acta physica slovaca vol. 49 No. 3, 345 – 350 June 1999

QUANTUM GENERATION OF THE NON-ABELIAN $SU(N)$ GAUGE FIELDS

P. I. Fomin, T. Yu. Kuzmenko[1]
*Bogolyubov Institute for Theoretical Physics, National Academy of Sciences of Ukraine,
14 b Metrologichna Str., Kyiv-143, 252143, Ukraine*

Received 27 October 1998, in final form 25 January 1999, accepted 2 February 1999

A generation mechanism of the non-Abelian gauge fields in the $SU(N)$ gauge theory is investigated. We show that the $SU(N)$ gauge fields ensuring the local invariance of the theory are generated at the quantum level only due to nonsmoothness of the scalar phases of the fundamental spinor fields. The expressions for the gauge fields are obtained in terms of the nonsmooth scalar phases.

PACS: 11.15.–q

Nowadays, the gauge principle occupies a significant place in quantum field theory. According to this principle, the fundamental interactions of elementary particles are transferred by gauge fields. The existence of these fields is considered to be necessary for ensuring the local gauge symmetries. The local $U(1)$ gauge symmetry in quantum electrodynamics was first discovered by Weyl [1]. The non-Abelian local gauge symmetries and corresponding gauge fields were introduced by Yang and Mills [2]. Based on this approach, later on the structure of weak and strong interactions was established [3,4].

It is commonly supposed that the gauge principle must necessarily be a consequence of the requirement of the gauge symmetry locality. However, it was shown [5] that in the framework of classical field theory, the local gauge invariance can be ensured without introduction of nontrivial gauge fields, i.e., vector fields with nonzero field strengths. It is sufficient to introduce only gradient vector field $\partial_\mu B(x)$, as a "compensative field", with zero strength $(\partial_\mu \partial_\nu - \partial_\nu \partial_\mu)B(x) = 0$. Such field does not contribute to dynamics [5]. From the viewpoint of the classification of fields by spin, the scalar field $B(x)$ corresponds to spin of zero and gradient vector field $\partial_\mu B(x)$ is longitudinal. True vector gauge fields A_μ are transversal fields corresponding to spin of unity. Gauge invariance of theory means that the longitudinal part of vector gauge fields does not contribute to dynamics.

If so, what is the real cause of the existence of gauge fields and interactions? In Ref.[6], the "quantum gauge principle" was formulated in the context of quantum electrodynamics. This principle holds that the Abelian $U(1)$ gauge fields are generated at the quantum level only and the generation of these fields is related to nonsmoothness of the field trajectories in the Feynman path integrals, by which the field quantization is determined. In this paper, we investigate the

[1] E-mail address: tanya@ap3.bitp.kiev.ua

mechanism of non-Abelian $SU(N)$ gauge field generation. It is shown that the non-Abelian nontrivial vector fields are generated because of nonsmoothness of the field trajectories for the scalar phases of the spinor fields in the $SU(N)$ gauge theory.

Let us consider the Lagrangian for free spinor fields

$$L = i\overline{\psi}^j \gamma^\mu \partial_\mu \psi^j - m\overline{\psi}^j \psi^j, \tag{1}$$

where $j = 1, 2, ..., N$. In what follows the index j will be omitted.

The Lagrangian (1) is invariant under global non-Abelian $SU(N)$-transformations

$$\psi'(x) = e^{it^a\omega_a}\psi(x), \quad \overline{\psi}'(x) = \overline{\psi}(x)e^{-it^a\omega_a}, \tag{2}$$

where t^a are $SU(N)$ group generators, $\omega_a = const$, $a = 1, 2, ..., N^2 - 1$.

In the framework of classical theory, physical fields are known to be described by sufficiently smooth functions. Considering smooth local $SU(N)$-transformations

$$\psi'(x) = e^{it^a\omega_a(x)}\psi(x), \quad \overline{\psi}'(x) = \overline{\psi}(x)e^{-it^a\omega_a(x)}, \tag{3}$$

we obtain that the transformed Lagrangian differs from the original one by the term:

$$\triangle L = i\overline{\psi}(x)e^{-it^a\omega_a(x)}\gamma^\mu(\partial_\mu e^{it^b\omega_b(x)})\psi(x). \tag{4}$$

In Ref.[5], it was shown that the local gauge invariance of the transformed Lagrangian can be ensured by introducing scalar fields $B_a(x)$. To put it another way, the Lagrangian

$$L = i\overline{\psi}\gamma^\mu\partial_\mu\psi + i\overline{\psi}(x)e^{-it^aB_a(x)}\gamma^\mu(\partial_\mu e^{it^bB_b(x)})\psi(x) - m\overline{\psi}\psi$$

is invariant under the transformations (3) provided that the fields $B_a(x)$ transform as:

$$e^{it^aB'_a(x)} = e^{it^aB_a(x)}e^{-it^b\omega_b(x)}.$$

The introduced scalar fields $B_a(x)$ do not contribute to dynamics, since they do not give rise to nonzero strengths and can be excluded by means of the smooth point transformations of the field variables $\psi \to \exp(it^aB_a)\,\psi$ [5]. Thus we need not compensate the term (4) by introducing nontrivial vector fields A^a_μ that do not reduce to gradients of scalar functions.

The situation changes in the quantum approach. In the Feynman formulation of quantum field theory the transition amplitudes are expressed by the path integrals that are determined on nonsmooth field trajectories [7]. In this context the Lagrangian (1) and its symmetries are determined on the class of nonsmooth functions $\psi(x)$, corresponding to nonsmooth trajectories in path integrals. In the strict sense, the derivatives involved in the Lagrangian (1) are discontinuous functions. From physics standpoint, field trajectory nonsmoothnesses are related to quantum fluctuations of the local fields. Feynman integrals, as a rule, are additionally specified by the implicit switch to "smoothed-out" approximations [8]. In this case the degrees of freedom corresponding to gauge vector fields are lost. Here we show that, as in quantum electrodynamics [6], in the non-Abelian $SU(N)$ gauge theory these degrees of freedom can be explicitly taken into account when "smoothing" of nonsmooth fields is more carefully carried out.

Let us approximate nonsmooth functions $\theta^a(x)$ by smooth functions $\omega^a(x)$:

$$\theta^a(x) = \omega^a(x) + ...$$

In order to write down the next term of the "smoothed-out" representation of the nonsmooth functions $\theta^a(x)$ it is necessary to consider the behaviour of the first derivatives of $\theta^a(x)$. The derivatives $\partial_\mu\theta^a(x)$ at nonsmoothness points of $\theta^a(x)$ are discontinuous functions. Since the derivatives $\partial_\mu\omega^a(x)$ are continuous functions, they approximate badly the behaviour of the derivatives of the "smoothed-out" $\theta^a(x)$. Let us denote a difference between them by $\theta^a_\mu(x)$ and write $\partial_\mu\theta^a(x)$ as follows:

$$\partial_\mu\theta^a(x) = \partial_\mu\omega^a(x) + \theta^a_\mu(x). \tag{5}$$

Since the nonsmooth fields $\theta^a_\mu(x)$ do not reduce to gradients of smooth scalar fields, they are the nontrivial vector fields that give rise to nonzero field strengths:

$$\partial_\mu\theta^a_\nu(x) - \partial_\nu\theta^a_\mu(x) \neq 0.$$

Therefore the fields $\partial_\mu\theta^a(x)$ involve the additional degrees of freedom which are related to nonsmoothness of the $\theta^a(x)$. It should be noted that the fields $\theta^a_\mu(x)$ are ambiguously determined due to ambiguity of choice of $\omega^a(x)$.

On integrating the left and right sides of Eq.(5) over space-like contour (P) we obtain:

$$\theta^a(x) = \omega^a(x) + \int\limits_{(P)}^{x} dy^\mu\theta^a_\mu(y).$$

Let us now consider $\theta^a(x)$ as scalar phases of the spinor fields $\psi(x)$ realizing the fundamental representation of the $SU(N)$ gauge group and separate out these phase degrees of freedom in an explicit form:

$$\psi(x) = e^{it^a\theta_a(x)}\psi_0(x), \tag{6}$$

where the spinor fields ψ_0 are the representatives of the class of gauge-equivalent fields [9], $e^{it^a\theta_a}$ is a unitary $N\times N$ matrix. Then, provided the Lagrangian (1) is determined on the class of nonsmooth functions $\psi(x)$, using Eq.(6) we obtain:

$$L = i\overline{\psi}_0\gamma^\mu\partial_\mu\psi_0 + i\overline{\psi}_0 e^{-it^a\theta_a}\gamma^\mu\left(\partial_\mu e^{it^b\theta_b}\right)\psi_0 - m\overline{\psi}_0\psi_0. \tag{7}$$

Represent the matrix $e^{it^a\theta_a}$ as a superposition of the unit matrix I and $SU(N)$ group generators t^a:

$$e^{it^a\theta_a} = CI + iS_a t^a. \tag{8}$$

Since t^a are traceless matrices normalized by $\mathrm{Tr}(t^a t^b) = \frac{1}{2}\delta^{ab}$, the coefficients C and S_a in Eq.(8) are given by:

$$C = \frac{1}{N}\mathrm{Tr}\left(e^{it^a\theta_a}\right), \quad S_a = -2i\mathrm{Tr}\left(t^a e^{it^b\theta_b}\right). \tag{9}$$

Then taking into account the commutation rules for $SU(N)$ group generators [10] we can write down:

$$e^{-it^a\theta_a}\partial_\mu e^{it^b\theta_b} = it^a\left\{\bar{C}\partial_\mu S_a - \bar{S}_a\partial_\mu C + (f_{abc} - id_{abc})\bar{S}^b\partial_\mu S^c\right\}, \tag{10}$$

where d_{abc} (f_{abc}) are totally symmetric (antisymmetric) structural constants of $SU(N)$-group, the overline denotes complex conjugation. It should be noted that the terms proportional to the unit matrix are absent in the right side of Eq.(10) because $\mathrm{Tr}\Big(e^{-it^a\theta_a}\partial_\mu e^{it^b\theta_b}\Big) = 0$.

Since the matrix $e^{it^a\theta_a}$ is unitary, the following equation is valid:

$$\bar{C}S_a - \bar{S}_a C + (f_{abc} - id_{abc})\bar{S}^b S^c = 0. \tag{11}$$

Differentiating the left and right sides of Eq.(11) and using the property of antisymmetry of f_{abc} we conclude that the expression in curly brackets in Eq.(10) is a real function. Thus this expression can be identified with the gauge fields:

$$A^a_\mu \equiv \bar{C}\partial_\mu S^a - \bar{S}^a\partial_\mu C + (f^{abc} - id^{abc})\bar{S}_b\partial_\mu S_c. \tag{12}$$

Unlike the gauge field in electrodynamics [6], the fields A^a_μ are nonlinear functions of $\theta^a(x)$. As a consequence of nonsmoothness of the phases $\theta^a(x)$ the fields A^a_μ are also not smooth. If we take into account only the first term in the right side of relation (5) we obtain that the fields A^a_μ do not contribute to the dynamics, as in classical field theory [5], and the degrees of freedom corresponding to gauge vector fields are lost. The account of $\theta^a_\mu(x)$ enables us to interpret the fields A^a_μ as nontrivial vector fields that give rise to nonzero field strenghths:

$$\partial_\mu A^a_\nu(x) - \partial_\nu A^a_\mu(x) \neq 0.$$

By way of illustration let us consider the Yang-Mills $SU(2)$ gauge group. In consequence of anti-commutativity of the $SU(2)$ group generators the coefficients C and S_a (see Eq.(9)) are given by:

$$C = \cos(\theta/2)\,, \quad S_a = 2n_a\sin(\theta/2)\,, \tag{13}$$

where

$$\theta = \sqrt{\theta_a\theta^a}, \quad n_a = \theta_a/\theta, \quad a = 1, 2, 3. \tag{14}$$

From Eqs.(13) and (14) it follows that the gauge fields A^a_μ can be written as:

$$A^a_\mu = n^a\partial_\mu\theta + \sin\theta(\partial_\mu n^a) + \sin^2(\theta/2)[\mathbf{n}\times\partial_\mu\mathbf{n}]^a. \tag{15}$$

Expression (15) demonstrates explicitly the relation between the Yang-Mills gauge fields and the nonsmooth scalar phases of the spinor fields.

Let us obtain the transformation law for the vector fields (12). For this purpose we consider the infinitesimal smooth local transformations for the spinor fields:

$$\psi'_0(x) = e^{it^a\omega_a(x)}\psi_0(x), \quad \overline{\psi}'_0(x) = \overline{\psi}_0(x)e^{-it^a\omega_a(x)}. \tag{16}$$

Then the Lagrangian (7) can be written as:

$$L = i\overline{\psi}'_0\gamma^\mu\partial_\mu\psi'_0 + i\overline{\psi}'_0 e^{it^a\omega_a}e^{-it^b\theta_b}\gamma^\mu\partial_\mu\left(e^{it^c\theta_c}e^{-it^l\omega_l}\right)\psi'_0 - m\overline{\psi}'_0\psi'_0. \tag{17}$$

Defining the gauge fields $A^{a\,\prime}_\mu(x)$ similarly to Eqs.(10) and (12) by the following equation:

$$it_a A^{a\,\prime}_\mu(x) = e^{it^a\omega_a}e^{-it^b\theta_b}\partial_\mu\left(e^{it^c\theta_c}e^{-it^l\omega_l}\right), \tag{18}$$

we find that the transformed gauge fields ${A^a_\mu}'(x)$ are related to the fields (12) as follows:

$$ {A^a_\mu}'(x) = A^a_\mu(x) - \partial_\mu \omega^a(x) - f_{abc}\omega^b(x)A^c_\mu(x). \tag{19} $$

Consequently, in the framework of considered scheme of the gauge field generation we derive the usual transformation law for the $SU(N)$ gauge fields, with the local gauge invariance of the Lagrangian (7) being not necessary.

Using Eqs.(10) and (12) we obtain that the Lagrangian (7) takes the form:

$$ L = i\overline{\psi}_0\gamma^\mu \hat{D}_\mu \psi_0 - m\overline{\psi}_0\psi_0, \tag{20} $$

where $\hat{D}_\mu \equiv \partial_\mu + iA^a_\mu t_a$ is the covariant derivative. It is easy to verify that the Lagrangian (20) is invariant under the transformations (16) and (19).

Therefore the gauge fields A^a_μ ensuring the local $SU(N)$ gauge invariance of the Lagrangian (20) are generated because of nonsmoothness of the field trajectories in Feynman path integral. The nonsmoothness of the fields A^a_μ corresponds to their quantum nature and means that these fields should also be quantized, i.e., continual integration is to be carried out over the variables $A^a_\mu(x)$. However the fields A^a_μ in the Lagrangian (20) do not exhibit all the properties of physical fields since they cannot propagate in space because of the absence of the kinetic term.

An expression similar to the kinetic term can be obtained by the calculation of the effective action for the spinor fields described by the Lagrangian (20). Using the results of the calculations performed in Ref.[11], we find the following expression for the kinetic term in the one-loop approximation

$$ L_{\text{eff}} = \kappa \ln\frac{\Lambda}{\mu_0} tr\hat{F}^2_{\mu\nu}, \quad \hat{F}_{\mu\nu} = [\hat{D}_\mu, \hat{D}_\nu], \tag{21} $$

where Λ and μ_0 are the momentum of the ultraviolet and infrared cut-off respectively; κ is the numerical coefficient.

The formula (21) takes the usual form [10]

$$ L_{\text{eff}} = \frac{\hbar c}{8g^2}\text{tr}F^2_{\mu\nu} $$

upon identifying

$$ g^2 = \frac{\hbar c}{8\kappa \ln\frac{\Lambda}{\mu_0}}. \tag{22} $$

The last equation relates the charge g with the parameters Λ and μ_0 as well as with the world's constants $\hbar$ and c, and thus demonstrates explicitly quantum origin of the charge.

We note in conclusion that the "compensating" gauge fields need not be artificially introduced for the local gauge invariance of the theory to be ensured. The vector gauge fields are generated through nonsmoothness of the scalar phases of the fundamental spinor fields. From the viewpoint of the described scheme of the gauge field generation, the gauge principle is an "automatic" consequence of field trajectory nonsmoothness in Feynman path integral.

Acknowledgement This work is supported in part by Swiss National Science Foundation Grant CEEC/NIS/96-98/7 IP 051219. One of the authors (PIF) is thankful to Professor H. Leutwyler for the kind hospitality at ITP of Bern University. We would like to thank Yu. Shtanov for several helpful comments and a careful reading of the manuscript.

References

[1] H. Weyl: *Z. Phys.* **56** (1929) 330

[2] C. N. Yang, R. L. Mills: *Phys. Rev.* **96** (1954) 191

[3] S. Weinberg: *Phys. Rev. Lett.* **19** (1967) 1264; A. Salam: *Elementary Particle Theory* (Ed. N. Svartholm). Almquist and Weacsell, Stockholm 1968

[4] H. Fritzsch, M. Gell-Mann, H. Leutwyler: *Phys. Lett.* **47 B** (1973) 365

[5] V. I. Ogievetski, I. V. Polubarinov: *Nuovo Cimento* **23** (1962) 173

[6] P. I. Fomin: *About the Nature of Gauge Fields and Interactions.* Proc. 10 Workshop on Problems on High Energy Physics and Field Theory, Protvino 1987 (Ed. S. N. Sokolov) Nauka, Moscow 1988, p. 229.

[7] F. A. Berezin: *Sov. Phys. Usp.* **23** (1981) 763

[8] I. M. Gelfand, A. M. Yaglom: *Usp. Mat. Nauk* **11** (1956) 77

[9] L. D. Faddeev, A. A. Slavnov: *Gauge Fields: Introduction to Quantum Theory.* Benjamin-Cummings, London 1980

[10] C. Itzykson, J. B. Zuber: *Quantum Field Theory*. McGraw-Hill, New York 1980

[11] I. L. Buchbinder, V. P. Gusynin, P. I. Fomin: *Yad. Fiz.* **44** (1986) 828