by the Schrödinger equation. The exact solution is given. Then it is applied to investigate the effects of the intrinsic decoherence on nonclassical effects (population inversion, oscillations of the number distribution, photon antibunching effect, the cross correlation and squeezing of the cavity field). We show how the intrinsic decoherence suppresses the nonclassical behaviour in this model, for the off-resonant case.

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## 1 Introduction

Due to the recent advances in cooling and trapping of ions [1] the motion of the center-of-mass (c.m.) of trapped ions has to be dealt with quantum mechanically. Laser irradiation [2–6] is used to control this motion coherently by coupling the ion's external and internal degrees of freedom. To describe a two-level ion undergoing quantized vibrational motion within a harmonic trapping potential and interacting with a classical light field many models have been constructed [2,3,7,8]. The dynamics of a trapped ion as it has been pointed out can be described by a Hamiltonian similar to a Jaynes-Cummings model [9] or its generalizations under certain regimes [4,5,10–12]. Various aspects of the dynamics of trapped ions have been studied within the framework of these Jaynes-Cummings-like models. For example, quantum nondemolition measurement of vibrational quanta of trapped ions has been analyzed theoretically [13] and several schemes have

[1] E-mail address: ammare@aun.eun.eg

been proposed [14] for the reconstruction of quantum-mechanical vibrational states of a trapped ion.

The Wigner function of nonclassical states of the vibrational mode of a trapped ion [15] has been reconstructed experimentally by applying one of these schemes. It is to be noted that ion trap experiments suffer from decoherence due to classical noise in the laser beams and trapping potential. Such effect has been seen in recent experiments [16]. This kind of decoherence may be described using the intrinsic decoherence models [17–20].

The intrinsic decoherence approach has been proposed and investigated in the frame-work of several models [21–25]. In particular, Milburn [17, 26] proposed a simple intrinsic decoherence model based on an assumption that on sufficiently short time steps the system evolves in a stochastic sequence of identical unitary transformations. This assumption modifies the von Neumann equation for the density operator of a quantum system through a simple modification of the usual Schrödinger evolution equation. The off-diagonal elements of the density operator in Milburn's model are intrinsically suppressed in the energy eigenstate basis, thereby intrinsic decoherence is realized without the usual dissipation associated with the normal decay. The decay is entirely of phase dependence only. Free evolution of a given quantum system has been discussed early [17] but investigations of interacting sub-systems followed [18–20]. The later were concerned with the Jaynes-Cummings model either with one-photon or multiphoton transitions. The Jaynes-Cummings model (JCM) [9] in quantum optics describes many pure quantum phenomena, such as collapses and revivals of the atomic inversion and oscillations of photon number distribution. It has been generally accepted that these nonclassical effects originate to form quantum coherences between the amplitudes. Therefore, it is an interesting topic to investigate the effects of the intrinsic decoherence on the nonclassical effects in the JCM, when we have two modes of the interacting field affecting the interaction, and hence nondegenerate two-quanta interaction [27, 28]. Such a model is discussed in this article when it is governed by the Milburn equation. It is shown later that the intrinsic decoherence in the particle (atom or trapped ion)-field interaction modifies the time evolution of the population inversion and oscillations of the number distribution of the quanta.

Decoherence due to normal decay is often said to be the most efficient effect in physics, to a point where observation comes too late after the effect has reached completion [29]. The effect in action has been observed in quantum optics where the decoherence phenomena transforming a Schrödinger-cat into a statistical mixture was observed while unfolding [30]. It is well known that the Jaynes-Cummings model (JCM) in quantum optics [31] and cavity QED with cold trapped ions [11] can describe many pure quantum phenomena, called nonclassical effects, such as collapses and revivals of population inversion, oscillations of number distributions for quanta and squeezing of the cavity field.

The paper is organized as follows: In Section II, we present the exact solution of the Milburn equation for the model and give the explicit expression of this solution in the two-dimensional basis of the particle. In Section III, we examine the effects of the intrinsic decoherence on the population inversion. Section IV is devoted to investigation of the influence of the intrinsic decoherence on photon statistics, the cross correlation between modes 1 and 2, amplitude-squared squeezing and two mode normal squeezing in this model. Concluding remarks are provided in the last section.

## 2 Exact solution of the Milburn equation

Consider a quantum system described by the evolution operator $\hat{\rho}$. Dynamics of the system is governed by the evolution operator $\hat{U}(t) = \exp[-\frac{i}{\hbar}t\hat{H}]$, where $\hat{H}$ is the Hamiltonian of the system. In standard quantum mechanics, the change in a state of the system in a time interval $(t, t+\tau)$ is given by

$$\hat{\rho}(t+\tau) = \hat{U}(\tau)\hat{\rho}(t)\hat{U}^{\dagger}(\tau) = \exp[-\frac{i}{\hbar}\tau\hat{H}]\hat{\rho}(t)\exp[\frac{i}{\hbar}\tau\hat{H}], \tag{1}$$

which is valid for any arbitrary value of $\tau$. Milburn [17] replaced the above paradigm with three new postulates:

**(1)** On a sufficiently small time scale the change in the state of the system is stochastic. The probability that the state of the system is changed is $p(\tau)$, which reflects quantum jumps in the state of the system.

**(2)** Assuming that the state of the system undergoes some changes, then the density operator changes according to the relation

$$\hat{\rho}(t+\tau) = \exp[-\frac{i}{\hbar}\hat{\theta}(\tau)\hat{H}]\hat{\rho}(t)\exp[\frac{i}{\hbar}\hat{\theta}(\tau)\hat{H}] \tag{2}$$

with $\theta(\tau)$ some function of $\tau$. In standard quantum mechanics $p(\tau)$ takes the value 1 while $\theta(\tau) = \tau$. Milburn's proposal: it is only required that $p(\tau) \to 1$ and $\theta(\tau) \to \tau$ for values of $\tau$ which are sufficiently large.

**(3)** In Milburn's model it is postulated that

$$\lim_{\tau \to 0} \theta(\tau) = \theta_0, \tag{3}$$

which effectively introduces a minimum time step in the universe [32], whose inverse is equal to the mean frequency of the unitary step, $\gamma = \frac{1}{\theta_0}$. Invoking the assumption that on a very short time scale the probability that the system evolves is $p(\tau) = \gamma\tau$, then the rate of change of $\hat{\rho}(t)$ in Milburn's model satisfies the following equation:

$$\frac{d}{dt}\hat{\rho}(t) = \gamma\left\{\exp[-\frac{i}{\hbar\gamma}\hat{H}]\hat{\rho}(t)\exp[\frac{i}{\hbar\gamma}\hat{H}] - \hat{\rho}(t)\right\}. \tag{4}$$

Obviously, the generalized Eq.(4) alters the Schrödinger dynamics. It reduces to the ordinary von Neumann equation for the density operator in the limit $\gamma \to +\infty$. Expanding Eq.(4) to first order in $\gamma^{-1}$, the following dynamical equation is obtained:

$$\frac{d}{dt}\hat{\rho}(t) = -\frac{i}{\hbar}[\hat{H}, \hat{\rho}] - \frac{1}{2\hbar^2\gamma}[\hat{H}, [\hat{H}, \hat{\rho}]], \tag{5}$$

the first order correction leads to diagonalization of the density operator in the energy eigenstate basis. Further, it introduces diffusion in variables that don't commute with the Hamiltonian. Earlier studies of this equation include studies of a harmonic oscillator and a precessing spin system [17,26], a simple JCM [18,19] and a multiphoton JCM [20] in the resonant case. Now, we shall consider the exact solution of the Milburn equation (Eq. 5) for a non-degenerate two-quanta off-resonant Jaynes-Cummings model. We write the Hamiltonian for this system as [27, 28].

$$\begin{aligned}\hat{H} &= \frac{\omega_0}{2}\hbar\hat{\sigma}_z + \hbar\sum_{j=1}^{2}\omega_j\hat{a}_j^\dagger\hat{a}_j + \lambda\hbar(\hat{a}_1^\dagger\hat{a}_2^\dagger\hat{\sigma}_- + \hat{a}_1\hat{a}_2\hat{\sigma}_+) \\ &= \omega_1\hbar[\hat{n}_1 + \frac{1}{2}(\hat{\sigma}_z + I)] + \omega_2\hbar[\hat{n}_2 + \frac{1}{2}(\hat{\sigma}_z + I)] - \frac{\hbar}{2}(\omega_1 + \omega_2)I \\ &+ \frac{\Delta}{2}\hbar\hat{\sigma}_z + \lambda\hbar(\hat{a}_1^\dagger\hat{a}_2^\dagger\hat{\sigma}_- + \hat{a}_1\hat{a}_2\hat{\sigma}_+) \end{aligned} \tag{6}$$

with the detuning parameter $\Delta = \omega_0 - \omega_1 - \omega_2$, where $\hat{a}_j$ $(\hat{a}_j^\dagger)$, is the annihilation (creation) operator of $j$th mode, $\lambda$ is the particle-field coupling constant, $\omega_1$ and $\omega_2$ are the frequencies for the two modes of the field, $\omega_0$ is the transition frequency of the particle (atom or trapped ion), $\hat{\sigma}_z$ is the atomic-inversion operator, and $\hat{\sigma}_\pm$ are the atomic "spin flip" operator which satisfy the relation $[\hat{\sigma}_+, \hat{\sigma}_-] = \hat{\sigma}_z$ and $[\hat{\sigma}_z, \hat{\sigma}_\pm] = \pm 2\hat{\sigma}_\pm$.

This Hamiltonian can be generated from a Raman coupling for an effective 3-level ion in a $\Lambda$-configuration confined with a 2-dimensional harmonic trap as described in [12]. An ion confined in an electro-magnetic trap can be regarded as a particle with quantized center-of-mass (c.m.) motion moving in a harmonic potential. Classical laser driving field changes the external states of the ion motion due to exciting or deexciting the internal atomic states of the trapped ion. This model (Eq. 6) can be obtained in the Lamb-Dicke approximation and in the limit of suitable trap anisotropy and specific sideband detunings of the laser. In this case the $\hat{a}$'s describe vibrational modes and $\hat{\sigma}$'s describe the ion internal states. As the coupling between the vibrational modes and the external environments is extremely weak, dissipative effects which are inevitable from cavity damping in the optical regime can be significantly suppressed for the ion motion. For simplicity, we take $\hbar = 1$ and neglect the constant $\frac{1}{2}(\omega_1 + \omega_2)I$ which amounts to a constant shift to the energy of the system, Eq.(6) becomes:

$$\hat{H} = \omega_1[\hat{n}_1 + \frac{1}{2}(\hat{\sigma}_z + I)] + \omega_2[\hat{n}_2 + \frac{1}{2}(\hat{\sigma}_z + I)] + \frac{\Delta}{2}\hat{\sigma}_z + \lambda(\hat{a}_1^\dagger\hat{a}_2^\dagger\hat{\sigma}_- + \hat{a}_1\hat{a}_2\hat{\sigma}_+). \tag{7}$$

Now, we express the solution for the density operator $\hat{\rho}(t)$ of the Milburn equation (5) applied to the Hamiltonian (6) in terms of three super operators $\hat{J}$, $\hat{S}$ and $\hat{L}$:

$$\hat{\rho}(t) = \exp(\hat{J}t)\exp(\hat{S}t)\exp(\hat{L}t)\hat{\rho}(0), \tag{8}$$

where $\hat{\rho}(0)$ is the density operator of the initial particle-field system. Initially we assume that the field is prepared in the coherent state $|\alpha_1, \alpha_2\rangle$ defined by

$$|\alpha_1, \alpha_2\rangle = \sum_{n_1, n_2}^{\infty} Q_{n_1} Q_{n_2} |n_1, n_2\rangle \tag{9}$$

with

$$Q_{n_i} = \mathrm{e}^{-\frac{\bar{n}_i}{2}}\sqrt{\bar{n}_i^{n_i}/n_i!}, \qquad (i = 1, 2), \tag{10}$$

where $|n_1, n_2\rangle = |n_1\rangle \otimes |n_2\rangle$ are the Fock states for the two modes and the particle is prepared in its excited state $|e\rangle$, so that

$$\hat{\rho}(0) = \begin{bmatrix} |\alpha_1, \alpha_2\rangle\langle\alpha_1, \alpha_2| & 0 \\ 0 & 0 \end{bmatrix}. \tag{11}$$

The superoperators $\hat{J}$, $\hat{S}$ and $\hat{L}$ are defined through their action on the density operator, namely

$$\hat{J}\hat{\rho} = \frac{1}{\gamma}\hat{H}\hat{\rho}\hat{H}, \tag{12}$$

$$\hat{S}\hat{\rho} = -i[\hat{H}, \hat{\rho}] = -i(\hat{H}\hat{\rho} - \hat{\rho}\hat{H}) \tag{13}$$

and

$$\hat{L}\hat{\rho} = -\frac{1}{2\gamma}\{\hat{H}^2, \hat{\rho}\} = -\frac{1}{2\gamma}(\hat{H}^2\hat{\rho} + \hat{\rho}\hat{H}^2). \tag{14}$$

The Hamiltonian (7) can be expressed as a sum of two terms which commute with each other, that is,

$$\hat{H} = \hat{H}_0 + \hat{H}_I, \qquad [\hat{H}_0, \hat{H}_I] = 0, \tag{15}$$

where

$$\hat{H}_0 = \begin{bmatrix} \hat{W}(\hat{n}_1 + 1, \hat{n}_2 + 1) & 0 \\ 0 & \hat{W}(\hat{n}_1, \hat{n}_2) \end{bmatrix}, \tag{16}$$

$$\hat{H}_I = \lambda \begin{bmatrix} \frac{\Delta}{2\lambda} & \hat{a}_1\hat{a}_2 \\ \hat{a}_1^\dagger\hat{a}_2^\dagger & -\frac{\Delta}{2\lambda} \end{bmatrix} \tag{17}$$

with

$$\hat{W}(\hat{n}_1, \hat{n}_2) = \omega_1\hat{n}_1 + \omega_2\hat{n}_2. \tag{18}$$

Similarly, the square of the Hamiltonian (7) can also be expressed as a sum of two operators, one of them is diagonal, in the form

$$\hat{H}^2 = \hat{A} + \hat{B}, \tag{19}$$

where

$$\hat{A} = \begin{bmatrix} \hat{W}^2(\hat{n}_1 + 1, \hat{n}_2 + 1) + \lambda^2\hat{\mu}^2(\hat{n}_1 + 1, \hat{n}_2 + 1) & 0 \\ 0 & \hat{W}^2(\hat{n}_1, \hat{n}_2) + \lambda^2\hat{\mu}^2(\hat{n}_1, \hat{n}_2) \end{bmatrix}, \tag{20}$$

$$\hat{B} = \begin{bmatrix} \frac{\Delta}{2\lambda}\hat{W}(\hat{n}_1 + 1, \hat{n}_2 + 1) & \hat{a}_1\hat{a}_2\hat{W}(\hat{n}_1, \hat{n}_2) \\ \hat{W}(\hat{n}_1, \hat{n}_2)\hat{a}_1^\dagger\hat{a}_2^\dagger & -\frac{\Delta}{2\lambda}\hat{W}(\hat{n}_1, \hat{n}_2) \end{bmatrix} \tag{21}$$

with

$$\hat{\mu}^2(\hat{n}_1,\hat{n}_2) = \hat{\nu}^2(\hat{n}_1,\hat{n}_2) + \left(\frac{\Delta}{2\lambda}\right)^2, \quad \text{and} \quad \hat{\nu}^2(\hat{n}_1,\hat{n}_2) = \hat{n}_1\hat{n}_2. \tag{22}$$

It is easily proved that $[\hat{A},\hat{B}] = 0$ by making use of the fact $\hat{a}_i f(\hat{n}_i) = f(\hat{n}_i+1)\hat{a}_i$ for any function $f$ of $n_i$. Taking into account the initial condition (11) we can write down the following expression

$$\begin{aligned}\hat{\rho}_2(t) &= \exp(\hat{S}t)\exp(\hat{L}t)\hat{\rho}(0) \\ &= \exp(-i\hat{H}_I t)\exp(-\frac{t}{2\gamma}\hat{B})\hat{\rho}_1(t)\exp(-\frac{t}{2\gamma}\hat{B})\exp(i\hat{H}_I t),\end{aligned} \tag{23}$$

where the auxiliary operator $\hat{\rho}_1(t)$ is defined by

$$\hat{\rho}_1(t) = \begin{bmatrix} |\Psi(t)\rangle\langle\Psi(t)| & 0 \\ 0 & 0 \end{bmatrix} \tag{24}$$

with

$$\begin{aligned}|\Psi(t)\rangle &= \exp\left[-\frac{t}{2\gamma}[\hat{W}^2(n_1+1,n_2+1) + \lambda^2\hat{\mu}^2(n_1+1,n_2+1)]\right] \\ &\times \exp[-iW(n_1+1,n_2+1)t]\,|\alpha_1,\alpha_2\rangle.\end{aligned} \tag{25}$$

If we notice that the powers of the operator $\hat{B}$ can be written as

$$\hat{B}^{2k} = \begin{bmatrix} [2\lambda\hat{W}(n_1+1,n_2+1)\hat{\mu}(n_1+1,n_2+1)]^{2k} & 0 \\ 0 & [2\lambda\hat{W}(n_1,n_2)\hat{\mu}(n_1,n_2)]^{2k} \end{bmatrix}, \tag{26}$$

$$\hat{B}^{2k+1} = \begin{bmatrix} \frac{\Delta}{2\lambda}\frac{[2\lambda\hat{W}(n_1+1,n_2+1)\hat{\mu}(n_1+1,n_2+1)]^{2k+1}}{\hat{\mu}(n_1+1,n_2+1)} & \hat{a}_1\hat{a}_2\frac{[2\lambda\hat{W}(n_1,n_2)\hat{\mu}(n_1,n_2)]^{2k+1}}{\hat{\mu}(n_1,n_2)} \\ \frac{[2\lambda\hat{W}(n_1,n_2)\hat{\mu}(n_1,n_2)]^{2k+1}}{\hat{\mu}(n_1,n_2)}\hat{a}_1^\dagger\hat{a}_2^\dagger & -\frac{\Delta}{2\lambda}\frac{[2\lambda\hat{W}(n_1,n_2)\hat{\mu}(n_1,n_2)]^{2k+1}}{\hat{\mu}(n_1,n_2)} \end{bmatrix}, \tag{27}$$

then we can write the operator $\exp[-\frac{t}{2\gamma}\hat{B}]$ in the form

$$\exp[-\frac{t}{2\gamma}\hat{B}] = \tag{28}$$

$$\begin{bmatrix} \hat{X}(n_1+1,n_2+1,t) - \frac{\Delta}{2\lambda}\frac{\hat{Y}(n_1+1,n_2+1,t)}{\hat{\mu}(n_1+1,n_2+1)} & -\hat{a}_1\hat{a}_2\frac{\hat{Y}(n_1,n_2,t)}{\hat{\mu}(n_1,n_2)} \\ -\frac{\hat{Y}(n_1,n_2,t)}{\hat{\mu}(n_1,n_2)}\hat{a}_1^\dagger\hat{a}_2^\dagger & \hat{X}(n_1,n_2,t) + \frac{\Delta}{2\lambda}\frac{\hat{Y}(n_1,n_2,t)}{\hat{\mu}(n_1,n_2)} \end{bmatrix},$$

where

$$\hat{X}(n_1,n_2,t) = \cosh\left[\frac{\lambda t}{\gamma}\hat{W}(n_1,n_2)\hat{\mu}(n_1,n_2)\right], \tag{29}$$

$$\hat{Y}(n_1, n_2, t) = \sinh\left[\frac{\lambda t}{\gamma}\hat{W}(n_1, n_2)\hat{\mu}(n_1, n_2)\right]. \tag{30}$$

Analogously, we can write the operator $\exp[-i\hat{H}_I t]$ in the two-dimensional basis for the particle as [27,28].

$$\exp[-i\hat{H}_I t] = \tag{31}$$
$$\begin{bmatrix} \hat{C}(n_1+1, n_2+1, t) - i\frac{\Delta}{2\lambda}\frac{\hat{S}(n_1+1,n_2+1,t)}{\hat{\mu}(n_1+1,n_2+1)} & -i\hat{a}_1\hat{a}_2\frac{\hat{S}(n_1,n_2,t)}{\hat{\mu}(n_1,n_2)} \\ -i\frac{\hat{S}(n_1,n_2,t)}{\hat{\mu}(n_1,n_2)}\hat{a}_1^\dagger\hat{a}_2^\dagger & \hat{C}(n_1, n_2, t) + i\frac{\Delta}{2\lambda}\frac{\hat{S}(n_1,n_2,t)}{\hat{\mu}(n_1,n_2)} \end{bmatrix}$$

with

$$\hat{C}(n_1, n_2, t) = \cos[\lambda t\hat{\mu}(n_1, n_2)], \qquad \hat{S}(n_1, n_2, t) = \sin[\lambda t\hat{\mu}(n_1, n_2)]. \tag{32}$$

Then from Eqs.(28) and (31) it follows that

$$\exp[-i\hat{H}_I t]\exp[-\frac{t}{2\gamma}\hat{B}] = \tag{33}$$
$$\begin{bmatrix} \hat{R}(n_1+1, n_2+1, t) - \frac{\Delta}{2\lambda}\frac{\hat{V}(n_1+1,n_2+1,t)}{\hat{\mu}(n_1+1,n_2+1)} & -\hat{a}_1\hat{a}_2\frac{\hat{V}(n_1,n_2,t)}{\hat{\mu}(n_1,n_2)} \\ -\frac{\hat{V}(n_1,n_2,t)}{\hat{\mu}(n_1,n_2)}\hat{a}_1^\dagger\hat{a}_2^\dagger & \hat{R}(n_1, n_2, t) + \frac{\Delta}{2\lambda}\frac{\hat{V}(n_1,n_2,t)}{\hat{\mu}(n_1,n_2)} \end{bmatrix},$$

where

$$\hat{R}(n_1, n_2, t) = \hat{C}(n_1, n_2, t)\hat{X}(n_1, n_2, t) + i\hat{S}(n_1, n_2, t)\hat{Y}(n_1, n_2, t), \tag{34}$$

$$\hat{V}(n_1, n_2, t) = \hat{C}(n_1, n_2, t)\hat{Y}(n_1, n_2, t) + i\hat{S}(n_1, n_2, t)\hat{X}(n_1, n_2, t). \tag{35}$$

Using Eq.(33), we can find an explicit expression for the operator $\hat{\rho}_2(t)$ of Eq.(23) as follows:

$$\hat{\rho}_2(t) = \begin{bmatrix} |\Psi_1(t)\rangle\langle\Psi_1(t)| & |\Psi_1(t)\rangle\langle\Psi_2(t)| \\ |\Psi_2(t)\rangle\langle\Psi_1(t)| & |\Psi_2(t)\rangle\langle\Psi_2(t)| \end{bmatrix} \tag{36}$$

with

$$|\Psi_1(t)\rangle = \left[\hat{R}(n_1+1, n_2+1, t) - \frac{\Delta}{2\lambda}\frac{\hat{V}(n_1+1, n_2+1, t)}{\hat{\mu}(n_1+1, n_2+1)}\right]|\Psi(t)\rangle, \tag{37}$$

$$|\Psi_2(t)\rangle = -\hat{a}_1^\dagger\hat{a}_2^\dagger\frac{\hat{V}(n_1+1, n_2+1, t)}{\hat{\mu}(n_1+1, n_2+1)}|\Psi(t)\rangle, \tag{38}$$

where $|\Psi(t)\rangle$ is given by Eq.(25). Taking into account the definition of the superoperator $\hat{J}$, it is straightforward to obtain the action of the operator $\exp(\hat{J}t)$ on the density operator $\hat{\rho}_2(t)$ as follows:

$$\hat{\rho}(t) = \sum_{k=0}^{\infty}\frac{1}{k!}\left(\frac{t}{\gamma}\right)^k \hat{H}^k\hat{\rho}_2(t)\hat{H}^k, \tag{39}$$

where the Hamiltonian $\hat{H}$ and the operator $\hat{\rho}_2(t)$ are given by Eq.(6) and (36), respectively. Equation (39) describes the exact solution of the Milburn equation (5) for the bimodal two-quanta Jaynes-Cummings model. Once the density operator is calculated all relevant statistical quantities can be computed.

## 3 Effect of the intrinsic decoherence on population inversion

In this section we investigate the effect of the intrinsic decoherence on population inversion for bimodal two-quanta JCM. The population inversion $W(t)$ is defined as the expectation value of the operator $\hat{\sigma}_z$, i.e.,

$$W(t) = \langle \hat{\sigma}_z(t) \rangle = \mathrm{Tr}[\hat{\rho}(t)\hat{\sigma}_z]. \tag{40}$$

By using the last equation and Eq.(39), we arrive at the final result of the population inversion in the simple form

$$\begin{aligned} W(t) \quad &= \quad \sum_{n_1,n_2=0}^{\infty} \frac{|Q_{n_1}|^2|Q_{n_2}|^2}{\mu^2(n_1+1,n_2+1)} \left\{ \left(\frac{\Delta}{2\lambda}\right)^2 + \nu^2(n_1+1,n_2+1) \right. \\ &\times \quad \left. \exp\left[\left(-\frac{2\lambda^2 t}{\gamma}\right)\mu^2(n_1+1,n_2+1)\right] \cos 2\lambda t\mu(n_1+1,n_2+1) \right\}, \end{aligned} \tag{41}$$

where $|Q_{n_1}|^2$, $|Q_{n_2}|^2$ are the initial values for the distribution function given by Eq.(10). We see that the above equations, when $n_2 = 0$, $\Delta = 0$ coincide with those of Ref. [18]. In particular, in the limit $\gamma \to +\infty$, the population inversion (41) reduces to

$$\begin{aligned} W(t) \quad &= \quad \sum_{n_1,n_2=0}^{\infty} \frac{|Q_{n_1}|^2|Q_{n_2}|^2}{\mu^2(n_1+1,n_2+1)} \\ &\times \quad \left[\left(\frac{\Delta}{2\lambda}\right)^2 + \nu^2(n_1+1,n_2+1)\cos 2\lambda t\mu(n_1+1,n_2+1)\right], \end{aligned} \tag{42}$$

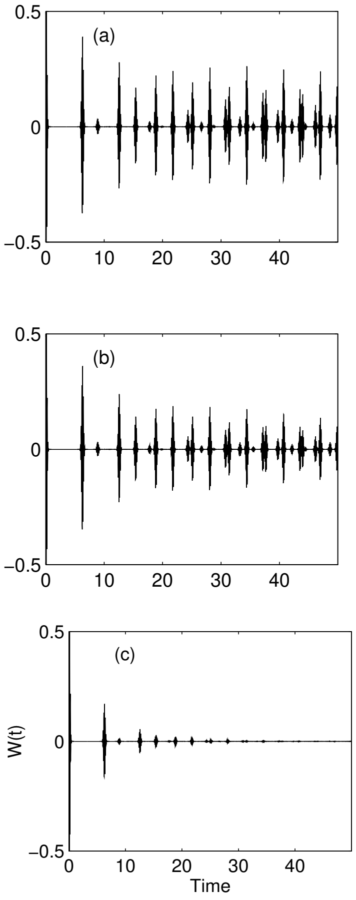
which is the formula obtained before [27,28].

We discuss the general behaviour of the population inversion for nondegenerate two-quanta JCM, when the particle (atom or trapped ion) initially starts in the excited state.

The numerical results are shown in Figs. 1 and 2, for various values of the parameter $\frac{\lambda^2}{\gamma}$, and different values of the detuning parameter $\frac{\Delta}{2\lambda}$, (namely 0, 10), and fixed initial mean numbers of quanta $\bar{n}_1$ and $\bar{n}_2$.

In Fig. 1 we plotted the population inversion $W(t)$ for three values of the parameter $\frac{\lambda^2}{\gamma}$ with the fixed initial mean numbers of quanta $\bar{n}_1 = \bar{n}_2 = 25$ in the case of the exact resonance i.e., the detuning parameter $\frac{\Delta}{2\lambda} = 0$.

In Fig. 2 we take the detuning parameter $\frac{\Delta}{2\lambda} = 10$ with the same values of the rest of the parameters in Fig. 1. These figures show that with the decrease of the parameter $\gamma$, i.e., with a more rapid suppression of quantum coherences we can observe rapid deterioration of revivals of the population inversion. Which means that the decay of quantum coherences is due to the very specific time evolution described by Eq.(5), i.e., due to the intrinsic decoherence. The amplitudes of the revivals are suppressed further by increasing the detuning parameter $\Delta$. However as time evolves we observe that $W(t)$ settles to a positive value which means energy is stored in the atomic system.



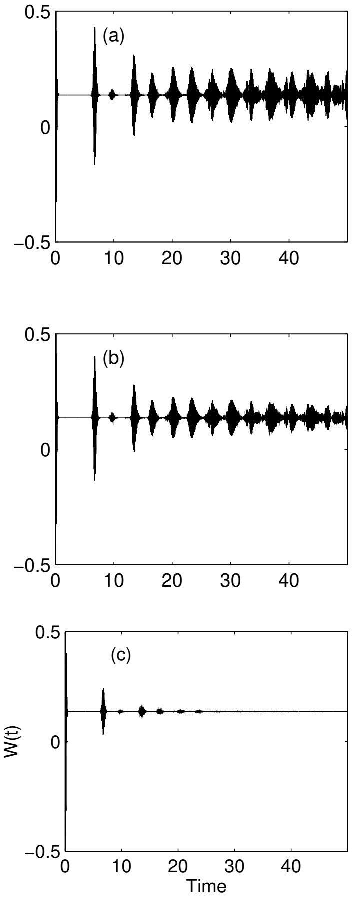




Fig. 1. Time evolution of the population inversion $W(t)$ of the particle initially prepared in the excited state interacting with the coherent state $|\alpha_1, \alpha_2\rangle$ ($\bar{n}_1 = \bar{n}_2 = 25$) for various values of the parameter $\frac{\lambda^2}{\gamma}$: (a) $\frac{\lambda^2}{\gamma} = 10^{-6}$, (b) $\frac{\lambda^2}{\gamma} = 10^{-5}$, and (c) $\frac{\lambda^2}{\gamma} = 10^{-4}$.
Fig. 2. The same as in Fig. 1 but with the detuning parameter $\frac{\Delta}{2\lambda} = 10$.

## 4 Effect of the intrinsic decoherence on nonclassical effects of the field

In this section we investigate the effect of the intrinsic decoherence on photon statistics and the cross correlation between modes 1 and 2 of the field in the non-degenerate two-quanta JCM.

### 4.1 Photon statistics

The field density operator $\hat{\rho_F}(t)$ is obtained from $\hat{\rho}(t)$ by tracing over the atomic states,

$$\hat{\rho}_F(t) = \mathrm{Tr}_A \hat{\rho}(t). \tag{43}$$

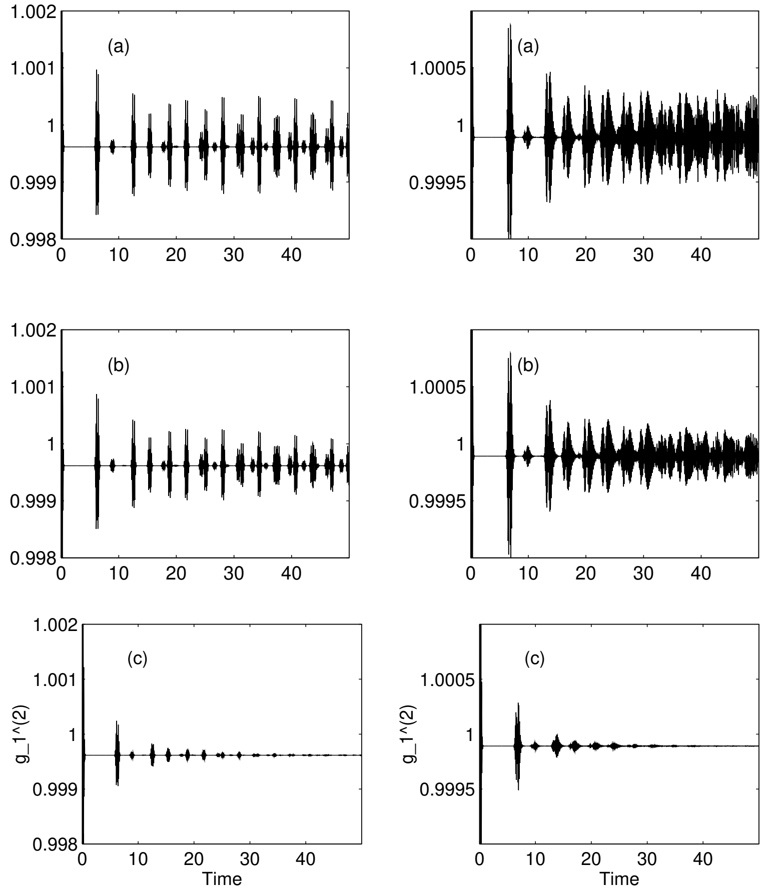



Fig. 3. Time evolution for the second order correlation for the first mode $g_1^{(2)}(t)$ of Eq.(46) for various values of the parameter $\frac{\lambda^2}{\gamma}$: (a) $\frac{\lambda^2}{\gamma} = 10^{-6}$, (b) $\frac{\lambda^2}{\gamma} = 10^{-5}$, and (c) $\frac{\lambda^2}{\gamma} = 10^{-4}$.
Fig. 4. The same as in Fig. 3 but with the detuning parameter $\frac{\Delta}{2\lambda} = 10$.

From the field density operator, the expectation value of the operator $\hat{n}_1^{s_1}\hat{n}_2^{s_2}$ is calculated from the formula

$$\langle \hat{n}_1^{s_1}\hat{n}_2^{s_2}\rangle = \mathrm{Tr}_{field}\left[\hat{\rho}_F(t)\hat{n}_1^{s_1}\hat{n}_2^{s_2}\right]. \tag{44}$$

The final result is

$$\begin{aligned}\langle \hat{n}_1^{s_1}\hat{n}_2^{s_2}\rangle &= \frac{1}{2}\sum_{n_1,n_2=0}^{\infty} |Q_{n_1}|^2|Q_{n_2}|^2\Big\{n_1^{s_1}n_2^{s_2}\left[1+\frac{\left(\frac{\Delta}{2\lambda}\right)^2}{\mu^2(n_1+1,n_2+1)}\right]^2 \\ &+ (n_1+1)^{s_1}(n_2+1)^{s_2}\frac{\nu^2(n_1+1,n_2+1)}{\mu^2(n_1+1,n_2+1)}+\frac{\nu^2(n_1+1,n_2+1)}{\mu^2(n_1+1,n_2+1)}\times\end{aligned} \tag{45}$$

$$[n_1^{s_1} n_2^{s_2} - (n_1+1)^{s_1}(n_2+1)^{s_2}] \exp\left[-\frac{2\lambda^2 t}{\gamma}\mu^2(n_1+1, n_2+1)\right] \cos 2\lambda t\mu(n_1+1, n_2+1)\Big\}.$$

Sub-Poissonian distribution of the field [33, 34] is one of the best known nonclassical effects of the light field. To understand the effect of the intrinsic decoherence on this in the nondegenerate two-quanta JCM, we examine the behaviour of the second order correlation functions for the two modes defined by [35].

$$g_i^{(2)}(t) = \frac{\langle \hat{n}_i^2(t)\rangle - \langle \hat{n}_i(t)\rangle}{(\langle \hat{n}_i(t)\rangle)^2}. \tag{46}$$

Mode $i$ is said to be sub-Poissonian if $g_i^{(2)}$ is less than 1, and super-Poissonian if $g_i^{(2)} > 1$. To establish if the system is sub- or super-Poissonian, we must calculate the two expectation values $\langle \hat{n}_i\rangle$ and $\langle \hat{n}_i^2\rangle$. These we can calculate from Eq.(45), we find that

$$\begin{aligned}\langle \hat{n}_i(t)\rangle &= \bar{n}_i + \frac{1}{2}\sum_{n_1,n_2=0}^{\infty} |Q_{n_1}|^2 |Q_{n_2}|^2 \frac{\nu^2(n_1+1,n_2+1)}{\mu^2(n_1+1,n_2+1)} \\ &\times \left[1 - \exp[-\frac{2\lambda^2 t}{\gamma}\mu^2(n_1+1,n_2+1)] \cos 2\lambda t\mu(n_1+1,n_2+1)\right], \end{aligned} \tag{47}$$

$$\begin{aligned}\langle \hat{n}_i^2(t)\rangle &= \bar{n}_i^2 + \bar{n}_i + \frac{1}{2}\sum_{n_1,n_2=0}^{\infty} |Q_{n_1}|^2 |Q_{n_2}|^2 (2n_i+1) \frac{\nu^2(n_1+1,n_2+1)}{\mu^2(n_1+1,n_2+1)} \\ &\times \left[1 - \exp[-\frac{2\lambda^2 t}{\gamma}\mu^2(n_1+1,n_2+1)] \cos 2\lambda t\mu(n_1+1,n_2+1)\right]. \end{aligned} \tag{48}$$

Now, it is easy to get the exact expression of $g_i^{(2)}(t)$ through substituting Eqs.(47) and (48) into Eq.(46).

Numerical results for various values of the parameter $\frac{\lambda^2}{\gamma}$, (namely $10^{-6}$, $10^{-5}$, $10^{-4}$), with different values of the detuning parameter $\frac{\Delta}{2\lambda}$, (namely 0, 10), and the fixed initial mean numbers of quanta $\bar{n}_1 = \bar{n}_2 = 25$ are shown in Figs. 3 and 4.

In Fig. 3 we have plotted the second order correlation function for the first mode $g_i^{(2)}(t)$ for the non-degenerate two-quanta JCM for three values of the parameter $\frac{\lambda^2}{\gamma}$, in the case of exact resonance, i.e., the detuning parameter $\frac{\Delta}{2\lambda} = 0$.

Fig. 4 has the same values as in the figure 3, but in the non-resonance case ($\frac{\Delta}{2\lambda} \neq 0$), i.e., with the detuning parameter $\frac{\Delta}{2\lambda} = 10$.

In view of figures 3 and 4, we observe that the revivals in the second order correlation function decay with the decrease of the decoherence parameter $\gamma$. However it settles to a sub-Poissonian distribution for $\Delta = 0$. For large detuning sub-Poissonian distribution is lost.

### 4.2 Cross correlation between the two modes

The cross correlation between modes 1 and 2 can be written in the form [35].

$$\Delta_{cross}(t) = \langle \hat{n}_1(t)\hat{n}_2(t)\rangle - \langle \hat{n}_1(t)\rangle\langle \hat{n}_2(t)\rangle. \tag{49}$$

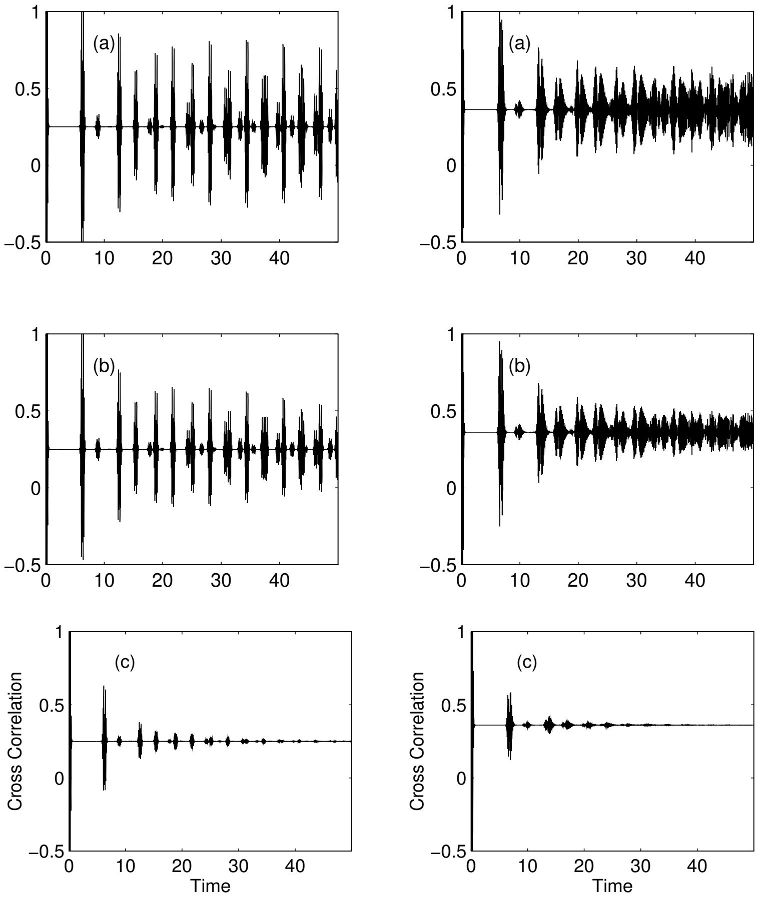



Fig. 5. Time evolution for the cross correlation $\Delta_{cross}(t)$ of Eq.(49) for various values of the parameter $\frac{\lambda^2}{\gamma}$: (a) $\frac{\lambda^2}{\gamma} = 10^{-6}$, (b) $\frac{\lambda^2}{\gamma} = 10^{-5}$, and (c) $\frac{\lambda^2}{\gamma} = 10^{-4}$.
Fig. 6. The same as in Fig. 5 but with the detuning parameter $\frac{\Delta}{2\lambda} = 10$.

The two modes are correlated if $\Delta_{cross}$ is positive and anticorrelated if otherwise. To establish if the two modes are correlated or anticorrelated, we must calculate the expectation values $\langle \hat{n}_i \rangle$ and $\langle \hat{n}_1 \hat{n}_2 \rangle$. The expectation values of $\langle \hat{n}_i \rangle$ have been evaluated in Eq.(47). The expectation value $\langle \hat{n}_1 \hat{n}_2 \rangle$ can be calculated from Eq.(45), namely:

$$\langle \hat{n}_1(t)\hat{n}_2(t) \rangle = \bar{n}_1\bar{n}_2 + \frac{1}{2}\sum_{n_1,n_2=0}^{\infty} |Q_{n_1}|^2|Q_{n_2}|^2(n_1+n_2+1)\frac{\nu^2(n_1+1,n_2+1)}{(\mu^2(n_1+1,n_2+1)} \times \left[1-\exp[-\frac{2\lambda^2 t}{\gamma}\mu^2(n_1+1,n_2+1)]\cos 2\lambda t\mu(n_1+1,n_2+1)\right]. \quad (50)$$

It is easy to get the exact expression of $\Delta_{\mathrm{cross}}$ through substituting Eqs.(47) and (50) into Eq.(49).

We now look at the cross correlation $\Delta_{\rm cross}(t)$ of Eq.(49). This quantity is measured in a Hanbury-Brown and Twiss type experiment for two-different beams [35,36]. In Figs. 5 and 6 we plot the time evolution of the cross correlation $\Delta_{\rm cross}(t)$ of Eq.(49) against $\lambda t$ for the two-quanta JCM for three values of the parameter $\frac{\lambda^2}{\gamma}$. In Fig. 5 we have plotted the cross correlation with the initial mean numbers $\bar{n}_1 = \bar{n}_2 = 25$ in the resonance case, i.e., $\frac{\Delta}{2\lambda} = 0$.

In Fig. 6 we plotted the previous relation but in the non-resonance case i.e., with the detuning parameter $\frac{\Delta}{2\lambda} = 10$.

It is observed from these figures, that the amplitudes of the revivals $\Delta_{cross}(t)$ decay with the decrease of the decoherence parameter $\gamma$ and we can conclude that the intrinsic decoherence in the atom interaction suppresses these non-classical effects in the non-degenerate two-quanta JCM. The two modes are mostly correlated for $\Delta \neq 0$, as $\gamma$ decreases.

### 4.3 Amplitude-squared squeezing of the field

Now, we study the amplitude-squared squeezing of the field of the nondegenerate bimodal two-quanta JCM governed by the Milburn equation and discuss effects of the intrinsic decoherence on the squeezing. We define the operators of the real and imaginary parts of the square of the amplitude [37].

$$\hat{X}_{j1} = \frac{1}{2}[\hat{A}_j^2(t) + \hat{A}_j^{\dagger 2}(t)], \quad \hat{X}_{j2} = \frac{1}{2i}[\hat{A}_j^2(t) - \hat{A}_j^{\dagger 2}(t)], \qquad (j = 1, 2), \tag{51}$$

where $\hat{A}_j(t) = \hat{a}_j \mathrm{e}^{i\omega_j t}$, $\hat{A}_j^\dagger(t) = \hat{a}_j^\dagger \mathrm{e}^{-i\omega_j t}$, are the slowly varying operators. These operators satisfy the commutation relation,

$$[\hat{X}_{j1}, \hat{X}_{j2}] = i(2\hat{n}_j + 1), \qquad (j = 1, 2), \tag{52}$$

which implies the uncertainty relation

$$(\Delta\hat{X}_{j1})^2(\Delta\hat{X}_{j2})^2 \geq \frac{1}{4}|\langle[\hat{X}_{j1}, \hat{X}_{j2}]\rangle|^2. \tag{53}$$

The state of the field is said to be amplitude-squared squeezed whenever one of the two quadratures satisfies the relation:

$$(\Delta\hat{X}_{j1 \text{ or } 2}) < \frac{1}{2}(2\hat{n}_j + 1), \qquad (j = 1 \text{ or } 2), \tag{54}$$

where $\hat{n}_j = \hat{a}_j^\dagger \hat{a}_j$, $(j = 1, 2)$. On the other hand, the condition (54) can be rewritten as

$$S_i^{(j)} = (\Delta\hat{X}_{ji})^2 - \frac{1}{2}(\langle 2\hat{n}_j + 1\rangle), \qquad (i = 1, 2,\ j = 1 \text{ or } 2), \tag{55}$$

and squeezing occurs when $S_1^{(j)}$ or $S_2^{(j)} < 0$. In terms of the photon annihilation and creation operators of the field, we get for the amplitude-squared squeezing factors $S^{(j)}$ the expression

$$S_1^{(j)} = \frac{1}{4}[2\langle\hat{A}_j^{\dagger 2}\hat{A}_j^2\rangle + \langle\hat{A}_j^4\rangle + \langle\hat{A}_j^{\dagger 4}\rangle - (\langle\hat{A}_j^2\rangle + \langle\hat{A}_j^{\dagger 2}\rangle)^2], \tag{56}$$

$$S_2^{(j)} = \frac{1}{4}[2\langle\hat{A}_j^{\dagger 2}\hat{A}_j^2\rangle - \langle\hat{A}_j^4\rangle - \langle\hat{A}_j^{\dagger^4}\rangle + (\langle\hat{A}_j^2\rangle - \langle\hat{A}_j^{\dagger^2}\rangle)^2]. \tag{57}$$

Now, we can obtain the amplitude-squared squeezing of the first or the second mode from the last expression if we take $j = 1$ or 2. By using the field density operators $\hat{\rho}_F(t)$, we obtain the expectation value in the general form for the field operators $\hat{A}_1^{\dagger r_1}\hat{A}_1^{s_1}\hat{A}_2^{\dagger r_2}\hat{A}_2^{s_2}$ as follows:

$$\begin{aligned}\langle \hat{A}_1^{\dagger r_1}\hat{A}_1^{s_1}\hat{A}_2^{\dagger r_2}\hat{A}_2^{s_2}\rangle &= \frac{1}{4}\sum_{n_1,n_2=0}^{\infty} Q_{n_1+s_1}Q_{n_2+s_2}Q^*_{n_1+r_1}Q^*_{n_2+r_2} \qquad (58)\\ \times &\left\{[J(1+D_s)(1+D_r)+J_1.L_1]\exp(i\lambda t a_-(n_1,n_2))\exp(-\frac{t}{2\gamma}b_+(n_1,n_2))\right.\\ + &[J(1+D_s)(1-D_r)-J_1.L_1]\exp(-i\lambda t a_+(n_1,n_2))\exp(-\frac{t}{2\gamma}c_-(n_1,n_2))\\ + &[J(1-D_s)(1+D_r)-J_1.L_1]\exp(i\lambda t a_+(n_1,n_2))\exp(-\frac{t}{2\gamma}c_+(n_1,n_2))\\ + &\left.[J(1-D_s)(1-D_r)+J_1.L_1]\exp(-i\lambda t a_-(n_1,n_2))\exp(-\frac{t}{2\gamma}b_-(n_1,n_2))\right\},\end{aligned}$$

where $J = \sqrt{\frac{(n_1+s_1)!(n_2+s_2)!}{(n_1)!(n_2)!}\frac{(n_1+r_1)!(n_2+r_2)!}{(n_1)!(n_2)!}}$, $J_1$ is $J$ with $n_1 \to n_1+1, n_2 \to n_2+1$, $L_1 = \frac{\nu(n_1+s_1+1,n_2+s_2+1)}{\mu(n_1+s_1+1,n_2+s_2+1)}\frac{\nu(n_1+r_1+1,n_2+r_2+1)}{\mu(n_1+r_1+1,n_2+r_2+1)}$, $D_s = \frac{(\frac{\Delta}{2\lambda})}{\mu(n_1+s_1+1,n_2+s_2+1)}$ and $D_r$ is $D_s$ with $s \to r$, and with

$$\begin{aligned} a_\pm(n_1,n_2) &= \mu(n_1+r_1+1,n_2+r_2+1) \pm \mu(n_1+s_1+1,n_2+s_2+1), && (59)\\ b_\pm(n_1,n_2) &= [(\omega_1(r_1-s_1)+\omega_2(r_2-s_2) \pm \lambda a_-(n_1,n_2)]^2, && (60)\\ c_\pm(n_1,n_2) &= [(\omega_1(r_1-s_1)+\omega_2(r_2-s_2) \pm \lambda a_+(n_1,n_2)]^2. && (61)\end{aligned}$$

By using Eqs.(58–61), and specifying the exponents $r$, $s$ we get the expression for the amplitude-squared squeezing of the first mode for the nondegenerate two-quanta Jaynes-Cummings model.

Now, we discuss the temporal behaviour of the $S_1^{(1)}$, which gives information on amplitude-squared squeezing, when we take $\bar{n}_1 = \bar{n}_2 = 10$ and different values of the detuning parameter $\frac{\Delta}{2\lambda}$. We study the effect of the decoherence parameter on amplitude-squared squeezing for the nonlinear JCM.

Numerical results for Eq. 56 are presented in Fig. 7, we plotted $S_1^{(1)}$ against $\lambda t$ for $\bar{n}_1 = \bar{n}_2 = 10$ and different values of the decoherence parameter $\frac{\lambda^2}{\gamma}$ (namely $10^{-6}$, $10^{-2}$, $10^{-1}$), and also the detuning parameter $\frac{\Delta}{2\lambda}$ (namely 0,5).

In Fig. 7(a) we display amplitude-squared squeezing (Eq. 56) with $\frac{\lambda^2}{\gamma} = 10^{-6}$ and $\frac{\Delta}{2\lambda} = 0$ (dashed curve) and $\frac{\Delta}{2\lambda} = 5$ (solid curve). Figures 7(b) and 7(c) are the same as Fig. 7(a), but with the decoherence parameter $\frac{\lambda^2}{\gamma} = 10^{-2}, 10^{-1}$, respectively.

From Fig. 7, we note that with the decrease of the parameter $\gamma$, i.e., with a more rapid decohering, we observe rapid decrease of the amount of squeezing for amplitude-squared squeezing. However it seems that the off-resonant model is more susceptible to decoherence. Thus we conclude that the effect of the intrinsic decoherence on the squeezing of the field is to suppress the amplitude-squared squeezing of the field in the JCM.

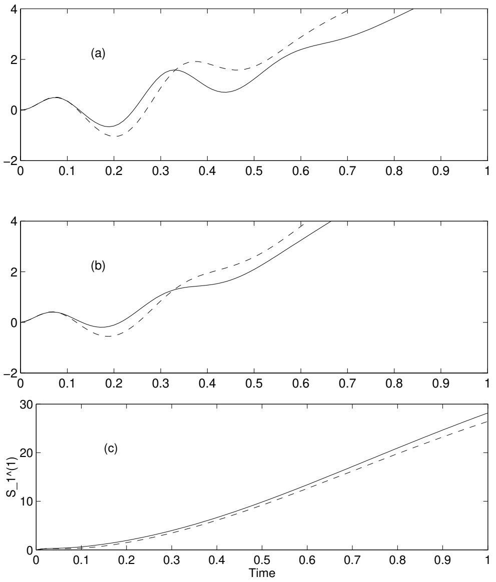



Fig. 7. Time evolution for the amplitude-squared squeezing parameter $S_1^{(1)}$ of Eq.(56) with the initial mean numbers $\bar{n}_1 = \bar{n}_2 = 10$, and the detuning parameter $\frac{\Delta}{2\lambda} = 0$ (dashed curve) and 5 (solid curve) for various values of the parameter $\frac{\lambda^2}{\gamma}$: (a) $\frac{\lambda^2}{\gamma} = 10^{-6}$, (b) $\frac{\lambda^2}{\gamma} = 10^{-2}$, and (c) $\frac{\lambda^2}{\gamma} = 10^{-1}$.

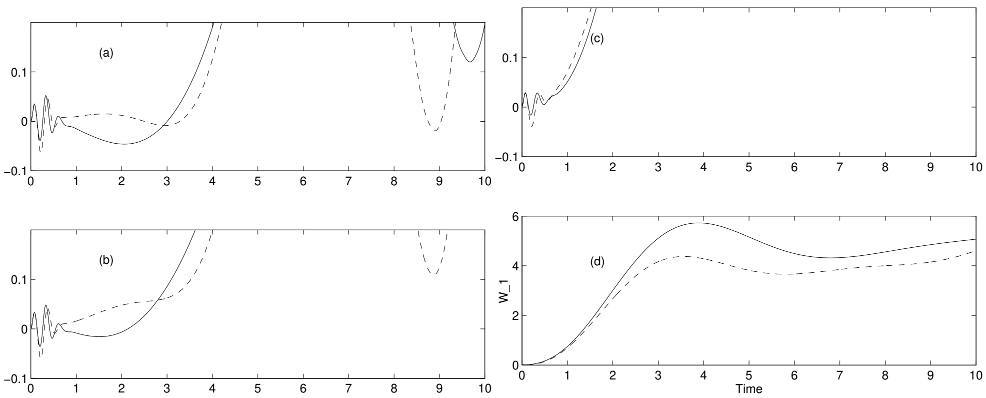



Fig. 8. Time evolution for the two-mode normal squeezing parameter $W_1$ of Eq.(67) with the initiall mean numbers $\bar{n}_1 = \bar{n}_2 = 10$, and the detuning parameter $\frac{\Delta}{2\lambda} = 0$ (dashed curve) and 5 (solid curve) for various values of the parameter $\frac{\lambda^2}{\gamma}$: (a) $\frac{\lambda^2}{\gamma} = 10^{-6}$, (b) $\frac{\lambda^2}{\gamma} = 10^{-3}$, (c) $\frac{\lambda^2}{\gamma} = 10^{-2}$, and (d) $\frac{\lambda^2}{\gamma} = 10^{-1}$.

### 4.4 Two-mode normal squeezing of the field

Now, we study the two-mode normal squeezing of the field of the nondegenerate bimodal two-quanta JCM governed by the Milburn equation and discuss effects of the intrinsic decoherence on this type of squeezing. The operators of the real parts of quadrature are defined by [38]

$$\hat{Z}_1(t) = \frac{1}{2\sqrt{2}}[\hat{A}_1 + \hat{A}_1^\dagger + \hat{A}_2 + \hat{A}_2^\dagger], \quad \hat{Z}_2(t) = \frac{1}{2i\sqrt{2}}[\hat{A}_1 - \hat{A}_1^\dagger + \hat{A}_2 - \hat{A}_2^\dagger], \tag{62}$$

where $\hat{A}_j = \hat{a}_j \mathrm{e}^{i\omega_j t}$, $\hat{A}_j^\dagger = \hat{a}_j^\dagger \mathrm{e}^{-i\omega_j t}$ $(j = 1, 2)$ are the slowly varying operators. These operators satisfy the commutation relation

$$[\hat{Z}_1, \hat{Z}_2] = \frac{i}{2} \tag{63}$$

and the uncertainty relation

$$(\Delta\hat{Z}_1)^2(\Delta\hat{Z}_2)^2 \geq \frac{1}{16}. \tag{64}$$

The state of the field is said to be squeezed whenever one of the two quadratures $\hat{Z}_1$ and $\hat{Z}_2$ satisfies the relation

$$(\Delta\hat{Z}_1)^2 \text{ or } (\Delta\hat{Z}_2)^2 < \frac{1}{4}. \tag{65}$$

On the other hand, the condition (65) can be rewritten as

$$W_j = (\Delta\hat{Z}_j)^2 - \frac{1}{4}, \qquad (j = 1 \text{ or } 2), \tag{66}$$

and squeezing occurs when $W_1$ or $W_2 < 0$. In terms of the photon annihilation and creation operators of the field, we readily find that

$$\begin{aligned} W_1 = \frac{1}{8}\Big\{ & 2\langle\hat{A}_1^\dagger\hat{A}_1\rangle + 2\langle\hat{A}_2^\dagger\hat{A}_2\rangle + \langle\hat{A}_1^2\rangle + \langle\hat{A}_1^{\dagger 2}\rangle + \langle\hat{A}_2^2\rangle + \langle\hat{A}_2^{\dagger 2}\rangle + \\ & 2\left(\langle\hat{A}_1\hat{A}_2\rangle + \langle\hat{A}_1^\dagger\hat{A}_2^\dagger\rangle + \langle\hat{A}_1^\dagger\hat{A}_2\rangle + \langle\hat{A}_1\hat{A}_2^\dagger\rangle\right) - [(\langle\hat{A}_1\rangle + \langle\hat{A}_1^\dagger\rangle) + (\langle\hat{A}_2\rangle + \langle\hat{A}_2^\dagger\rangle)]^2\Big\}, \end{aligned} \tag{67}$$

$$\begin{aligned} W_2 = \frac{1}{8}\Big\{ & 2\langle\hat{A}_1^\dagger\hat{A}_1\rangle + 2\langle\hat{A}_2^\dagger\hat{A}_2\rangle - \langle\hat{A}_1^2\rangle - \langle\hat{A}_1^{\dagger 2}\rangle - \langle\hat{A}_2^2\rangle - \langle\hat{A}_2^{\dagger^2}\rangle - \\ & 2\left(\langle\hat{A}_1\hat{A}_2\rangle + \langle\hat{A}_1^\dagger\hat{A}_2^\dagger\rangle - \langle\hat{A}_1^\dagger\hat{A}_2\rangle - \langle\hat{A}_1\hat{A}_2^\dagger\rangle\right) + [(\langle\hat{A}_1\rangle - \langle\hat{A}_1^\dagger\rangle) + (\langle\hat{A}_2\rangle - \langle\hat{A}_2^\dagger\rangle)]^2\Big\}. \end{aligned} \tag{68}$$

By using Eqs.(58–61), we obtain $W_1$ and $W_2$. Now, we discuss the temporal behaviour of $W_1$, which gives information on two-mode normal squeezing, when we take $\bar{n}_1 = \bar{n}_2 = 10$ and different values of the detuning parameter $\frac{\Delta}{2\lambda}$. We study the effect of the decoherence parameter $\gamma$ on two-mode normal squeezing for the nonlinear JCM.

Numerical results for Eq.(67) are presented in Fig. 8. We plotted two-mode normal squeezing (67) against $\lambda t$ for $\bar{n}_1 = \bar{n}_2 = 10$ and different values of the decoherence parameter $\frac{\lambda^2}{\gamma}$ (namely $10^{-6}$, $10^{-3}$, $10^{-2}$, $10^{-1}$), and also the detuning parameter $\frac{\Delta}{2\lambda}$ (namely 0,5). In Fig. 8(a) we plotted two-mode normal squeezing (Eq. 67) with $\frac{\lambda^2}{\gamma} = 10^{-6}$ and $\frac{\Delta}{2\lambda} = 0$ (dashed curve) and $\frac{\Delta}{2\lambda}$ (solid curve).

Figures 8(b), 8(c) and 8(d) are the same as Fig. 8(a), but with the decoherence parameter $\frac{\lambda^2}{\gamma} = 10^{-3}, 10^{-2}, 10^{-1}$, respectively.

In view of Figs. 8, we observe that the amount of squeezing rapidly decreases with the decrease of the decoherence parameter $\gamma$. Thus we conclude that the effect of the intrinsic decoherence on the squeezing of the field is to suppress the squeezing of the field in the nondegenerate two-quanta Jaynes-Cummings model.

## 5 Conclusions

In general the phenomena that depend on the diagonal elements (related to the photon number operators) are greatly effected by decreasing the decoherence parameter $\gamma$. However, it must be pointed out that the decrease of the decoherence parameter leaves the two modes correlated and more energy stored in the atomic system for the off-resonant ($\Delta \neq 0$) model; while sub-Poissonian distribution is persistent for the resonant ($\Delta = 0$) model. While the phenomena that depend on the non-diagonal elements (those related to the operators $\hat{A}$, or $\hat{A}^{\dagger}$ and its powers) are not affected as much. Furthermore, the off-resonant ($\Delta \neq 0$) model is much more affected than the resonant ($\Delta = 0$) model.

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