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OPEN PROBLEMS IN QUANTUM-MECHANICAL APPROACHES TO MULTISTEP DIRECT NUCLEAR REACTIONS

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Open problems in the theoretical formulation and computational implementation of multistep direct preequilibrium reactions are discussed. Recommendations for future research on both theoretical and experimental aspects of the problem are given.

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1 Introduction

This paper provides an overview of discussions and conclusions from a workshop "Open Problems in Quantum-Mechanical Approaches to Multistep Direct Nuclear Reactions" held at the ECT* (European Centre for Theoretical Studies in Nuclear Physics and Related Areas), Trento, Italy, July 27 - August 1, 1998. The focus was on multistep direct (MSD) theories, but related approaches that shed light on the applicability and limitations of MSD were considered. The entire energy range in which quantum multistep reactions using only nucleon degrees of freedom are dominant was considered; *i.e.*, up to energies of a few hundred MeV.

The following open problems were addressed: Fundamental issues in the formulations of multistep processes; Effective interactions and optical potentials relevant to multistep direct calculations; Level densities for MSD calculations; Experimental database relevant to multistep problems and identification of key future experiments; Implementation and computer code issues; Connections to other approaches (*e.g.* multiple scattering theories and semiclassical models).

Although it was not possible to answer all of the questions raised during the discussions in the short time available at the workshop, a number of suggestions were made for further work in understanding the formal basis of the MSD reaction process and its implementation. Many of these are indicated below.

2 Formulations of multistep scattering theory

With few exceptions, calculations of multistep direct processes [1] have been carried out using two approaches: the Feshbach, Kerman, and Koonin (FKK) theory [2] and that of Tamura, Udagawa, and Lenske (TUL) [3]. A third approach by Nishioka, Weidenmüller, and Yoshida (NWY) [4] relaxes some of the statistical assumptions in FKK and TUL but is more difficult to implement. A discussion of the statistical assumptions in these three approaches together with some intercomparisons has been given by Koning and Akkermans [5]. Discussions at the workshop primarily concentrated on comparisons of the FKK and TUL theories, although it was recognized that a better understanding of the connections with NWY must be carried out in the future.

A crucial difference between TUL and FKK as usually implemented is the choice of boundary conditions in the intermediate states. This issue has been extensively documented (see, *e.g.* [6]) but not conclusively resolved. Since the choice of boundary conditions has a major effect on the numerical results obtained from FKK, the implications of this choice were discussed in the workshop and are indicated below. Discussions of the connections between the TUL and FKK approaches for the most part assumed use of the same boundary conditions in the two approaches; *i.e.* those that are indicated by insertion of a complete set of intermediate states without additional assumptions or approximations such as those made by Feshbach (see [6]).

Many ingredients are common to both the TUL and FKK approaches. It is clear that this should be so if the two-step part of FKK is viewed as an approximation to TUL, in which the full Green's function is retained in the latter while it is approximated by its on-shell part in the former. Among these common ingredients is the nature of the intermediate states. Another is the type of interaction used in the transition matrix elements, together with a closely related question, the choice of optical potential.

Simple particle-hole intermediate states have been used in FKK calculations while TUL calculations have used simple macroscopic form factors and in many cases have included collectivity in the intermediate states via the random phase approximation. However, this distinction is not fundamental. We recommend that the more detailed RPA treatment be included in the FKK model as well. In any case, the same treatment of intermediate states should be used when numerical intercomparisons are made. We also note that the Brink-Axel hypothesis is used in the characterization of intermediate states; this deserves further examination since it is essential to practical implementation of the models. An additional issue that remains an important open problem is the treatment of unbound intermediate states and the related question of multiple particles in the continuum, which have been only partially addressed [7, 8]. Also related to the treatment of intermediate states is the subject of level densities appropriate for MSD calculations; this is discussed below.

Since the publication of the original paper on the TUL approach [3] considerable developments have been made both in the foundation and in the practical implementation [9-11]. While in Ref. [3] many arguments were still rather intuitive, a more systematic derivation has been presented by Wolter. In particular, the statistical assumptions employed have been made more explicit, showing that the approach involves assumptions on the statistics of the "fast" degrees of freedom of the residual nucleus, but takes into account the full quantal propagation of the system. Since the observables in multistep theories are heavily averaged, the main information is in the magnitude and the energy dependence of the continuum cross sections. Thus in TUL the magnitude of the cross sections is free of any adjustable parameters, and strictly related to effective interactions and known optical potentials, in a consistent fashion, using RPA and Green function techniques. It is thus a microscopic statistical approach. However, in the practical implementation approximations have to be made, some of which are discussed below. The TUL approach has only been calculated up to two steps. However, it should be possible with modern computers to calculate additional steps, and this would be of interest not only in applications but also in studies of the boundary condition problem discussed below.

Hussein suggested the possibility that a significant contribution to the averaged MSD cross-section may arise from intermediate Green's function fluctuations. In a study of a typical two-step direct reaction, such as the excitation of the double giant dipole resonance in heavy ions, the fluctuation contribution can be as large as the average value [12]. This contributes to filling the gap between theory and experiment. In the context of MSD, these fluctuations will result in an increase in the exciton number before the next interaction in the chain. Thus an effective number of excitons (not necessarily integer) may be required.

For the most part the DWBA matrix elements appearing in the theory have been calculated with real, energy-independent form factors (see, however, [13]). On the other hand, very comprehensive investigations over the past two decades have yielded quantitative understanding of one-step inelastic scattering and charge exchange reactions through the use of complex, energy- and density-dependent interactions in the framework of the distorted wave impulse approximation (DWIA) with nuclear medium corrections. Since it seems reasonable that this treatment should be extended to multistep reactions, this subject was extensively discussed at the workshop and a summary is included below.

To understand how realistic effective interactions should be applied to DWIA matrix elements and optical potentials in the context of MSD, it is important to derive a theory of MSD in a way that includes the essential elements of multiple scattering theory (*e.g.* KMT [14]) leading to the impulse approximation. Carrying this out is challenging but important.

The workshop recognized the importance of testing the adequacy of the on-shell approximation in the Green's functions appearing in the FKK treatment of MSD. Since numerical computations beyond the second step with TUL are very difficult with presently available computers, the validity of this approximation appears necessary to the existence of a practical implementation of MSD when several steps are required. Addressing this issue is likely to require detailed numerical studies of realistic problems.

3 Boundary conditions for MSD reactions

There has been a longstanding controversy regarding the boundary conditions to be used in the intermediate-state wave functions in the FKK version of MSD theory. Until recently, nearly all calculations have used normal DWBA matrix elements as advocated by Feshbach [6, 15], which conveniently allows MSD processes to be calculated by convoluting 1-step cross sections. In part, the widespread use of this approach has been due to the availability of codes that compute normal DWBA matrix elements. However, Feshbach's arguments for this procedure have not been universally accepted. An alternative approach, in which the boundary conditions are those naturally appearing in the complete set of states inserted in the evaluation of the intermediate state optical Green's function, has been advocated by Kawai and others, and noted in Koning and Akkermans' derivation [5].

Below are described some results, first presented in Ref. [16], that are relevant to this issue, along with some new calculated results that address the implementation of modified DWBA, referred to as MDWBA, in a calculation at 80 MeV.

A version of DWUCK has been modified by Dietrich that calculates MDWBA matrix elements. Compared to ordinary DWBA calculations, the MDWBA approach leads to enhanced multistep contributions. This code has been used in multistep direct calculations [16, 17] with the following results:

1. In the 10–30 MeV incident energy range, the MDWBA approach describes measured data well. That is, 14 MeV (n,xn) spectra were calculated for ^{93}Nb and ^{107}Ag, and the multistep preequilibrium contributions, together with equilibrium decay contributions, accounted for the experimental data. Even though the multistep contributions are larger using MDWBA compared to DWBA, they are still small in magnitude compared to the 1-step process [16, 17].
2. In order to obtain a substantial coupling between MSD and MSC preequilibrium chains ("$P \to Q$" transitions), which appears to be required by measured emission spectra data [18, 19], it is necessary to use MDWBA [17].

However, a problem occurs when the MDWBA approach is implemented for higher energy reactions. An example is shown in Fig. 1 (solid line) for the emission spectrum of neutrons following bombardment of 80 MeV protons on ^{90}Zr [16]. The residual interaction strength was chosen to account for the highest emission energies where 1-step scattering dominates. Clearly, the calculated spectrum has the wrong shape, since the multistep contributions are too large, and a breakdown of flux conservation occurs.

Kerman has argued that the failure of theory in Fig. 1 may not be due to the use of MDWBA, but may be due to other simplifications and inadequacies in the numerical implementation of MSD theory. Specifically, Kerman notes that: (a) multistep contributions are overpredicted because use is made of single-particle levels high into the continuum, whereas in the unbound region above Coulomb and angular momentum barriers, these states are poorly defined; (b) use of phenomenological optical potentials may be inappropriate. This point is discussed in more detail below.

It may be inconsistent to use the phenomenological single-channel optical potential to calculate the scattering wavefunctions. Since the MSD formalism explicitly calculates the direct nonelastic processes that occur at these energies, it appears to be incorrect to use an imaginary term obtained from phenomenological fits to the total and elastic scattering data. This is well known in the context of standard coupled channel calculations, where the imaginary potential must be reduced from its single-channel value. Indeed, for energies above approximately 50 MeV, the compound nucleus contribution to the nonelastic processes in the early stages of the reaction is expected to be small. Making use of a significantly reduced imaginary component in the imaginary potential may solve the numerical problem in the implementation of MDWBA. The "blow-up" (Fig. 1, solid line) that is obtained in the magnitude of multistep components would be significantly reduced since the effects of using the complex-conjugate potential in the modified DWBA matrix elements would be reduced.

A counter-argument to the above point has been made by Wolter: in coupled channels approaches the reduction of the imaginary part of the optical potential is due to the increase of flux by the back-coupling of the reaction channels into the elastic channel. In multistep theories, on the other hand, this does not occur, since the "never-come-back" hypothesis is used always. Thus the optical potential should be chosen to represent the elastic scattering in the particular channel. However, the difficulty is that one needs an excitation energy-dependent optical potential, which is only accessible to calculation. For example, it can be obtained using the excited state density which can be obtained from a multistep calculation, and the folding model, but this needs to be investigated.

To assess the effect of using a reduced imaginary optical potential in MSD calculations using MDWBA, we have performed a simplified "toy calculation" where we have used a conventional complex potential for the first stage, but have reduced the imaginary potential to zero for the second and higher stages of the multistep MDWBA calculation. The results are shown in Fig. 1 as a dashed line. It is evident that the results are in better agreement with measurements, though there is still an overprediction at the lowest emission energies. We note, though, that if a reduced basis of single-particle states is used, as discussed above, the calculated multistep contributions would fall further, in better agreement with experiment.

In summary, through numerical calculations we have found evidence suggesting that the MDWBA approach may be consistent with preequilibrium emission spectra measurements, at all energies including those where multistep processes play a significant role. However, modifications are needed to the optical potentials as well as the particle-hole level densities used in the calculations. Future progress will require understanding what optical potentials are appropriate for multistep direct reactions where many channels are explicitly coupled, and will also require an improved treatment of level densities in the unbound part of the spectrum.

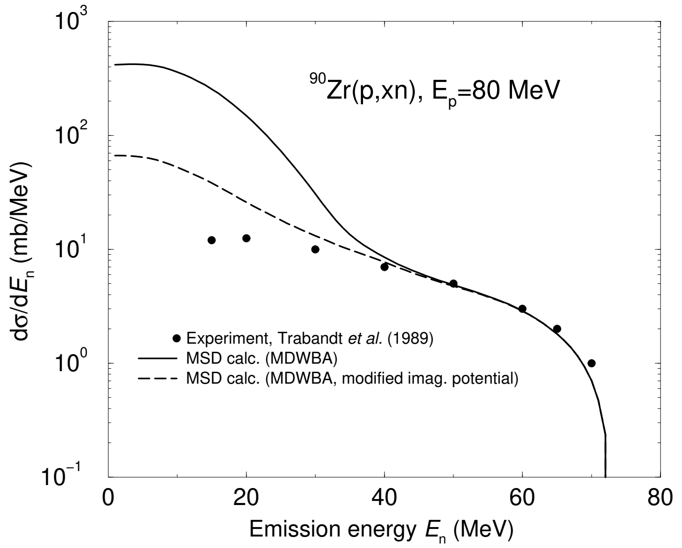

Fig. 1. Calculated preequilibrium emission spectra when MDWBA is used. Agreement with experiment [20] is improved when no imaginary components are used in the optical potentials beyond the first stage.

4 Effective Interactions for Multi-Step Direct Reactions

The discussions of effective interactions and optical potentials addressed roughly three primary topics associated with reactions induced by nucleons. These were:

1. Formal considerations of the perturbation series used to define the "steps" in MSD reaction calculations, and the related problem of choosing an effective interaction and optical potential consistent with that series;
2. Practical issues in choosing and calculating the effective interaction for MSD calculations; and
3. Recent calculations of observables using nucleon-nucleon (NN) interactions based on *G*-matrix calculations in nuclear matter.

Progress was made in identifying the most relevant aspects of these topics, but many issues remain open pending a more complete derivation of the theory.

4.1 Formal Considerations

The formal considerations focused on the choice of perturbation series used to define the "elementary steps", which are the primary elements in MSD calculations [2, 6] and on the related problem of making a reasonable choice of the "active" *P*-space [21] in which the theory is to be implemented and the complementary passive Q-space. Formally, the series may be developed by starting with the exact expression for the projectile-target T-matrix given by:

$$T_{fi} = \langle \phi(\vec{k}'),\ \psi_F \mid V \mid \Psi_I^{(+)}(\vec{k})\rangle, \tag{1}$$

where $\phi(\vec{k}')$ is the final-state plane-wave of relative motion, ψ_F is the final-state target wave function, V is the projectile-target coupling potential and $\Psi_I^{(+)}(\vec{k})$ is the exact wave function for the colliding system. Introducing an exit channel distorting potential U_F (to be defined) and using the two-potential formula of Gell-Mann and Goldberger [22, 23] gives an alternate form of the still exact T-matrix,

$$T_{fi} = \langle \chi^{(-)}(\vec{k}'),\ \psi_F \mid U_F \mid \psi_I,\ \phi(\vec{k})\rangle + \langle \chi^{(-)}(\vec{k}'),\ \psi_F \mid V - U_F \mid \Psi_I^{(+)}(\vec{k})\rangle. \tag{2}$$

Here $\chi^{(-)}(\vec{k}')$ denotes the distorted wave function corresponding to U_F and

$$\Psi_I^{(+)}(\vec{k}) = [1 + G(V - U_I)]\psi_I \chi^{(+)}(\vec{k})\ , \tag{3}$$

where G is the full Green's function for the interacting system, and the incident channel distorted wave $\chi^{(+)}(\vec{k})$ is generated from the optical potential U_I. A multi-step series useful for the present discussion can be obtained by inserting Eq. 3 into Eq. 2, and expanding the full Green's function in terms of a distorted-wave Green's function, provided the incident and exit channel distorting potentials are chosen to be the same ($U_I = U_F$) and to be diagonal in the nuclear states. In this case the residual or perturbing projectile-target effective interaction is $V - U_I$. If these restrictions on the distorting potentials are not met one gets a more generalized series, and one in which the first term in Eq. 2 may not vanish.

By definition, any effective interaction (or operator) depends manifestly on the subspace (P space) in which it is to operate. Therefore, in the above expressions the coupling V should be regarded as effective in the sense that it is to act only within the P space which is treated explicitly. In principle, the Q space contains at least the compound states of the projectile-target system. In practice, the Q space also contains high-lying components of the wave functions of relative motion associated with short-range correlations. Because of the very strong short-range repulsion in the nucleon-nucleon (NN) interaction, the short-range correlations are important at essentially all energies for which the use of an NN potential is appropriate. Identifying the appropriate P space and a scheme for calculating the associated effective interaction and optical potential are two important objectives that will need to be addressed in a more complete derivation of MSD processes. At the higher energies of interest one of the multiple scattering series approaches developed in references [14, 24, 25] appears most promising. At the lower energies of interest where the compound states are relatively more important, the calculation of the effective interaction will be even more challenging.

4.2 Issues concerning the treatment of the intermediate states

An important issue is the treatment of the intermediate states in multistep reactions. In particular, the specification of the form factors to be used in the multistep calculation and their statistical properties. This is already important for the first step calculation, but the statistics are of particular relevance for the higher steps.

As is discussed below, in FKK approaches one commonly uses a schematic Yukawa form factor of fixed range and a fitted strength. This is most likely too simple in view of the fact that it is the magnitude of continuum cross sections that is most crucial. Recently this procedure has been improved by explicitly calculating many particle-hole matrix elements with a simple interaction [26]. While it is questionable whether this is necessary for highly averaged quantities, these results are interesting for an explicit study of the statistics of the form factors. In TUL, an approach is used to split the average transition matrix element to continuum states c at excitation energy E_x into two factors $< c|V^{res}|0 >=< c|O_\lambda|0 > F_\lambda(r;E_x)$, where O_λ is a multipole operator, the matrix element of which is an average transition strength function which is calculated using RPA. $F_\lambda(r;E_x)$ is the average radial form factor which is consistently defined using average RPA transition densities, but in actual calculations is assumed to have shapes characteristic of collective form factors, *e.g.* derivatives of optical potentials. Both factors are thus in principle connected to a microscopic interactions and represent the statistical and quantal features of the process. This approach assumes that the radial shape of the form factors does not fluctuate. This is reasonable in the first step, since the collective features of the process dominate; however, fluctuations may reduce the transition strength in higher order. This issue should be clarified, perhaps by using simple model calculations.

It was further discussed whether it is justifiable to use collective strength functions in the intermediate steps at all. This is connected to the question of the intermediate state statistics which was already discussed thoroughly in Ref. [5]. It has also been discussed in Ref. [4] in terms of "adiabatic vs. sudden" approaches, meaning whether there is time enough to develop collectivity in the intermediate states. However, time arguments, *e.g.* in terms of the width of states, are not really helpful, since time scales are not sufficiently well defined and distinct from each other. On the other hand, it was argued that the intermediate Green function $G_m = 1/(E - H_\gamma^{opt} - H_\gamma^{nuc})$ contains the full intrinsic Hamiltonian of the intermediate nuclear states H_γ^{nuc}. Its diagonal representation is written in terms of the real states and thus contains collectivity in a natural way.

4.3 Practical Issues in Determining the Effective Interaction

Historically, most MSD calculations have been carried out using a very simple residual interaction, i.e. typically of a single Yukawa form [6]. This appears to be much too simplistic, since the large body of work now available on effective forces clearly demonstrates the necessity of using more realistic interactions. Some of the specific issues discussed along these lines were

1. the *spin and isospin* dependence of the effective interaction t;
2. the *range* of t; and
3. the *density dependence* of t.

Two additional questions considered briefly were:

(d) For large energy losses, what energy in $t(E)$ should be used?

(e) Which types of data might be most sensitive to the above effects?

It was generally agreed, based on experience gained in treating discrete excitations, that for energies below pion production medium effects, such as those included in calculations of the *G*-matrix in nuclear matter, should be included. Moreover, it was pointed out that the inclusion of medium effects in the form of density dependence has become routine [27-29] and that there are programs available for doing so [30]. Although the theory for the inclusion of such medium effects in MSD calculations is at present imprecise, the general expectation is that such effects are very likely to be important at a level similar to that seen for transitions to discrete states and that it is probably better to include them within the framework used for discrete states than to leave them out.

4.4 Studies Based on *G*-matrix Calculations in Nuclear Matter

Delaroche presented an extensive set of comparisons of nucleon-nucleus elastic scattering data with calculations based on optical potentials derived from the nuclear matter approach as developed by JLM [31] for incident nucleon energies below 200 MeV. Overall, the agreement between the measured and calculated cross sections and analyzing powers was quite impressive and further confirmed the importance of including medium corrections in this energy range. Moreover, the study also supports the idea that nuclear medium effects are rather reliably included via the nuclear matter approach supplemented by the local density approximation, at least in the case of elastic scattering. The details of the results described above can be found in Ref. [32].

In addition to interactions based on JLM, a number of effective interactions are readily available including a recent nuclear-matter *G*-matrix interaction developed by the Melbourne group [33]. Such interactions can be usefully applied in semiclassical calculations as well, as reported by Watanabe [34] using the Love-Franey interaction [35]. The MSD results may not be as sensitive to some of the details of the effective interaction as are discrete transitions, and calculations should be performed to understand what parts of the interaction are indeed important for MSD.

Calculations have also been performed at the University of Stellenbosch, in collaboration with Amos' group at the University of Melbourne, which aim to use more realistic interactions in multistep direct calculations [36]. With such an interaction, which takes into account the spin dependence of the strong nuclear force, the analyzing power in (p, p') reactions may be calculated and compared to recent polarization data obtained at the National Accelerator Centre, Faure, South Africa. In calculations of the angular distributions for ^{181}Ta and ^{90}Zr using the parameter-free Melbourne interaction [33], cross-sections are found which are significantly larger than the experimental cross-sections. Similar differences have been found using the M3Y interaction [37]. The reasons for this overprediction of measurements have not been established, though this may point to weaknesses in other parts of the computation of MSD processes.

5 Level densities in MSD calculations

A great deal of effort has been invested in the calculation of level densities [1] over the past twenty-five years. Progress has been extraordinarily rapid in the area of particle-hole level densities [38], which is particularly relevant to multistep calculations.

Beginning with the formula of Ericson, numerous refinements have been added. The effects of shell closures, pairing and the distinguishability of protons and neutrons have all received considerable attention within the framework of the equidistant spacing model. In addition, the effects of utilizing "realistic" single-particle energy sets have also been examined.

In the earliest pre-equilibrium calculations, all exciton states of a given particle and hole number were weighted equally. With the introduction of the distinction between multi-step direct and multi-step compound, it has been necessary to include collective effects. The TUL model is now routinely used with levels generated from an RPA calculation, which also yields coupling strengths.

Parallel developments have occurred in modeling total level densities. Some years ago, realistic single-particle schemes were used with the statistical mechanical formalism to calculate total level densities. This was then extended to models including pairing using the BCS quasiparticle approach. New computers and algorithms make it possible to calculate these level densities without using the saddle point approximation.

A remaining problem is the lack of collective effects in such calculations. Many investigators add the collective levels to the calculation for non-interacting levels [39, 40]. This clearly improves the fit to data but does require additional input. Other efforts begin with the full two-body interaction and expand the level density based on the moments of the Hamiltonian [41]. Improvements in computer performance have made these calculations feasible in large bases.

The principal remaining questions involve consistency and quality of input. Calculations can now be made using single-particle energies from a Hartree-Fock calculation. Results from the use of these levels in calculations with the RPA and with moment methods need to be compared to obtain a check on the consistency of current models. Methods used to fold together $1p$-$1h$ densities to get mp-mh densities involve additional assumptions; it would be desirable to check and see if the sum matches the total level density calculated another way. The use of more fundamental methods for calculating level densities will also be of importance as studies of reactions off of the stability line are undertaken.

6 Status of nuclear reaction MSD codes

Multistep direct calculations using the FKK theory are mostly performed using codes based upon ECIS, DWBA-91, and DWUCK. The main codes used to date for MSD calculations are those developed by Bonetti and collaborators, by Chadwick *et al.*, by Hodgson and collaborators, by Marcinkowski, by Kawano, and by Koning. A version of DWUCK that calculates modified DWBA matrix elements ("non-normal" matrix elements that include a 1/*S* factor) is available from Dietrich or Chadwick.

As has been stressed by Koning [26] and by Kawano, when using the DWUCK-4 code, one should take care of the $(2j_h + 1)$ factor for holes, which in DWUCK-4 is treated differently compared to treatments in other codes.

For a deeper understanding of multistep direct reactions, these codes will need to be further developed to include the improved physics treatments discussed in this report, including: more than one particle in the continuum; better level densities; more sophisticated effective interactions; optical potentials appropriate for MSD computations, *etc.*

A system of two codes (ORION and TRISTAN by Lenske and Wolter) is available for the TUL-type MSD calculations. ORION provides the DWBA-like cross sections for the first two steps using macroscopic form factors. TRISTAN calculates QRPA response functions and convolutes them with the ORION results to give double-differential cross sections. These calculations involve correct S-matrix elements and account for collective effects. At present they are limited to two MSD steps. Versions of the codes for the calculation of charge exchange reactions exist but are not readily available.

ORION and TRISTAN have been incorporated into the EMPIRE II code by Herman. This code also includes multistep direct and compound nucleus models. It was successfully applied for calculation of the double-differential cross sections for the recent update of the European EFF-3.0 evaluation for ^{56}Fe. The results show net improvement over the previous ones which were obtained with the classical preequilibrium model. In addition, new calculations turn out to be easier and much faster, in spite of a more complex theoretical formulation.

7 Experimental data relevant to MSD

A number of experimental programs at laboratories around the world have contributed to the development and testing of multistep direct reaction theory. Lack of space prohibits a comprehensive description of all such programs. However, it is worth noting that measurements at the National Accelerator Centre (South Africa), Indiana University (USA), Los Alamos National Laboratory (USA), the University of Maryland (USA), Oak Ridge National Laboratory (USA), Ohio University (USA), Tohoku University (Japan), Kyushu University (Japan), Milan University (Italy), Osaka University (Japan), Louvain-la-Neuve (Belgium) and Uppsala University (Sweden), have provided valuable information on reactions in the continuum region for incident energies in the 20 – 250 MeV range.

Additional experiments will be needed to advance developments in the theory. The following key areas for future experiments were proposed:

- Complete sets of double-differential emission spectra data including complex particles, e.g., (p, p'), (p, n), (p, d), (p, t), (p, α), and $(p,{}^3\mathrm{He})$ for the same target and incident energy. Similar neutron data are also required. These data will play an important role as standard data sets for model analyses.
- High resolution (p, p') energy spectra which will facilitate a more reliable extraction of collective components in the continuum region.
- Spin observables in the continuum (*e.g.*, analyzing powers and spin transfer coefficients), which will serve as a severe test of any MSD theory and help test the effective interaction used in MSD calculations. Analyzing powers have recently been measured at NAC, and these data (together with earlier measurements from Osaka University) will play an important role in testing multistep theories.
- Coincidence experiments using a multi-detector system, *e.g.*, $(p, 2p)$ reaction, for the study of multiple particle emission mechanism in MSD process.
- (p, γ) and (n, γ) experiments for the study of radiative capture mechanisms incorporating the direct-semidirect (DSD) model. Such data will guide the development of theories describing multistep processes in gamma-ray emission [42, 43]

8 Connections between Quantal and Semiclassical Models

Semiclassical theories, such as the exciton model, the hybrid model, the intranuclear cascade model, Boltzmann master equation models, and the semiclassical distorted-wave model (SCDW, described in more detail below) have played an important role in the development of the theory of preequilibrium reactions. Typically (as exemplified by the work of Gadioli presented at the workshop), they exhibit a good predictive capability, and allow a comprehensive description of the many competing reaction mechanisms. They have also provided insights into reaction mechanisms and theoretical treatments that need to be included within the quantum theories. Semiclassical theories are still the only approaches available when one considers complex processes such as those one may encounter in heavy ion reactions at some tens of MeV/nucleon where one might have the emission of several pre-equilibrium ejectiles [44]. In these cases there is an extra averaging (over the many particles which are emitted or concur in originating a cluster) and, furthermore, the angular distribution of the emitted particles calculated by the semiclassical models is often in very reasonable agreement with the data, even at very backward angles.

The semiclassical approaches do not, however, compute amplitudes for transitions in order to determine decay probabilities. Therefore they are unable to predict with high accuracy observables that are particularly sensitive to quantum effects, such as diffraction patterns in angular distributions, analyzing powers, and cross sections for collective excitations.

Whilst the semiclassical models were not the central concern of the workshop, some interesting developments and insights into the SCDW theory [45-47] were presented. These insights provided a useful theoretical basis for the widely-used intranuclear cascade (INC) model, and are repeated below.

The semi-classical distorted wave model (SCDW) for nucleon induced MSD starts from a DWBA or DWIA expansion of the T-matrix, as other models do, but works in 3-dimensional coordinate space rather than with angular momentum eigenfunctions. The argument for the notion of spatially localized nucleon-nucleon (NN) collisions, which is basic to INC, is roughly the following (it is sufficient to explain it for one-step processes): After taking the average over the random final nuclear states, the double differential inclusive cross section is given by a squared modulus of the T-matrix element of one-particle one-hole excitation summed over all the hole (particle) states below (above) the Fermi level, multiplied by a delta function which ensures energy conservation. It is this summation over the particle and hole states and the short range of the NN interaction which cancels the interference between outgoing waves generated at different points of the nucleus. Use of the local semi-classical approximation for the distorted waves leads to an expression for the cross section in the form of a sum of local average NN collision cross sections times the nucleon density weighted by the squared moduli of the distorted waves, *viz.* the probability of the incident particle reaching and the outgoing particle escaping from the point. For more than 2-step processes, the Eikonal approximation to the Green function is made for the propagation of the leading particle in intermediate states. It results in a proper desciption of the absorption and the geometrical damping of the flux of the leading particle in going from one point of collision to the next. Thus, the whole expression for the SCDW cross section is in accordance with INC except that the quantum-mechanical distorted waves now take account of all effects of the distorting potentials and the contributions from regions which are classically inaccessible for the leading particle.

9 Conclusions

Even after two decades of activity the field of multistep reactions remains very active. There are several reasons for this continued interest: the physics is an interesting mix of direct reactions and statistical theory; such a theory is needed for the interpretation of existing and anticipated experimental data; and there are important applications to medical physics, accelerator-based transmutation, and other problems. However, the state of the theory is not as happy as suggested by the high quality of fits to experimental data that have so far been obtained, since there remain open questions in the foundations of the theory, the approximations that have been developed to implement the theory, and the intercomparison of different implementations of the theory.

The workshop whose results are reported here was proposed with the objective of improving this situation by opening a discussion among practitioners of multistep calculations, experts on the fundamentals of reaction theory, and experts on various ingredients of nuclear reactions (*e.g.* effective interactions, level densities) that can reduce the number of free parameters and otherwise improve MSD calculations. We believe this objective was met, since a number of important problems were identified that should be addressed in future work. These are noted in the body of this report and in the following remarks.

A recurring theme was the importance of developing a new formulation of MSD reactions starting from a microscopic basis and incorporating key elements of multiple scattering theory. This is necessary for a correct understanding of how the effective interactions and optical potentials in MSD relate to those in well-understood simpler processes such as 1-step DWIA and standard coupled channels calculations.

Additional theoretical efforts should be directed toward understanding the consequences of statistical assumptions (*i.e.* randomness of matrix elements) on the available formulations of MSD, particularly TUL and FKK, as well as the effects of the on-shell approximation in the intermediate states in FKK. One promising approach is numerical modeling of carefully chosen simplified ("toy") problems that are realistic enough to test the issues at hand.

Along with the theoretical developments, it would be very desirable to identify and perform new experiments that test the theory more stringently than the currently available data base consisting primarily of double differential cross sections. As noted in Section 7, these should include exclusive experiments and measurements of spin variables. Such measurements have proven important in testing the theories of 1-step processes (see, *e.g.* [43] and the extensive body of data from the program of elastic and inelastic proton measurements at Indiana University).

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