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## PROTON SPIN STRUCTURE AND QUARK-PARTON MODEL*

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The alternative to the standard formulation of the quark-parton model is proposed. Our approach is based on the solution of the master equations relating the structure and distribution functions, which consistently takes into account the intrinsic quark motion - in contradistinction to the standard infinite momentum approach, in which this motion is latently suppressed. The model well reproduces the experimental data on the both polarized and unpolarized structure functions, assuming that only the valence quarks term contributes to the nucleon spin. It is shown, the combined analysis of the polarized and unpolarized data can give an information about the effective masses and intrinsic motion of the quarks inside the nucleon. Simultaneously, it is shown that the rate of the nucleon energy carried by the quarks can be less, than estimated from the standard approach. As an addition, a prediction for the proton spin function $g_2$ is given.

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### 1 Introduction

Measuring of the structure functions is a unique tool for the study of the nucleon internal structure - together with the quark-parton model (QPM) giving the relations between the structure functions and the parton distributions, which represent a final, detailed picture of the nucleon. In this sense, these relations, obtained under definite assumptions, are extraordinary important, since the distribution functions themselves are not directly measurable. At the same time, the standard, simple formulas relating the structure and distribution functions are ordinarily considered so self-evident, that in some statements, the both are identified.

The experiments dedicated to the Deep Inelastic Scattering (DIS), are oriented to the measuring either *unpolarized* or *polarized* structure functions. The results on the unpolarized functions are well compatible with our expectations based on the QPM and QCD, but the situation for the polarized functions is much more complicated. Until now, it is not well understood, why the integral of the proton spin structure function $g_1$ is substantially less, than expected

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from very natural but possibly equally naive assumption, that the nucleon spin is created by the valence quarks. Presently, there is a strong tendency to explain the missing part of the nucleon spin as a result of the considerable contributions of the sea quarks (particularly strange quarks) and the gluons. It has been also suggested, that the quark orbital momentum can play some role as well. Nevertheless, a consistent explanation of the underlying mechanisms is still missing. During the last years, the hundreds of papers have been devoted to the nucleon spin structure, for the present status see e.g. [1,2], the comprehensive overviews [3,4] and citations therein.

In the present paper we summarize and actualize our discussion started in [5]- [7], where we have shown, that the standard formulation of the QPM, conceptually firmly connected with the infinite momentum frame (IMF), oversimplifies the parton kinematics. In [6] we demonstrated that the effect of oversimplified kinematics in IMF can have an impact particularly on the spin structure function $g_1$, or more exactly, it can substantially modify the relation between the distribution and spin structure functions.

The paper is organized as follows. In the following section the basic kinematical quantities related to the DIS are introduced and particularly the meaning of variable $x_B$ is discussed. In the Sec. 3 we consider proton as an idealized system of the quasifree, massive partons with the four-momenta on the mass shell. In the covariant formulation we deduce the relations between the structure and distribution functions for the unpolarized and polarized case. At the same time, the obtained relations are compared with those derived in the standard, IMF approach. In Sec. 4, using the results obtained in the previous sections, we propose a more realistic model of the proton, in which the internal motion of quarks is consistently taken into account. In Sec. 5 the results of the model on the polarized and unpolarized proton structure functions are compared with the experimental data and some free parameters are fixed. Some additional comments on the model and obtained results are done in the Sec. 6. The last section is devoted to the overall summary and concluding remarks.

## 2 Kinematics

First of all let us recall some basic notions used in the description of DIS and the interpretation of the experimental data on the basis of QPM. The process is usually described (see Fig. 1) by the variables

$$q^2 \equiv -Q^2 = (k-k')^2, \qquad x_B = \frac{Q^2}{2Pq}. \tag{1}$$

As a rule, lepton mass is neglected, i.e. $k^2 = k'^2 = 0$. Important assumption of QPM is that struck parton remains on-shell, that implies

$$q^2 + 2pq = 0. \tag{2}$$

Bjorken scaling variable $x_B$ can be interpreted as the fraction of the nucleon momentum carried by the parton in the nucleon infinite momentum frame (IMF). The motivation of this statement can be explained as follows. Let us denote

$$p(lab) \equiv (p_0, p_1, p_2, p_3), \qquad P(lab) \equiv (M, 0, 0, 0), \qquad q(lab) \equiv (q_0, q_1, q_2, q_3), \tag{3}$$

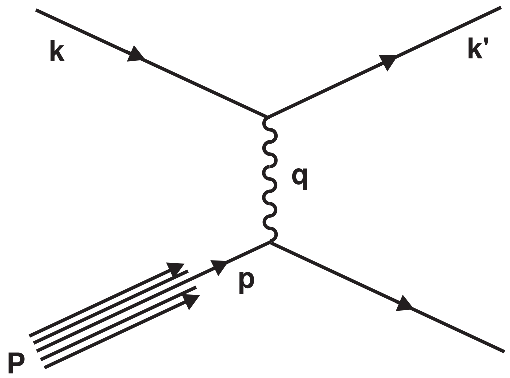


Fig. 1. Diagram describing DIS as a one photon exchange between the charged lepton and parton.

the four-momenta of the parton, nucleon and exchanged photon in the nucleon rest system (LAB). The Lorentz boost to the IMF (in the direction of collision axis) gives

$$p(\mathrm{inf}) \equiv (p_0', p_1', p_2, p_3), \qquad P(\mathrm{inf}) \equiv (P_0', P_1', 0, 0), \qquad q(\mathrm{inf}) \equiv (q_0', q_1', q_2, q_3), \tag{4}$$

where for $\beta \to -1$

$$p_0' = p_1' = \gamma(p_0 + p_1), \quad P_0' = P_1' = \gamma M, \qquad \gamma = 1/\sqrt{1-\beta^2}. \tag{5}$$

If we denote

$$x \equiv \frac{p_0'}{P_0'} = \frac{p_1'}{P_1'} = \frac{p_0 + p_1}{M}, \tag{6}$$

then one can write

$$p(\mathrm{inf}) = xP(\mathrm{inf}) + (0, 0, p_2, p_3). \tag{7}$$

Now let the lepton have initial momentum $k(lab) \equiv (k_0, -k_0, 0, 0)$. If we denote $\nu \equiv k_0 - k_0'$ and $q_L \equiv q_1$, then $q_L < 0$ and from Eqs. (1, 2) it follows

$$x_B = \frac{pq}{Pq} = \frac{p_0\nu + \mid q_L \mid p_1}{M\nu} - \frac{\vec{p}\vec{q}}{M\nu}, \tag{8}$$

where $\vec{p}$, $\vec{q}$ are the parton and photon transversal momenta . Obviously

$$k'^2 = (k-q)^2 = k^2 + q^2 - 2k_0\nu + 2k_0 \mid q_L \mid = 0,$$

$$\frac{\mid q_L \mid}{\nu} = 1 + \frac{Q^2}{2k_0\nu} = 1 + \frac{M}{k_0}x_B. \tag{9}$$

Using this relation the Eq. (8) can be modified

$$x_B = \frac{p_0 + p_1}{M} + \frac{p_1}{k_0}x_B - \frac{\vec{p}\vec{q}}{M\nu}; \tag{10}$$

therefore, if the lepton energy is sufficiently high, so $p_1/k_0 \approx 0$, one can write

$$x_B = x - \frac{p_T\ q_T}{M\nu}\cos\varphi, \tag{11}$$

where $\varphi$ is the angle between the parton and photon momenta in the transversal plane.

So, if parton transversal momenta are neglected, $x_B$ really represents fraction of momentum (6). In a higher approximation the experimentally measured $x_B$ being an integral over $\varphi$ is effectively smeared with respect to the fraction $x-$ which is not correlated with $\varphi$. An estimation of the second term in the last equation can be done as follows. Because

$$q^2 = \nu^2 - |\vec{q}|^2,$$

$$\left(\frac{\vec{q}}{\nu}\right)^2 = 1 + \frac{Q^2}{\nu^2} = 1 + \frac{4M^2}{Q^2}x_B^2, \tag{12}$$

then Eqs. (9, 12) give

$$\frac{q_T}{\nu} = \sqrt{\left(\frac{\vec{q}}{\nu}\right)^2 - \left(\frac{q_L}{\nu}\right)^2} = \sqrt{\left(\frac{4M^2}{Q^2} - \frac{M^2}{k_0^2}\right)x_B^2 - \frac{2M}{k_0}x_B} < \frac{2Mx_B}{\sqrt{Q^2}}; \tag{13}$$

therefore, for $M/k_0 \approx 0$ we obtain

$$\frac{pq}{M\nu} = \frac{p_0 + p_1}{M} - \frac{2p_T x_B}{\sqrt{Q^2}}\cos\varphi \tag{14}$$

and

$$x_B = x - \frac{2p_T x_B}{\sqrt{Q^2}}\cos\varphi, \tag{15}$$

which gives

$$\frac{x}{x_B} = 1 + \frac{2p_T}{\sqrt{Q^2}}\cos\varphi. \tag{16}$$

Therefore $x_B$ can be at sufficiently high $Q^2$ considered as a good approximation of $x$ (and vice versa). Accordingly, in the next sections we shall frequently use the approximation

$$\frac{pq}{M\nu} \approx \frac{p_0 + p_1}{M}, \tag{17}$$

which will considerably simplify calculation of some integrals. The effect of this approximation is well under control, all relevant relations could be calculated without use of it, but probably at a price, that the sense of some obtained expressions would be quite apparent only after a numeric calculation.

Now, if we assume parton phase space is spherical (in LAB) and an idealized scenario in which the parton has a mass $m^2 = p_0^2 - p_1^2 - p_2^2 - p_3^2$, then further relations can be obtained.

1) *variable x*

From Eq.(6) and the condition $x \leq 1$ it can be shown

$$x \geq \frac{m^2}{M^2}, \tag{18}$$

$$\sqrt{p_1^2+p_2^2+p_3^2} \leq p_m \equiv \frac{M^2-m^2}{2M}, \qquad p_T^2 \leq M^2(x-\frac{m^2}{M^2})(1-x). \tag{19}$$

Obviously, the highest value of $p_1$ is reached if $p_T = 0$ and

$$x = \frac{\sqrt{p_1^2+m^2}+p_1}{M} = 1 \tag{20}$$

which gives

$$p_{1\,\max} = p_m \equiv \frac{M^2-m^2}{2M}. \tag{21}$$

Then spherical symmetry implies

$$\sqrt{p_1^2+p_2^2+p_3^2} \leq p_m, \tag{22}$$

i.e. the first relation in (19) is proved. Apparently, the minimal value of $x$ is reached for $p_1 = -p_m$ and $p_T = 0$. After inserting to (6) one gets (18). Finally, the relation (6) implies

$$p_1 = \frac{M^2x^2-m^2-p_T^2}{2Mx} \tag{23}$$

which, inserted to modified relation (22)

$$p_1^2+p_T^2 \leq \left(\frac{M^2-m^2}{2M}\right)^2 \tag{24}$$

after some computation gives the second relation in (19).

2) *variable $x_B$*

Let us express $x_B$ in the LAB

$$x_B = \frac{pq}{Pq} = \frac{p_0\nu-\vec{p}\,\vec{q}}{M\nu} = \frac{1}{M}\left(\sqrt{m^2+|\vec{p}|^2}-\frac{\vec{p}\,\vec{q}}{\nu}\right) \tag{25}$$

and estimate its minimal value. With the use of (12) we obtain

$$x_B \geq \frac{1}{M}\left(\sqrt{m^2+p_m^2}-p_m\sqrt{1+\frac{4M^2}{Q^2}x_B^2}\right). \tag{26}$$

Since

$$\sqrt{1+\frac{4M^2}{Q^2}x_B^2} \leq 1+\frac{2M^2}{Q^2}x_B^2 \tag{27}$$

and

$$\frac{1}{M}\left(\sqrt{m^2+p_m^2}-p_m\right)=\frac{m^2}{M^2}, \tag{28}$$

relation (26) can be rewritten

$$x_B \geq \frac{m^2}{M^2}-\frac{2Mp_m}{Q^2}x_B^2 \geq \frac{m^2}{M^2}-\frac{2Mp_m}{Q^2}\frac{m^4}{M^4}=\frac{m^2}{M^2}\left(1-\frac{2p_m}{M}\frac{m^2}{Q^2}\right). \tag{29}$$

i.e. for $m^2 \ll Q^2$ lower limit of $x_B$ coincides with the limit (18).

## 3 Idealized scenario: quasifree partons on mass shell

In this section we imagine partons as a gas (or a mixture of gases) of quasi free particles filling up the nucleon volume. The prefix *quasi* means that partons bounded inside the nucleon behave at the interaction with external photon probing the nucleon as free particles having the four-momenta on mass shell.

### 3.1 Deconvolution of the distribution function

Let us suppose $F(x)$ is the distribution function of some sort of partons given in terms of variable $x$ according to Eq. (6) and these partons are assumed to have the mass $m$. If the spherical symmetry is assumed in the nucleon rest system and $G(p_0)d^3p$ is the number of partons in the element of the phase space, then the distribution function $F(x)$ can be expressed as the convolution

$$F(x)=\int \delta\left(\frac{p_0+p_1}{M}-x\right)G(p_0)d^3p, \qquad p_0=\sqrt{m^2+p_1^2+p_2^2+p_3^2}. \tag{30}$$

Using the set of integral variables $h, p_0, \varphi$ instead of $p_1, p_2, p_3$

$$p_1=h, \qquad p_2=\sqrt{p_0^2-m^2-h^2}\sin\varphi, \qquad p_3=\sqrt{p_0^2-m^2-h^2}\cos\varphi, \tag{31}$$

the integral (30) can be rewritten

$$F(x)=2\pi\int_m^{E_{\max}}\int_{-H}^{+H}\delta\left(\frac{p_0+h}{M}-x\right)G(p_0)p_0\,dh\,dp_0, \qquad H=\sqrt{p_0^2-m^2}. \tag{32}$$

First, let us calculate inner integral within limits $\pm H$ depending on $p_0$. For given $x$ and $p_0$ there contributes only $h$ for which

$$p_0+h=Mx, \tag{33}$$

but simultaneously $h$ must be inside the limits

$$-\sqrt{p_0^2-m^2}\leq h\leq\sqrt{p_0^2-m^2} \tag{34}$$

which means, that for

$$p_0 + \sqrt{p_0^2 - m^2} < Mx \tag{35}$$

or equivalently for

$$p_0 < \xi \equiv \frac{Mx}{2} + \frac{m^2}{2Mx}. \tag{36}$$

considered integral gives zero. For $p_0 > \xi$, when the both conditions (33), (34) are compatible for some value $h$ the integral can be evaluated

$$\int_{-H}^{+H} \delta\left(\frac{p_0 + h}{M} - x\right) G(p_0) p_0 dh = MG(p_0) p_0. \tag{37}$$

Therefore the integral (32) can be expressed

$$F(x) = 2\pi M \int_{\xi}^{E_{\max}} G(p_0) p_0 dp_0. \tag{38}$$

Let us note, the equation similar to this appears already in [8] but with the structure function $F_2(x)$ instead of the distribution one. We shall deal with the $F_2$ in the next section, where it will be shown, that the corresponding equation is more complicated. For a comparison see also [9], where on the place of $G(p_0)$ the statistical distribution characterized by some temperature and chemical potential is used.

Next, from the relation (36) we can express $x$ as a function $\xi$

$$x_{\pm} = \frac{\xi \pm \sqrt{\xi^2 - m^2}}{M}. \tag{39}$$

Using the relations (18), (36) one can easily check

$$1 \geq x_+ \geq \frac{m}{M} \geq x_- \geq \frac{m^2}{M^2}, \qquad E_{\max} = \frac{M^2 + m^2}{2M} \geq \xi \geq m. \tag{40}$$

First let us insert $x_+$ into (38)

$$F\left(\frac{\xi + \sqrt{\xi^2 - m^2}}{M}\right) = 2\pi M \int_{\xi}^{E_{\max}} G(p_0) p_0 dp_0. \tag{41}$$

Differentiation in respect to $\xi$ gives

$$G(\xi) = -\frac{1}{2\pi M^2} F'\left(\frac{\xi + \sqrt{\xi^2 - m^2}}{M}\right)\left(\frac{1}{\xi} + \frac{1}{\sqrt{\xi^2 - m^2}}\right). \tag{42}$$

Now we integrate the density $G(p_0)$ over angular variables obtaining

$$P(p_0) dp_0 \equiv \int_{\Omega} G(p_0) d^3 p = 4\pi G(p_0) p_0 \sqrt{p_0^2 - m^2} dp_0 \tag{43}$$

and after inserting into (42) we get

$$P(p_0)dp_0 = -2F'\left(\frac{p_0+\sqrt{p_0^2-m^2}}{M}\right)\frac{p_0+\sqrt{p_0^2-m^2}}{M}\frac{dp_0}{M}. \tag{44}$$

Second root $x_-$ gives very similar result

$$P(p_0)dp_0 = +2F'\left(\frac{p_0-\sqrt{p_0^2-m^2}}{M}\right)\frac{p_0-\sqrt{p_0^2-m^2}}{M}\frac{dp_0}{M}. \tag{45}$$

From the definition

$$x_\pm \equiv \frac{p_0 \pm \sqrt{p_0^2-m^2}}{M}, \tag{46}$$

the useful relations easily follow

$$x_+x_- = \frac{m^2}{M^2}, \qquad x_+ + x_- = \frac{2p_0}{M}, \qquad x_+ - x_- = \frac{2\sqrt{p_0^2-m^2}}{M}, \tag{47}$$

$$\frac{dp_0}{M} = \frac{1}{2}(1-\frac{m^2}{M^2x_\pm^2})dx_\pm, \qquad \frac{dx_+}{x_+} = -\frac{dx_-}{x_-}. \tag{48}$$

Now, the equations (44), (45) can be joined

$$P(p_0) = \mp\frac{2}{M}F'(x_\pm)x_\pm. \tag{49}$$

How to understand the two different partial intervals (40) of $x$ give independently the complete distribution $P(p_0)$ in Eq. (49)? It is due to the fact that e.g. $x_-$ represents in the integral (30) the region

$$\frac{\sqrt{p_1^2+p_T^2+m^2}+p_1}{M} = x_- \leq \frac{m}{M}. \tag{50}$$

given by the paraboloid

$$p_T^2 \leq 2m|p_1|, \qquad p_1 \leq 0, \tag{51}$$

containing complete information about $G(p_0)$ which is spherically symmetric. The similar argument is valid for $x_+$ representing the rest of sphere. The Eqs. (44, 45) imply the similarity of $F(x)$ in both intervals

$$\frac{F'(x_+)x_+}{F'(x_-)x_-} = -1, \tag{52}$$

which with the use of second relation (48) can be easily shown to be equivalent to

$$F(x_+) = F(x_-). \tag{53}$$

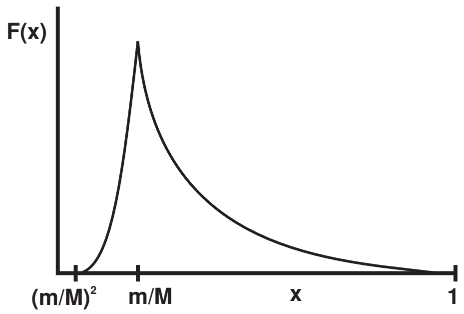


Fig. 2. Example of the function obeying Eqs. (52, 53).

The relation (49) implies the distribution function $F(x)$ should be increasing for $(m/M)^2 < x < m/M$ and decreasing for $m/M < x < 1$ e.g. as shown in Fig. 2. Now let us calculate the following integrals.

*The total number N of partons:*

$$\begin{aligned} N &= \int_m^{E_{\max}} P(p_0)dp_0 = -\int_{m/M}^1 F'(x_+)(x_+ - \frac{m^2}{M^2 x_+})dx_+ \\ &= -\int_{m/M}^1 F'(x_+)x_+dx_+ + \int_{m/M}^1 F'(x_+)x_-dx_+ \end{aligned} \tag{54}$$

The last integral can be modified with the use of (48), (52)

$$\int_{m/M}^1 F'(x_+)x_-dx_+ = -\int_{m/M}^1 F'(x_-)x_-^2\frac{dx_+}{x_+} = \int_{m/M}^{m^2/M^2} F'(x_-)x_-dx_-. \tag{55}$$

Then integration by parts gives

$$N = -\int_{m^2/M^2}^1 F'(x)xdx = \int_{m^2/M^2}^1 F(x)dx. \tag{56}$$

*The total energy E of partons:*

$$\begin{aligned} E &= \int_m^{E_{\max}} P(p_0)p_0dp_0 = -\int_{m/M}^1 F'(x_+)(x_+ - \frac{m^2}{M^2 x_+})\frac{M}{2}(x_+ + x_-)dx_+ \\ &= -\frac{M}{2}\int_{m/M}^1 F'(x_+)(x_+^2 - x_-^2)dx_+. \end{aligned} \tag{57}$$

A similar procedure as for $N$ then gives the result

$$E = -\frac{M}{2}\int_{m^2/M^2}^1 F'(x)x^2dx = M\int_{m^2/M^2}^1 F(x)xdx. \tag{58}$$

Therefore, the both descriptions based either on IMF variable $x$ or the parton energy $p_0$ in the LAB give the consistent results on the total number of partons and the fraction of energy carried by the partons.

### 3.2 Structure functions $F_2, F_1$

An important connection between the structure and distribution functions can be within QPM derived by a few (equivalent) ways, see e.g. textbooks [10-12]. In this section we shall consider the electromagnetic unpolarized structure functions assuming quasifree partons with spin 1/2. The general form of cross section for the scattering *electron + proton* and *electron + point like, Dirac particle* can be written

$$d\sigma(e^- + p) = \frac{e^4}{q^4}\frac{1}{4\sqrt{(kP)^2 - m_e^2M^2}}K^{\alpha\beta}W_{\alpha\beta}4\pi M\frac{d^3k'}{2k_0'(2\pi)^3}, \tag{59}$$

$$d\sigma(e^- + l) = \frac{e^4}{q^4}\frac{1}{4\sqrt{(kp)^2 - m_e^2m_l^2}}K^{\alpha\beta}L_{\alpha\beta}2\pi\delta((p+q)^2 - m^2)\frac{d^3k'}{2k_0'(2\pi)^3}, \tag{60}$$

where electron tensor has the standard form

$$K^{\alpha\beta} = 2(k^\alpha k'^\beta + k'^\alpha k^\beta + g^{\alpha\beta}\frac{q^2}{2}) \tag{61}$$

and the remaining hadron and lepton tensors $W_{\alpha\beta}$, $L_{\alpha\beta}$ can be written in the 'reduced' shape

$$W_{\alpha\beta} = \frac{P_\alpha P_\beta}{M^2}W_2 - g_{\alpha\beta}W_1, \tag{62}$$

$$L_{\alpha\beta} = 4p_\alpha p_\beta - 2g_{\alpha\beta}pq. \tag{63}$$

General assumption that the scattering on proton is realized via scattering on the partons implies

$$d\sigma(e^- + p) = \int F(\xi)d\sigma(e^- + l)d\xi, \tag{64}$$

where $F(\xi)$ is a probabilistic function describing distribution of partons according to some parameter(s) $\xi$. Now, if $F(\xi)$ is substituted by the usual distribution function and we assume

$$p_\alpha = \xi P_\alpha. \tag{65}$$

then it is obvious, that Eq. (64) will be satisfied provided that

$$P_\alpha P_\beta\frac{W_2}{M^2} - g_{\alpha\beta}W_1 = \frac{1}{M}\int\frac{F(\xi)}{\xi}(2\xi^2P_\alpha P_\beta - g_{\alpha\beta}\xi Pq)\delta((\xi P + q)^2 - m^2)d\xi. \tag{66}$$

For simplicity in this equation, and anywhere in this section, the weighting by the parton charges is omitted. In fact the Eq. (66) is just a master equation in [10](lesson 27, Eq. (27.4)), from which the known relations are derived

$$2MW_1(q^2,\nu) = \frac{F_2(x)}{x}, \qquad xF(x) = F_2(x) \equiv \nu W_2(q^2,\nu), \qquad x \equiv \frac{-q^2}{2M\nu}. \tag{67}$$

Here, let us point out, this result is based on the approximation (65), which is currently accepted in IMF. In fact, relation (65) in the covariant formulation is equivalent to the assumption, that the partons are static with respect to the nucleon, therefore there are suppressed not only the transversal momenta, but also longitudinal ones. In the LAB this relation implies $\xi = m/M$, so in the case of our quasifree partons, corresponding distribution function would describe rather some distribution of the parton masses.

Before repeating the above procedure for our distribution $G(p_0)d^3p$ in LAB, one has correctly account for the flux factor corresponding to partons moving inside the proton volume with velocity $\vec{v} = \vec{p}/p_0$. If this velocity has the opposite direction to the probing electron, then after passing through the whole subset $G(p_0)d^3p$ the electron has not still reached backward boundary of the proton, where meanwhile the new partons appeared. And on contrary, if the velocity of subset has the same direction as the electron, then not all of these partons have the same chance to meet this electron. Namely, the partons close to the backward boundary are excluded from the game sooner than the electron reaches them. Quantitatively, in a subset of quarks $G(p_0)d^3p$, the number of quarks limited by the proton volume and having chance to meet the probing electron will be

$$dN = (1 - v_1/v_e)G(p_0)d^3p, \tag{68}$$

where $v_1 = p_1/p_0$ is the component of quark velocity in the direction of the passing electron, $v_e \approx -1$ is the electron velocity, if we assume electron four-vector $k = (k_0, -k_0, 0, 0)$. Therefore, one can put

$$d\sigma(e+p) = \int (1 + p_1/p_0)G(p_0)d\sigma(e+q)d^3p. \tag{69}$$

If we neglect the electron mass, then inserting from Eqs. (59, 60) gives

$$K^{\alpha\beta}W_{\alpha\beta} = K^{\alpha\beta}\frac{kP}{2M}\int \frac{(1+p_1/p_0)}{kp}G(p_0)L_{\alpha\beta}\delta((p+q)^2 - m^2)d^3p. \tag{70}$$

The flux factors expressed in the proton rest frame $kP = k_0M$, $kp = k_0(p_0 + p_1)$ and tensors $W_{\alpha\beta}, L_{\alpha\beta}$ from Eqs. (62, 63) inserted to the last equation give

$$K^{\alpha\beta}\left\{P_\alpha P_\beta \frac{W_2}{M^2} - g_{\alpha\beta}W_1\right\}$$

$$= K^{\alpha\beta}\left\{\int G(p_0)(2p_\alpha p_\beta - g_{\alpha\beta}pq)\delta((p+q)^2 - m^2)\frac{d^3p}{p_0}\right\}. \tag{71}$$

The last equation can be rewritten

$$K^{\alpha\beta}(l_{\alpha\beta} - r_{\alpha\beta}) = 0, \tag{72}$$

where $l, r$ are corresponding tensors $\{...\}$ in the l.h.s. and r.h.s. of Eq. (71). Since the tensor $K^{\alpha\beta}$ obeys the current conservation

$$q_\alpha K^{\alpha\beta} = q_\beta K^{\alpha\beta} = 0, \tag{73}$$

Eq. (72) will be satisfied, if the difference $(l-r)_{\alpha\beta}$ has the form $A(P_\alpha q_\beta + P_\beta q_\alpha) + Bq_\alpha q_\beta$. In this way we get the tensor equation

$$P_\alpha P_\beta \frac{W_2}{M^2} - g_{\alpha\beta}W_1 + A(P_\alpha q_\beta + P_\beta q_\alpha) + Bq_\alpha q_\beta =$$
$$= \int \frac{G(p_0)}{p_0}(2p_\alpha p_\beta - g_{\alpha\beta}\, pq)\delta((p+q)^2 - m^2)d^3p, \quad p_0 = \sqrt{m^2 + p_1^2 + p_2^2 + p_3^2}, \tag{74}$$

for which (65) *is not required*. One can prove, that the tensors $W, L$ in the form satisfying (73), would obey Eq. (74) in which $A = B = 0$. The terms with the functions $A$ and $B$ do not contribute to the cross section. Also let us note, the correction similar to (68) was not used in the Eq. (66) since due to (65) the all partons in the applied approach have the same velocity as the proton.

Now the contracting of (74) with tensors $g^{\alpha\beta}$, $q^\alpha q^\beta$, $P^\alpha P^\beta$, $P^\alpha q^\beta$ gives in the result set of four equations

$$W_2 - 4W_1 + 2M\nu(A - xB) = \frac{1}{M\nu}\int \frac{G(p_0)}{p_0}[m^2 - 2Mx\nu]\delta\left(\frac{pq}{M\nu} - x\right)d^3p, \tag{75}$$

$$\frac{\nu}{2Mx}W_2 + W_1 - 2M\nu(A - xB) = \frac{1}{M\nu}\int \frac{G(p_0)}{p_0}[Mx\nu]\delta\left(\frac{pq}{M\nu} - x\right)d^3p, \tag{76}$$

$$W_2 - W_1 + \nu(2MA + \nu B) = \frac{1}{M\nu}\int \frac{G(p_0)}{p_0}[p_0^2 - \frac{Mx\nu}{2}]\delta\left(\frac{pq}{M\nu} - x\right)d^3p, \tag{77}$$

$$W_2 - W_1 + (M\nu - 2M^2x)A - 2Mx\nu B =$$
$$= \frac{1}{M\nu}\int \frac{G(p_0)}{p_0}[p_0Mx - \frac{Mx\nu}{2}]\delta\left(\frac{pq}{M\nu} - x\right)d^3p, \tag{78}$$

in which the $\delta$–function from the integral (74) is expressed

$$\delta((p+q)^2 - m^2) = \delta(2pq + q^2) =$$
$$= \delta\left(2M\nu\left(\frac{pq}{M\nu} - \frac{Q^2}{2M\nu}\right)\right) = \frac{1}{2M\nu}\delta\left(\frac{pq}{M\nu} - x\right). \tag{79}$$

If we define

$$V_j(x) \equiv \int G(p_0)\left(\frac{p_0}{M}\right)^j \delta\left(\frac{pq}{M\nu} - x\right)d^3p, \tag{80}$$

then the solution of the set (75)-(78) reads

$$2MW_1 = \frac{\nu}{2Mx + \nu}\cdot\left\{V_{-1}(x)\left[x - \frac{M}{\nu}\left(\frac{m^2}{M^2} - x^2\right) - 2\frac{m^2x}{\nu^2}\right]\right.$$
$$\left. + V_0(x)\frac{2Mx}{\nu} + V_1(x)\frac{2M^2x}{\nu^2}\right\}, \tag{81}$$

$$\nu W_2 = x\left(\frac{\nu}{2Mx+\nu}\right)^2 \cdot \left\{V_{-1}(x)\left[x - \frac{M}{\nu}\left(\frac{m^2}{M^2}+x^2\right) - 2\frac{m^2 x}{\nu^2}\right]\right.$$

$$\left. + V_0(x)\frac{6Mx}{\nu} + V_1(x)\frac{6M^2 x}{\nu^2}\right\}, \tag{82}$$

$$\nu^2 MA = -\left(\frac{\nu}{2Mx+\nu}\right)^2 \cdot \left\{V_{-1}(x)\left[\frac{1}{2}\left(\frac{m^2}{M^2}+3x^2\right) + \frac{m^2 x}{M\nu}\right]\right.$$

$$\left. - V_0(x)2x\left(1-\frac{Mx}{\nu}\right) - V_1(x)\frac{3Mx}{\nu}\right\}, \tag{83}$$

$$\nu^3 B = \left(\frac{\nu}{2Mx+\nu}\right)^2 \cdot \left\{V_{-1}(x)\left[\frac{1}{2}\left(\frac{m^2}{M^2}+3x^2\right) + \frac{m^2 x}{M\nu}\right]\right.$$

$$\left. - V_0(x)3x + V_1(x)\left(1-\frac{Mx}{\nu}\right)\right\}. \tag{84}$$

For next discussion we assume $\nu \gg M$, then

$$\nu W_2 \equiv F_2(x) = x^2 V_{-1}(x), \qquad MW_1 \equiv F_1(x) = \frac{x}{2}V_{-1}(x), \tag{85}$$

so it is obvious the Callan-Gross relation $2xF_1 = F_2$ holds in this approximation.

In the next step, with the use of the approximation (17), we express the integrals (80):

$$V_j(x) = \int G(p_0)\left(\frac{p_0}{M}\right)^j \delta\left(\frac{p_0+p_1}{M} - x\right) d^3p. \tag{86}$$

This relation with the use of (30),(49) implies

$$\left(\frac{p_0}{M}\right)^j P(p_0) = \mp\frac{2}{M}V_j'(x_\pm)x_\pm, \tag{87}$$

where $x_\pm$ is defined in (46). The relations (87) and (47) give

$$\frac{V_j'(x)}{V_k'(x)} = \left(\frac{p_0}{M}\right)^{j-k} = \left(\frac{x_+ + x_-}{2}\right)^{j-k} = \left(\frac{x}{2}+\frac{x_0^2}{2x}\right)^{j-k}, \qquad x_0 = \frac{m}{M}. \tag{88}$$

In the previous section we have shown such functions as Eq. (86) obey the relation (53), which means in particular, that the functions have a maximum at $x_0$ and vanish for $x \leq x_0^2$. Therefore the same statement is valid also for functions $F_2/x^2$ and $F_1/x$ from Eq. (85)

$$\frac{F_2(x_+)}{x_+^2} = \frac{F_2(x_-)}{x_-^2}, \qquad \frac{F_1(x_+)}{x_+} = \frac{F_1(x_-)}{x_-}. \tag{89}$$

This means that the structure functions of our idealized nucleon also have the maximum at $x_0$ or higher, if the peak of $V_{-1}(x)$ is not rather sharp.

Further, our considerations have started to move in previous section from the distribution function $F(x)$ for which we have obtained relation (49). The combination of this equation with (85), (87) and (47) gives

$$P(p_0) = -\frac{1}{M}\left(\frac{F_2(x)}{x^2}\right)'(x^2+x_0^2), \qquad x = \frac{p_0+\sqrt{p_0^2-m^2}}{M}, \tag{90}$$

$$F'(x) = \frac{1}{2}\left(\frac{F_2(x)}{x^2}\right)'\left(x+\frac{x_0^2}{x}\right). \tag{91}$$

How do we compare the last equation with the standard relation (67) for $F$ and $F_2$? As we have already told, the standard approach (66) would be exact in the case when the partons are static with respect to the nucleon, i.e. when $x = m/M$. The Eq. (74) itself is more exact, but the further procedure with it requires the masses of all the partons in the considered subset being equal. Therefore for a comparison let us consider first the extreme scenario when the parton distribution functions $F(x)$ and $P(p_0)$ are (see Eq. (49)) rather narrowly peaked around the points $x_0 = m/M$ and $p_0 = m$. Then for $x \approx x_0$ Eq. (91) gives

$$F'(x) = \frac{1}{2}\left(\frac{F_2(x)}{x^2}\right)'\left(x+\frac{x_0^2}{x}\right) \simeq \frac{1}{2}\frac{F_2'(x)}{x_0^2}(x_0+x_0) = \frac{F_2'(x)}{x_0} \tag{92}$$

from which the second relation (67) follows as a limiting case of (91)

$$x_0F(x_0) \approx F_2(x_0). \tag{93}$$

Now, in the realistic case when the distribution functions are broad, the exact validity of Eq. (66) again requires static partons, therefore the corresponding distribution function represents also a spectrum of masses. But then obviously the above procedure for a single $m$ can be repeated with spectrum of masses $F(x_0)$ giving in the result instead of Eq. (93) the relation

$$\int x_0F(x_0)\delta(x-x_0)dx_0 = \int F_2(x_0)\delta(x-x_0)dx_0, \tag{94}$$

which implies

$$xF(x) = F_2(x). \tag{95}$$

In this sense the approach based on Eq. (66) can be understood as a limiting case of that based on Eq. (74).

### 3.3 Spin structure functions $g_1, g_2$

In the previous section the master equation (74) has been based on the standard symmetric tensors (62) and (63) corresponding to the unpolarized DIS. After introduction the spin terms into both the tensors (see e.g. [10], Eqs. (33.9, 33.10)) our spin equation reads

$$P_\alpha P_\beta\frac{W_2}{M^2} - g_{\alpha\beta}W_1 + i\varepsilon_{\alpha\beta\lambda\sigma}q^\lambda\left[s^\sigma MG_1 + (Pqs^\sigma - sqP^\sigma)\frac{G_2}{M}\right] + A(P_\alpha q_\beta + P_\beta q_\alpha) + Bq_\alpha q_\beta$$

$$= \int G(p_0)(2p_\alpha p_\beta - g_{\alpha\beta} pq)\delta((p+q)^2 - m^2)\frac{d^3p}{p_0}$$

$$+i\varepsilon_{\alpha\beta\lambda\sigma}q^\lambda \int H(p_0) m w^\sigma \delta((p+q)^2 - m^2)\frac{d^3p}{p_0}, \tag{96}$$

where $G$ and $H$ are related to the polarized quark distributions

$$G(p_0) = \sum_j e_j^2(h_j^\uparrow(p_0) + h_j^\downarrow(p_0)), \tag{97}$$

$$H(p_0) = \sum_j e_j^2(h_j^\uparrow(p_0) - h_j^\downarrow(p_0)) \tag{98}$$

and the spin four-vectors satisfy

$$s_\mu s^\mu = w_\mu w^\mu = -1, \qquad s_\mu P^\mu = w_\mu p^\mu = 0. \tag{99}$$

The Eq. (96) requires for the spin terms

$$s^\sigma M G_1 + (Pqs^\sigma - sqP^\sigma)\frac{G_2}{M} = \frac{m}{2M\nu}\int \frac{H(p_0)}{p_0} w^\sigma \delta\left(\frac{pq}{M\nu} - x\right) d^3p, \tag{100}$$

where we use for the $\delta-$function the relation (79).

Now, to be more definite, let us consider first a extremely simple scenario (in LAB) assuming the following.
1) To the function $H$ in Eq. (98) only the valence quark term contributes.
2) Momenta distributions have the same (spherically symmetric) shape for $u$ and $d$ quarks

$$h_d(p_0) = \frac{1}{2}h_u(p_0) \equiv h(p_0). \tag{101}$$

and both the quarks have the same mass.
3) All the three quarks contribute to the proton spin equally

$$h_d^\uparrow - h_d^\downarrow = \frac{1}{2}(h_u^\uparrow - h_u^\downarrow) \equiv \Delta h(p_0) = \frac{1}{3}h(p_0), \qquad p_0 = \sqrt{m^2 + p_1^2 + p_2^2 + p_3^2}, \tag{102}$$

i.e. in a first step we ignore constraints due to axial vector current operators on the spin contribution from different flavors. Since all the three quarks are assumed to give the proton spin, the last equation implies

$$3\int \Delta h(p_0) d^3p = 1. \tag{103}$$

The combination with (98) gives

$$H(p_0) = 2\frac{4}{9}\Delta h(p_0) + \frac{1}{9}\Delta h(p_0) = \Delta h(p_0) \tag{104}$$

and

$$\int H(p_0) d^3p = \frac{1}{3}. \tag{105}$$

Now, let us assume the proton is polarized in the direction of the collision axis (coordinate one), then Eqs. (99, 100) require for the proton at rest

$$s=(0,1,0,0) \tag{106}$$

and for the quark with four-momentum $p$,

$$w=\left(\frac{p_1}{\sqrt{p_0^2-p_1^2}},\frac{p_0}{\sqrt{p_0^2-p_1^2}},0,0\right). \tag{107}$$

More rigorous derivation of this form of the quark polarization vector, which is based on the requirement of the relativistic covariance, is done in the next section. The contracting of Eq. (100) with $P_\sigma$ and $s_\sigma$ (or equivalently, simply taking $\sigma=0,1$) gives the equations

$$q_1G_2=\frac{m}{2M\nu}\int\frac{H(p_0)}{p_0}\frac{p_1}{\sqrt{p_0^2-p_1^2}}\delta\left(\frac{pq}{M\nu}-x\right)d^3p, \tag{108}$$

$$MG_1+\nu G_2=\frac{m}{2M\nu}\int\frac{H(p_0)}{p_0}\frac{p_0}{\sqrt{p_0^2-p_1^2}}\delta\left(\frac{pq}{M\nu}-x\right)d^3p. \tag{109}$$

In the next step we apply the approximations from the Eqs. (9) and (17)

$$q_1\simeq-\nu, \qquad \frac{pq}{M\nu}\simeq\frac{p_0+p_1}{M}. \tag{110}$$

Let us note, the negative sign in the first relation is connected with the choice of the lepton beam direction giving the Eq. (17). The opposite choice should give

$$q_1\simeq+\nu, \qquad \frac{pq}{M\nu}\simeq\frac{p_0-p_1}{M} \tag{111}$$

and one can check the both alternatives result in the equal pairs $G_1,G_2$, which read

$$2g_1(x)\equiv 2M^2\nu G_1=m\int\frac{H(p_0)}{p_0}\frac{p_0+p_1}{\sqrt{p_0^2-p_1^2}}\delta\left(\frac{p_0+p_1}{M}-x\right)d^3p, \tag{112}$$

$$2g_2(x)\equiv 2M\nu^2 G_2=-m\int\frac{H(p_0)}{p_0}\frac{p_1}{\sqrt{p_0^2-p_1^2}}\delta\left(\frac{p_0+p_1}{M}-x\right)d^3p. \tag{113}$$

Let us remark the integration of Eqs. (108) and (113) over $x$ gives on r.h.s. the integral

$$\int\frac{H(p_0)}{p_0}\frac{p_1}{\sqrt{p_0^2-p_1^2}}d^3p=0, \tag{114}$$

which is zero due to spherical symmetry. Therefore in this approach the first moment of $g_2(x)$ is zero as well. Now we shall pay attention particularly to the function $g_1$, which can be rewritten

$$2g_1(x)=\frac{x_0}{3}\int h(p_0)\frac{M}{p_0}\sqrt{\frac{p_0+p_1}{p_0-p_1}}\delta\left(\frac{p_0+p_1}{M}-x\right)d^3p, \qquad x_0=\frac{m}{M}. \tag{115}$$

What our assumptions 1)-3) do mean in the language of the standard IMF approach? In the previous section we have suggested that our approach is equivalent to the standard one [based on the approximation (65)], for the static quarks described by the distribution function $h(p_0)$ sharply peaked around $m$. In such a case the last equation for $p_0 \approx m$, $p_1 \approx 0$ after combining with (98) and (30) gives

$$2g_1(x) = \int \sum_j e_j^2 (h_j^{\uparrow}(p_0) - h_j^{\downarrow}(p_0)) \delta\left(\frac{p_0 + p_1}{M} - x\right) d^3p =$$

$$= \sum_j e_j^2 (f_j^{\uparrow}(x) - f_j^{\downarrow}(x)), \tag{116}$$

where $f_j(x)$ are corresponding distribution functions in the IMF, so in this limiting case our spin equation (115) is also identical to the standard one, see Eq. (33.14) in [10]. On the other hand the last equation can be in our simplified scenario rewritten

$$2g_1(x) = \frac{1}{3}\int h(p_0)\delta\left(\frac{p_0 + p_1}{M} - x\right) d^3p = \frac{1}{3}f(x) = \frac{F_{2val}(x)}{3x}. \tag{117}$$

This relation says, what our assumptions 1)-3) mean in the terms of the IMF approach, in particular we obtain

$$\Gamma_{IMF} \equiv \int g_1(x)dx = \frac{1}{6}\int f(x)dx = \frac{1}{6}. \tag{118}$$

The Eqs. (115) and (117) are equivalent for the static quarks, but how they differ for the nonstatic ones? In accordance with (86) let us denote

$$V_j(x) \equiv \int h(p_0)\left(\frac{p_0}{M}\right)^j \delta\left(\frac{p_0 + p_1}{M} - x\right) d^3p, \tag{119}$$

then (85) and (117) give

$$2g_1(x) = \frac{xV_{-1}(x)}{3}, \qquad \Gamma_{IMF} = \frac{1}{6}\int xV_{-1}(x)dx. \tag{120}$$

Now, let us calculate the corresponding integral from our rest frame equation (115)

$$\Gamma_{lab} = \frac{x_0}{6}\int\int h(p_0)\frac{M}{p_0}\sqrt{\frac{p_0 + p_1}{p_0 - p_1}}\delta\left(\frac{p_0 + p_1}{M} - x\right) d^3p dx. \tag{121}$$

Due to the $\delta$-function, the square root term in the integral can be rewritten

$$\sqrt{\frac{p_0 + p_1}{p_0 - p_1}} = \sqrt{\frac{Mx}{2p_0 - Mx}} =$$

$$= \sqrt{\frac{Mx}{2p_0}}\left(1 - \frac{Mx}{2p_0}\right)^{-\frac{1}{2}} = \left(\frac{Mx}{2p_0}\right)^{\frac{1}{2}} \sum_{j=0}^{\infty} \binom{-\frac{1}{2}}{j} (-1)^j \left(\frac{Mx}{2p_0}\right)^j \tag{122}$$

and with the using of Eq. (119) the integral correspondingly

$$\Gamma_{lab} = \frac{x_0}{6} \int \sum_{j=0}^{\infty} \binom{-\frac{1}{2}}{j} (-1)^j V_{-j-3/2}(x) \left(\frac{x}{2}\right)^{j+1/2} dx. \tag{123}$$

The integration by parts combined with the relations (88) gives

$$\int V_{-j-3/2}(x) \left(\frac{x}{2}\right)^{j+1/2} dx =$$

$$= \int V'_{-j-3/2}(x) \frac{2\,(x/2)^{j+3/2}}{j+3/2} dx = \int V'_0(x) \left(\frac{x}{2} + \frac{x_0^2}{2x}\right)^{-j-3/2} \frac{2\,(x/2)^{j+3/2}}{j+3/2} dx$$

$$= \int V'_0(x) \frac{2}{j+3/2} \left(\frac{1}{1+x_0^2/x^2}\right)^{j+3/2} dx$$

$$= \int V_0(x) 2 \left(\frac{1}{1+x_0^2/x^2}\right)^{j+1/2} \frac{2x_0^2/x^3}{\left(1+x_0^2/x^2\right)^2} dx.$$

If we denote $t \equiv x_0^2/x^2$ and $z \equiv 1/(1+t^2)$ then (123) can be rewritten

$$\Gamma_{lab} = \frac{1}{6} \int V_0(x) 4t^3 z^{5/2} \sum_{j=0}^{\infty} \binom{-\frac{1}{2}}{j} (-1)^j z^j dx = \frac{1}{6} \int V_0(x) 4t^3 z^2 \sqrt{\frac{z}{1-z}} dx,$$

which implies

$$\Gamma_{lab} = \frac{1}{6} \int_{x_0^2}^{1} \frac{4x_0^2/x^2}{(1+x_0^2/x^2)^2} V_0(x) dx. \tag{124}$$

Simultaneously, since

$$\begin{aligned} \int_{x_0^2}^{1} V_0(x) dx &= -\int_{x_0^2}^{1} x V'_0(x) dx = -\int_{x_0^2}^{1} x V'_{-1}(x) \left(\frac{x}{2} + \frac{x_0^2}{2x}\right) dx \\ &= -\int_{x_0^2}^{1} V'_{-1}(x) \left(\frac{x^2}{2} + \frac{x_0^2}{2}\right) dx = \int_{x_0^2}^{1} V_{-1}(x) x dx, \end{aligned}$$

the integral (120) can be rewritten

$$\Gamma_{IMF} = \frac{1}{6} \int_{x_0^2}^{1} V_0(x) dx. \tag{125}$$

Let us express the last integral as

$$\int_{x_0^2}^{1} V_0(x) dx = \int_{x_0^2}^{x_0} V_0(x) dx + \int_{x_0}^{1} V_0(x) dx$$

and modify the first integral on r.h.s. using substitution $y = x_0^2/x$

$$\int_{x_0^2}^{x_0} V_0(x)dx = \int_{x_0}^{1} V_0\left(\frac{x_0^2}{y}\right)\frac{x_0^2}{y^2}dy.$$

Now let us recall the general shape of the functions (119) obeying Eq. (53), which implies

$$V_0\left(\frac{x_0^2}{y}\right) = V_0(y),$$

therefore instead of (125) one can write

$$\Gamma_{IMF} = \frac{1}{6}\int_{x_0}^{1} V_0(x)\left(\frac{x^2+x_0^2}{x^2}\right)dx. \tag{126}$$

Similar modification of Eq. (124) gives

$$\Gamma_{lab} = \frac{1}{6}\int_{x_0}^{1} V_0(x)\left(\frac{4x_0^2}{x^2+x_0^2}\right)dx. \tag{127}$$

Obviously, both the integrals are equal for $V_0$ sharply peaked around $x = x_0$, but generally, for nonstatic quarks

$$\Gamma_{lab} < \Gamma_{IMF}. \tag{128}$$

What can our result (128) mean quantitatively? Obviously, it will depend on the function $V_0$, on its width. to get some feeling, we use for the $V_0$ the following parameterization. According to the Eq. (49) for $x > x_0$ one can write

$$xV_0'(x) = -\frac{M}{2}P(p_0), \qquad p_0 = \frac{M}{2}\left(x + \frac{x_0^2}{x}\right). \tag{129}$$

Now, for $p_0$ close to $m$ let us parameterize the energy distribution by

$$P(p_0) = \frac{\alpha\exp(\alpha)}{m}\exp\left(-\alpha\frac{p_0}{m}\right), \tag{130}$$

which satisfies the normalization

$$\int_{m}^{\infty} P(p_0)dp_0 = 1. \tag{131}$$

Obviously, the distribution (130) means the average quark kinetic energy equals to $m/\alpha$. Inserting (130) into (129) gives

$$V_0'(x) = -\frac{\alpha\exp(\alpha)}{2x_0x}\exp\left(-\frac{\alpha}{2}\left[\frac{x}{x_0} + \frac{x_0}{x}\right]\right). \tag{132}$$

Let us note, for $|y| \ll 1$

$$(1+y)^a \approx \exp(ay),$$

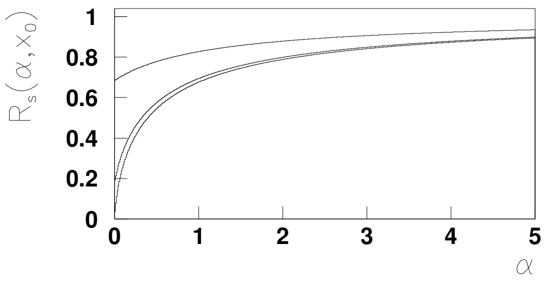


Fig. 3. Ratio $R_s$ plotted for values $x_0$=0.2, 0.02, 0.0002, in order from top to bottom.

therefore if we substitute the exponential function in (132) by

$$\exp\left(-\frac{\alpha}{2}\left[\frac{x}{x_0}+\frac{x_0}{x}\right]\right) \sim \left[(1-x)\left(1-\frac{x_0^2}{x}\right)\right]^{\alpha/2x_0} \equiv f(x,x_0),$$

$$x_0^2 \leq x \leq 1, \tag{133}$$

the resulting $V_0(x)$ will coincide with (132) in a vicinity of $x_0$, but moreover will obey the global kinematical constraint outlined in Fig. 2. The ratio of integrals (127) and (126) calculated by parts with the use of Eqs. (132) and (133) gives

$$R_s(\alpha,x_0) \equiv \frac{\Gamma_{lab}}{\Gamma_{IMF}} = \frac{4\int_{x_0}^{1} x_0/x\,(\arctan[x/x_0]-\pi/4)\,f(x,x_0)dx}{\int_{x_0}^{1}\left(1-x_0^2/x^2\right)f(x,x_0)dx}, \tag{134}$$

the results of the numerical computing are plotted in the Fig. 3. What do these curves mean? It is obvious, that for static quarks, i.e. $\alpha \to \infty$, then $R_s \to 1$, so both the approaches are equivalent, as we have already shown. On the other hand, it is also apparent, that for nonstatic quarks, with small ratio $\alpha = m/\langle E_{kin}\rangle$, both the approaches can differ substantially.

### 3.4 Covariant formulation

Master equation (96) is assembled for quark momenta distributions $G$, $H$ in the nucleon rest frame, but despite of that, the equation is relativistically covariant. Its manifestly covariant form follows immediately from (96) after the substitution

$$G(p_{0,lab}) = G\left(\frac{pP}{M}\right), \qquad H(p_{0,lab}) = H\left(\frac{pP}{M}\right). \tag{135}$$

For moving nucleon we have

$$P = (P_0,P_1,0,0), \qquad \beta = \frac{P_1}{P_0}, \qquad \gamma = \frac{P_0}{M} \tag{136}$$

and

$$\frac{pP}{M} = \frac{p_0P_0 - p_1P_1}{M} = \gamma(p_0 - \beta p_1) = p_{0,lab} \tag{137}$$

which means, that the phase space of the subset of quarks with $p_{0,lab}$ fixed, represented by the sphere

$$p_1^2 + p_2^2 + p_3^2 = p_{0,lab}^2 - m^2 \tag{138}$$

in the nucleon rest frame, is in a boosted system correctly represented by the ellipsoid with the shape defined by the Lorentz transform (137). Let us make a remark, in the same way as the Eq. (96) a similar equation can be obtained and solved also for the set of the neutrino structure functions, nevertheless in this paper we consider only the electromagnetic ones.

Further, one can obtain also covariant solution of Eq. (96) for the spin functions $G_1$, $G_2$, but first it is necessary to define correct form of the quark polarization vector $w$. Generally, this vector should be constructed from the proton momentum $P$, proton polarization vector $s$ and the quark momentum $p$:

$$w_\mu = AP_\mu + Bs_\mu + Cp_\mu, \tag{139}$$

where $A,B,C$ are invariant functions of $P,s,p$. Then contracting of Eq. (100) with $P_\sigma, q_\sigma$ and $s_\sigma$ and using relations (99) give the equations

$$-sqMG_2 = \frac{m}{2M\nu}\int H\left(\frac{pP}{M}\right)\left(AM^2 + CpP\right)\delta\left(\frac{pq}{M\nu} - x\right)\frac{d^3p}{p_0}, \tag{140}$$

$$sqMG_1 = \frac{m}{2M\nu}\int H\left(\frac{pP}{M}\right)(AM\nu + Bsq + Cpq)\,\delta\left(\frac{pq}{M\nu} - x\right)\frac{d^3p}{p_0}, \tag{141}$$

$$-MG_1 - \nu G_2 = \frac{m}{2M\nu}\int H\left(\frac{pP}{M}\right)(-B + Cps)\,\delta\left(\frac{pq}{M\nu} - x\right)\frac{d^3p}{p_0}. \tag{142}$$

Elimination of $G_1, G_2$ gives

$$\int H\left(\frac{pP}{M}\right)Cp\left(\frac{\nu P/M - q}{sq} - s\right)\delta\left(\frac{pq}{M\nu} - x\right)\frac{d^3p}{p_0} = 0 \tag{143}$$

and since $P,q,s$ are independent, $C$ must be zero. The remaining invariants $A,B$ follow from Eq. (99), which implies

$$A^2M^2 - B^2 = -1, \qquad ApP + Bps = 0 \tag{144}$$

and solution of these equations reads

$$A = \mp\frac{ps}{\sqrt{(pP)^2 - (ps)^2M^2}}, \qquad B = \pm\frac{pP}{\sqrt{(pP)^2 - (ps)^2M^2}}. \tag{145}$$

So the quark polarization vector has the form

$$w_\mu = \pm\frac{(pP)s_\mu - (ps)P_\mu}{\sqrt{(pP)^2 - (ps)^2M^2}}. \tag{146}$$

Contributions of both possible solutions [sign $+(-)$ means that quark spin is parallel (antiparallel) to the proton spin in its rest frame] are in our calculation taken into account by the difference in Eq. (98). Apparently, for the proton rest frame and polarization $s = (0,1,0,0)$ the last equation is identical to Eq. (107). Now, obtained invariants $A,B,C$ give spin structure functions from Eqs. (140, 141) in covariant form

$$G_1 = \frac{m}{2M(Pq)(sq)} \int H\left(\frac{pP}{M}\right) \frac{(pP)(sq) - (ps)(Pq)}{\sqrt{(pP)^2 - (ps)^2M^2}} \delta\left(\frac{pq}{Pq} - x\right) \frac{d^3p}{p_0}, \tag{147}$$

$$G_2 = \frac{mM}{2(Pq)(sq)} \int H\left(\frac{pP}{M}\right) \frac{ps}{\sqrt{(pP)^2 - (ps)^2M^2}} \delta\left(\frac{pq}{Pq} - x\right) \frac{d^3p}{p_0}. \tag{148}$$

Apparently, according to these relations the structure functions can depend also on mutual orientation of $s$ and $q$. Of course, this dependence is more complicated, apart the factor $sq$ ahead of the integrals, integration involves also the terms $ps$ and $pq$. This question is being studied and will be discussed in a separate paper. Our further considerations will be based on the results obtained in the previous section, which follow from Eqs. (147, 148) applied in the proton rest frame for the longitudinal polarization $s = (0,1,0,0)$. Obviously, for this case the last two equations are equivalent to Eqs. (108, 109).

The scheme based on the Eqs. (96) and (146) with all their implications suggested in the previous sections can be a priori valid for quasi-free quarks (on mass shell) filling up the nucleon volume. In this sense the scheme represents a covariant formulation of the naive QPM. We have shown that Eq. (96) in which the quark internal motion is consistently taken into account imply relations between the structure and distribution functions different from those obtained in the standard procedure relying on the preferred reference frame, IMF, which is based on the approximation $p_\mu = x_\mu P$. In the covariant formulation this approximation is equivalent to the assumption, that the partons are static with respect to the nucleon. Of course, this consequence is somewhat obscured just in the IMF, where all the relative motion is frozen, since all the processes run infinitely slowly - including the passing of the probing lepton through the nucleon. Let us remark, the standard relations (e.g. $F_2 = x\sum e_i^2 q_i$) obtained in the naive QPM with static quarks are currently applied even in the standard approach based on QCD improved QPM, which is not a consistent procedure, since it means that correct dynamics is combined with incorrect kinematics.

In this way we have shown, that the relations between the structure and distribution functions can be, at least on the level of the naive QPM, strongly modified (particularly for the polarized case) by the parton internal motion. This result can be instructive by itself. Let us remark, the impact of the quark intrinsic motion on the function $g_1(x)$ has been discussed also in some other approaches [13]-[18] and necessity of the covariant formulation for the spin structure functions has been pointed out in [19].

## 4 Model

To illustrate, that the scheme suggested in the previous section can be really valid for quasifree fermions, let us look at the Fig. 4, where the 'structure function' of the deuteron measured in

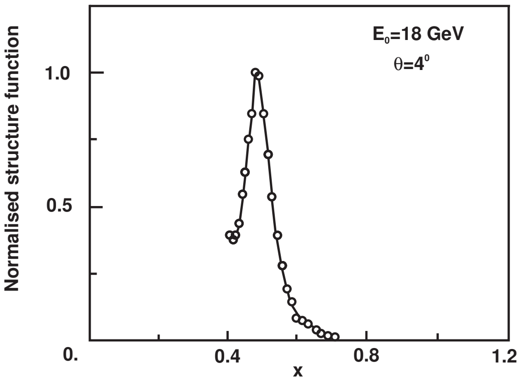


Fig. 4. The structure function for quasi-elastic $e^- d$ scattering, see text.

quasi-elastic $e^- d$ scattering [20] is shown, clearly proving the presence of two nucleons in the nucleus. The similarity with general picture Fig. 2 is well seen.

Of course, in the case of partons inside the nucleon the situation is much more delicate. The interaction among the quarks and gluons is very strong, partons themselves are mostly in some shortly living virtual state, is it possible to speak about their mass at all? Strictly speaking probably not. The mass in the exact sense is well defined only for free particles, whereas the partons are never free by definition. However let us try to assume the following. The relations obtained within the scheme suggested in the previous sections can be used as a good approximation even for the interacting quarks, but provided that the term 'mass of quasi-free parton' is substituted by the term 'parton effective mass'. By this term we mean the mass, which a free parton would have to have to interact with the probing photon equally as the real, bounded one. Intuitively, this mass should correlate to $Q^2$: a lower $Q^2$ allows more time and space for the struck parton to interact with some others, as a result the energy is transferred to a greater system than the parton itself. And on contrary, the higher $Q^2$ should mediate interaction with more 'isolated' parton. Moreover, we accept that the value of the effective mass even for a fixed $Q^2$ can fluctuate - e.g. in a dependence on the actual QCD process accompanying the photon momentum transfer. This means, that the terms in the relations involving the mass of quasi-free parton $x_0 = m/M$ will be substituted by their convolution with some 'mass distribution' $\mu$ :

$$f(x_0) \to \int \mu(x_0, Q^2) f(x_0) dx_0. \tag{149}$$

In the following we shall propose a simple, but sufficiently general model for the unknown distributions $\mu, G, H$, in which all the dynamics of the system is absorbed. Then, these distributions will be used as an input for the calculating of the corresponding structure functions. Construction of the model is based on the following assumptions and considerations:

1) Parton distribution $P(\varepsilon)d\varepsilon$ representing the number of quarks in the energy interval $\langle \varepsilon, \varepsilon + d\varepsilon \rangle$ can be formally expressed :

$$P(\varepsilon) = \sum_j r_j j \rho_j(\varepsilon), \qquad \sum_j r_j = 1, \tag{150}$$

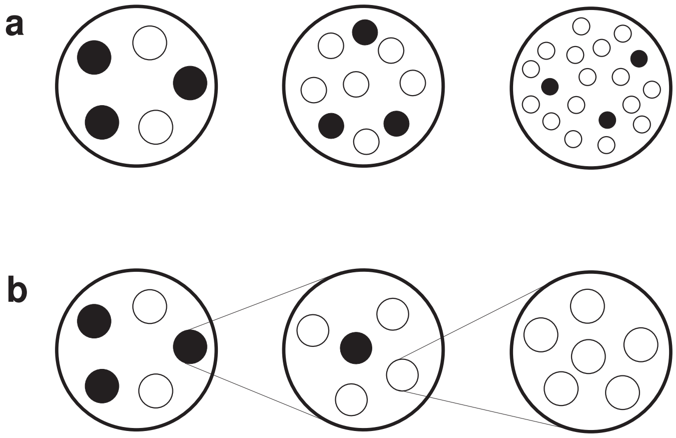


Fig. 5. Nucleon consisting of the valence and sea quarks - with different resolutions, see text.

where $r_j$ is a probability that the nucleon is in the state with $j$ partons (quarks + antiquarks) of various flavors, and $\rho_j$ is the corresponding average one-parton distribution, which satisfies

$$\int \rho_j(\varepsilon)d\varepsilon = 1. \tag{151}$$

2) Nucleon consists of the three quarks and partons (gluons + $q\overline{q}$ pairs) mediating the interaction between them, as sketched in Fig. 5$a$, where the individual pictures represent some terms in the sum (150). The flavors and spins of all the quarks in each the picture are mutually cancelled, up to the three quarks giving additively the corresponding nucleon quantum numbers. These three quarks are in the figure marked by black and in our approach are identified with the *valence quarks*. The reason, that such identification is quite sensible, is the following. Apparently, the sum (150) can be split into quark and antiquark parts $P_q(\varepsilon), P_{\overline{q}}(\varepsilon)$, then our valence term reads

$$P_{val}(\varepsilon) = P_q(\varepsilon) - P_{\overline{q}}(\varepsilon), \tag{152}$$

which in the $x-$representation exactly corresponds to the current definition of the valence quarks. Correspondingly, the unmarked quarks are identified with the *sea quarks*. But both the kinds of quarks have the same energy distributions $\rho_j(\varepsilon)$ entering the Eq. (150), in this sense they are completely equivalent. On the other hand, it is obvious, that for the valence quarks, in Eq. (150) only 'black' quarks from the figure contribute, therefore if $\rho_j$ is assumed in the first approximation independent on the flavor, then

$$P_{val}(\varepsilon) = 3\sum_{j=3}^{\infty} r_j \rho_j(\varepsilon). \tag{153}$$

3) The quarks carry only part of the nucleon energy (mass),

$$\int P(\varepsilon)\varepsilon d\varepsilon = c_q M, \tag{154}$$

where the factor $c_q$ equals roughly one half, the rest is carried by the gluons. In the first approximation we shall assume this factor is valid also for any term in the sum (150),

$$j\int\rho_j(\varepsilon)\varepsilon d\varepsilon = c_q M, \tag{155}$$

which in other words means the ratio of the total energies of quarks and gluons, together constituting the nucleon mass, is the same for all possible states sketched in Fig. 5*a*.

4) We assume all the quarks in the nucleon state $j$ have approximately the same effective mass $x_0 = m_j/M$. One can expect, for higher $j$ the parameter $x_0$ will drop and so the sum (153) can be substituted by the integral

$$P_{val}(\varepsilon) = 3\int\mu_V(x_0)\rho(\varepsilon,x_0)dx_0, \qquad \int\mu_V(x_0)dx_0 = 1. \tag{156}$$

Obviously, Eq. (150) can be with the use of Eq. (155) rewritten in a similar way:

$$P(\varepsilon) = \int\mu(x_0)\rho(\varepsilon,x_0)dx_0, \tag{157}$$

where

$$\mu(x_0) = \mu_V(x_0)\frac{c_q M}{\overline{\varepsilon}(x_0)}, \qquad \overline{\varepsilon}(x_0) = \int\rho(\varepsilon,x_0)\varepsilon d\varepsilon. \tag{158}$$

The physical meaning of the distributions $\mu_V,\mu$ is the following. The distribution $\mu(x_0)$ represents a probability, that the effective mass of the quark, which the probing lepton interacts with, is $x_0$ or alternatively, $\mu(x_0)dx_0$ is the number of quarks in the interval $\langle x_0, x_0+dx_0\rangle$, which the lepton has chance to interact with. On the other hand, the (normalized) distribution $\mu_V(x_0)$ can be interpreted as a probability, that the exchanging photon 'distinguishes the quarks with the effective mass $x_0$ - as expressed by the pictures with different granularity in Fig. 5*a.* In this sense, each picture in the Fig. 5*a* can be labeled by some $x_0$, equally as the corresponding term $\rho(\varepsilon,x_0)$ in the integral (156). Obviously, at the same time the $\mu_V(x_0)$ represents also the distribution of effective masses corresponding to the valence quark term. Intuitively, the probability of different contributions in Fig. 5*a* should depend also on $Q^2$ (higher $Q^2$ ='better resolution'), so we expect

$$\mu(x_0)\rightarrow\mu(x_0,Q^2), \qquad \mu_V(x_0)\rightarrow\mu_V(x_0,Q^2). \tag{159}$$

In the next, we shall identify these distributions with that introduced in Eq. (149).

5) The relations (85), (88) and (49) give the recipe how to obtain the structure function $F_2$ from a given energy distribution of the partons with some fixed value $x_0$ and charge $e_q$ :

$$F_2(x,x_0) = e_q^2\varphi(x,x_0), \qquad \varphi(x,x_0) = -x^2\int_x^1\frac{2V_0'(\xi)\xi}{\xi^2+x_0^2}d\xi,$$

$$2V_0'(\xi)\xi = \pm M\rho(\varepsilon,x_0), \qquad \varepsilon = \frac{M}{2}\left(\xi+\frac{x_0^2}{\xi}\right), \tag{160}$$

where the sign $+(-)$ in the second relation refers to the region $\xi < x_0$ $(\xi > x_0)$. For the application of this procedure to Eqs. (156, 157) one has to weight the contributions integrated over

$x_0$ by the corresponding (mean) charge squared. Apparently, the charge weight of the valence quarks is the constant

$$w_{val} = \frac{1}{3}\left[\left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^2\right] = \frac{1}{3} \tag{161}$$

for the proton and similarly for the neutron, $w_{val} = 2/9$. For the sea we assume in the first approximation the 'equilibrated mixture' of the quarks $u : d : s = 1 : 1 : 1$, so

$$w_{sea} = \frac{1}{3}\left[\left(\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^2\right] = \frac{2}{9}. \tag{162}$$

Then for the nucleon with $j$ quarks we get

$$w_j = \frac{3w_{val} + (j-3)w_{sea}}{j} = w_{sea} + \frac{3(w_{val} - w_{sea})}{j}, \tag{163}$$

or in terms of $x_0$

$$w(x_0) = w_{sea} + 3(w_{val} - w_{sea})\frac{\overline{\varepsilon}(x_0)}{c_q M}. \tag{164}$$

Therefore, the energy distributions (156), (157) generate the corresponding structure functions:

$$F_{2val}(x,Q^2) = 3w_{val}\int \mu_V(x_0,Q^2)\varphi(x,x_0)dx_0, \tag{165}$$

$$F_2(x,Q^2) = \int \left(\frac{c_q M}{\overline{\varepsilon}(x_0)} w_{sea} + 3(w_{val} - w_{sea})\right)\mu_V(x_0,Q^2)\varphi(x,x_0)dx_0. \tag{166}$$

6) Now, let us pay attention to the spin structure functions. According to the concept suggested in item 2), only valence quarks contribute to the nucleon spin. First, we shall consider the spin functions generated by the valence quarks with some fixed effective mass $x_0$, then we shall easily proceed to the case with the distribution $\mu_V(x_0)$.

In Sec. 3.3, for sake of simplicity, we have assumed all the three valence quarks contribute to the proton spin equally [Eq. (102)]. On the other hand it is obvious the quark symmetry group can impose an extra constraint on the contributions of different quark flavors as it follows e.g. from the philosophy of the well known Bjorken [21] and Ellis-Jaffe [22] sum rules based on the symmetries *U(6)* and *SU(3)*. In our consideration we shall not strictly assume any particular group of symmetry, but the different spin contributions of $u$- and $d$-quarks will be expressed by a free parameter $a$, $0 \leq a \leq 1$, having in the notation of Eq. (102), e.g. for the proton, the following sense

$$\Delta h_u(p_0) = 2ah(p_0), \qquad \Delta h_d(p_0) = (1-2a)h(p_0), \tag{167}$$

where $h$ is the valence distribution

$$u(p_0) = d(p_0) \equiv h(p_0), \qquad \int h(p_0)d^3p = 1, \tag{168}$$

which is not, due to different normalization, identical with the distribution $\rho(\varepsilon)$, but the both are simply related

$$\rho(\varepsilon) = 4\pi\varepsilon\sqrt{\varepsilon^2 - m^2}h(\varepsilon), \tag{169}$$

in the same way, as the distributions $P, G$ in Eq. (43).

In the case of proton, there are the particular cases:
a) $a = 0$ corresponds to the mutual spin orientation of the three valence quarks $(s_u, s_u, s_d) = (-1, +1, +1)$.
b) $a = 1/3$ corresponds to the oversimplified scenario studied in Sec. 3.3, assuming the equal contribution of all the three quarks; $(s_u, s_u, s_d) = (+1/3, +1/3, +1/3)$.
c) $a = 2/3$ corresponds to the non-relativistic *SU(6)* approach. From the wave function

$$| p, \uparrow\rangle = \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{6}}|(ud+du)u - 2uud\rangle \otimes \frac{1}{\sqrt{6}}|(\uparrow\downarrow + \downarrow\uparrow)\uparrow -2\uparrow\uparrow\downarrow\rangle\right.$$

$$\left. + \frac{1}{\sqrt{2}}|(ud-du)u\rangle \otimes \frac{1}{\sqrt{2}}|(\uparrow\downarrow - \downarrow\uparrow)\uparrow\rangle\right) \tag{170}$$

one can easily show the mean value of the spin carried by the $d(u)-$ quarks is $-1/3(4/3)$, i.e. $(s_u, s_u, s_d) = (+2/3, +2/3, -1/3)$, which agrees with $a = 2/3$ in Eq. (167).
d) $a = 1$ corresponds to the mutual orientation of the three quarks $(s_u, s_u, s_d) = (+1, +1, -1)$.

So, the proton spin function $H(p_0)$ entering the master equation (96) and expressed in terms of the functions (167) reads

$$H^p(p_0) = \frac{8}{9}au(p_0) + \frac{1}{9}(1-2a)d(p_0) = \frac{1}{9}(1+6a)h(p_0). \tag{171}$$

Assuming the neutron is isospin symmetric, its corresponding spin function will be

$$H^n(p_0) = \frac{4}{9}(1-2a)u(p_0) + \frac{2}{9}ad(p_0) = \frac{1}{9}(4-6a)h(p_0), \tag{172}$$

therefore the corresponding equations for the nucleon spin structure functions read

$$g_j^p(x, x_0) = w_{spin}^p \psi_j(x, x_0), \qquad g_j^n(x, x_0) = w_{spin}^n \psi_j(x, x_0), \qquad j = 1, 2,$$

$$w_{spin}^p = \frac{1}{9}(1+6a), \qquad w_{spin}^n = \frac{1}{9}(4-6a), \tag{173}$$

where, in accordance with Eqs. (112, 113)

$$\psi_1(x, x_0) = \frac{m}{2}\int \frac{h(p_0)}{p_0}\sqrt{\frac{p_0+p_1}{p_0-p_1}}\delta\left(\frac{p_0+p_1}{M} - x\right)d^3p, \tag{174}$$

$$\psi_2(x, x_0) = -\frac{m}{2}\int \frac{h(p_0)}{p_0}\frac{p_1}{\sqrt{p_0^2 - p_1^2}}\delta\left(\frac{p_0+p_1}{M} - x\right)d^3p. \tag{175}$$

The function $\psi_1(x,x_0)$ can be with the use of Eqs. (119, 122) expanded

$$\psi_1(x,x_0) = \frac{x_0}{2}\sum_{j=0}^{\infty}\binom{-\frac{1}{2}}{j}(-1)^j V_{-j-3/2}(x)\left(\frac{x}{2}\right)^{j+1/2}. \tag{176}$$

Since Eq. (88) implies

$$V_{-j-3/2}(x) = -\int_x^1 V_0'(\xi)\left(\frac{2\xi}{\xi^2+x_0^2}\right)^{j+3/2}d\xi, \tag{177}$$

one can easily show the sum in Eq. (176) gives

$$\psi_1(x,x_0) = -x_0\int_x^1 \frac{V_0'(\xi)\xi}{\xi^2+x_0^2}\sqrt{\frac{x\xi}{\xi^2+x_0^2-x\xi}}d\xi. \tag{178}$$

Similar manipulation with the function $\psi_2$ gives the result

$$\psi_2(x,x_0) = -\frac{x_0}{2}\int_x^1 \frac{V_0'(\xi)\xi}{\xi^2+x_0^2}\frac{\xi^2+x_0^2-2x\xi}{\sqrt{x\xi(\xi^2+x_0^2-x\xi)}}d\xi. \tag{179}$$

Obviously, for the case with the distribution $\mu_V$, the corresponding spin structure functions read

$$g_j(x,Q^2) = w_{spin}\int \mu_V(x_0,Q^2)\psi_j(x,x_0)dx_0, \qquad j=1,2. \tag{180}$$

Let us note, the structure functions $F_2, F_{2val}, g_1, g_2$ are not independent, all of them are in the corresponding way generated by the distributions $\mu_V$ and $V_0$ (or, equivalently by $\rho$).

7) Now, to make the construction suggested above applicable for some quantitative comparison with the experimental data, we have to use some reasonable, simple and sufficiently flexible parameterization for the unknown functions $\mu_V$ and $V_0$. We suggest the following.
a) Normalized distribution $\mu_V$ is assumed in the form

$$\mu_V(x_0,Q^2) = \frac{\Gamma(r+s+2)}{\Gamma(r+1)\Gamma(s+1)}\cdot x_0^r(1-x_0)^s, \qquad 0<x_0<1, \tag{181}$$

where the $Q^2-$dependence is involved in the parameters $r,s$.
b) For the function $V_0'(x)x$ we shall use the parameterization suggested in Eqs. (130-133)

$$V_0'(x)x = \mp c_{norm}\cdot f(x,x_0), \qquad f(x,x_0) \equiv \left[(1-x)\left(1-\frac{x_0^2}{x}\right)\right]^{\alpha/2x_0}, \tag{182}$$

where the upper (lower) sign in the first relation refers to the region $x > x_0 \quad (x < x_0)$ and

$$c_{norm} = \left[\int_{x_0}^1 f(x,x_0)\left(1-\frac{x_0^2}{x^2}\right)dx\right]^{-1}, \tag{183}$$

which follows e.g. from Eqs. (130, 54). Now, apparently one has to accept the parameter $\alpha \approx m/\langle E_{kin}\rangle$ depends on $x_0$ as well. Let us consider the following. Sequence of the pictures in Fig. 5$a$ can be understood as the pictures of the one and the same nucleon, but 'seen with different resolutions' as outlined in Fig. 5*b*. Then, it is natural to assume the four-momentum $P$ of the parton from some picture can be obtained from four-momenta $p_\lambda$ of $n$ partons in a picture more rightwards, representing the parton 'seen with better resolution':

$$P = \sum_\lambda p_\lambda. \tag{184}$$

Obviously, the mean values satisfy

$$\langle P_0 \rangle = n \langle p_{\lambda 0} \rangle, \qquad \left\langle \left| \vec{P} \right| \right\rangle = c_{corr} \cdot n \langle |\vec{p}_\lambda| \rangle, \qquad 0 \leq c_{corr} \leq 1, \tag{185}$$

where $c_{corr}$ equals $0(1)$ for the extreme case, when the motion of the partons in the corresponding subset is completely uncorrelated (correlated). The last relations imply the effective masses and kinetic energies obey

$$m(P) \geq n \cdot m(p), \qquad \langle E_{kin}(P) \rangle \leq n \cdot \langle E_{kin}(p) \rangle, \tag{186}$$

which means the quantity $\alpha$ is a non-decreasing function of $x_0$. In this moment we know nothing more about this function, in the next section we shall show a reasonable agreement with the experimental data can be obtained with

$$\alpha(x_0) = \alpha_1 \left(-\ln(x_0)\right)^{-\alpha_2}. \tag{187}$$

Since we parameterize the function $V_0'$ rather than the function $\rho$, it will be useful the quantity $\overline{\varepsilon}(x_0)$, defined in Eq. (157) and afterwards entering the important Eq. (166), to express also in terms of $V_0'$. Obviously, using Eqs. (160) and (182) one gets

$$\overline{\varepsilon}(x_0) = \int \rho(\varepsilon, x_0) \varepsilon d\varepsilon = -\int_{x_0}^1 V_0'(\xi) \xi \left( \xi + \frac{x_0^2}{\xi} \right) \frac{M}{2} \left( 1 - \frac{x_0^2}{\xi^2} \right) d\xi$$

$$= c_{norm} \frac{M}{2} \int_{x_0}^1 f(\xi, x_0) \left( \xi - \frac{x_0^4}{\xi^3} \right) d\xi. \tag{188}$$

Now, we can our results shortly summarize. If there are given some values of the free parameters $c_q, a, r, s, \alpha_1, \alpha_2$, then the corresponding proton and neutron structure functions can be directly calculated according to Eqs. (165, 166), 180), where the distribution $\mu_V$ is given by Eq. (181), the function $\overline{\varepsilon}(x_0)$ by Eq. (188) with the use of Eqs. (182, 183, 187) and the functions $\varphi, \psi_1, \psi_2$ are:

$$\varphi(x, x_0) = 2x^2 \int_x^1 \eta(\xi, x_0) d\xi, \tag{189}$$

$$\psi_1(x, x_0) = x_0 \int_x^1 \eta(\xi, x_0) \sqrt{\frac{x\xi}{\xi^2 + x_0^2 - x\xi}} d\xi, \tag{190}$$

$$\psi_2(x, x_0) = \frac{x_0}{2} \int_x^1 \eta(\xi, x_0) \frac{\xi^2 + x_0^2 - 2x\xi}{\sqrt{x\xi(\xi^2 + x_0^2 - x\xi)}} d\xi, \tag{191}$$

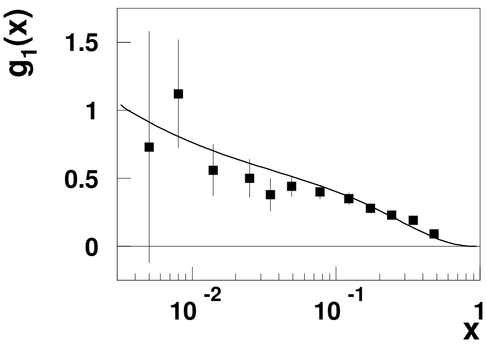


Fig. 6. Proton spin structure function $g_1(x)$ at $Q^2 = 10\ \ \mathrm{GeV}^2$. The points represent experimental data [24], the curve is the result of our calculation.

where

$$\eta(\xi, x_0) = c_{norm}\theta(\xi - x_0)\frac{f(\xi, x_0)}{\xi^2 + x_0^2}. \tag{192}$$

The last expression is calculated from Eqs. (182, 183) and (187) with the use of the step function $\theta(x) = +1(-1)$ for $x > 0$ $(x < 0)$.

## 5 Comparison with the experimental data

Now we shall try to compare our formulas for the structure functions with the existing data. We shall not attempt to make a consistent, global fit of the free parameters based on some rigorous fitting procedure, but only show the set of optimal parameters obtained by their tentative varying on the computer 'by hand'. Moreover, our constraint will be only agreement with the proton structure functions $F_2$ and $g_1$. It means that the parameter $a$, controlling asymmetry between the proton and neutron spin functions, must be somehow fixed before the fitting. For the first approximation we use the $SU(6)$ value, $a = 2/3$ [see item 6c) in the previous section].

For a comparison with $F_2$ we use the parameterizations of the world data suggested in [23] and [24], both taken for $Q^2 = 10\ \mathrm{GeV}^2$. The data for $g_1$ are taken over from the recent paper [24] of the SMC Collaboration. After some checking on the computer, the optimal set of the free parameters is considered:

$$c_q = 0.43, \quad r = -0.49, \quad s = 6.5, \quad \alpha_1 = 1.6, \quad \alpha_2 = 1.5 \tag{193}$$

Results of the calculation of the proton structure functions $g_1$ and $F_2$ with these parameters are shown in Figs. 6, 7 together with the data. Let us remark, the experimental points for $g_1$ correspond to the values evolved in [24] to $Q^2 = 10\ \mathrm{GeV}^2$. In the error bars all the quoted

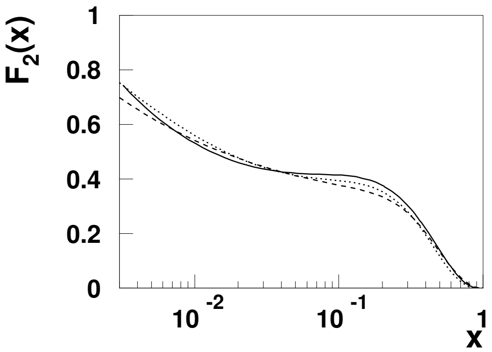


Fig. 7. Proton structure function $F_2(x)$ at $Q^2 = 10\ \text{GeV}^2$. The dotted and dashed curves represent the fits of the experimental data suggested in [23] and [24]. The full curve is the result of our calculation.

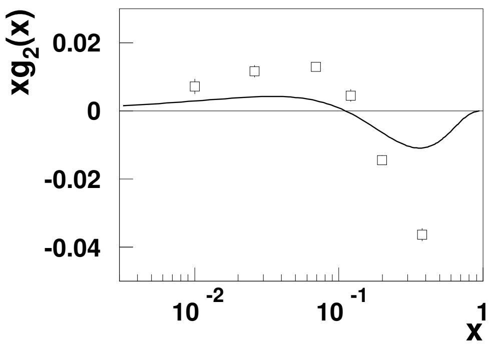


Fig. 8. Proton structure function $xg_2(x)$ at $Q^2 = 10\ \text{GeV}^2$. The points represent the term $xg_2^{WW}(x)$ (see text), the curve is the result of our calculation.

errors (statistical, systematic and those due to uncertainty of QCD evolution) are combined. Obviously, the agreement with the experimental data in both the figures can be considered very good, particularly if we take into account that our parameterization of the unknown distributions is perhaps the simplest possible and moreover, the parameters (193) still may not be optimal.

Now, having 'tuned' the free parameters by the $g_1$ and $F_2$, one can calculate the remaining functions $g_2$ and $F_{2val}$. The results are shown in Figs. 8, 9. Our $xg_2$ surely does not contradict

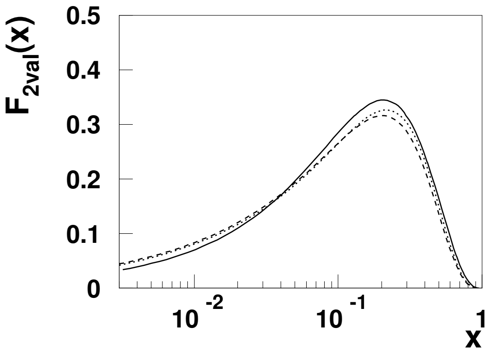


Fig. 9. Proton structure function $F_{2val}(x)$. The dashed and dotted curves represent the functions based on the standard global analysis according to the relations (194) and (197), see text. The full curve is the result of our calculation.

the experimental data [25], which are compatible with zero - with statistical errors substantially bigger, than the vertical range of the figure. But instead of the data, the comparison is done with Wandzura-Wilczek [26] twist-2 term for $xg_2^{WW}$, which is evaluated in [25] from the corresponding $g_1$. It is obvious, that two completely different approaches give at least qualitatively very similar results. The proton valence function $F_{2val}$ in Fig. 9 is compared with the corresponding combination of the distributions $xu_V(x)$ and $xd_V(x)$ obtained (for $Q^2 = 4$ GeV) in [27] by the standard global analysis:

$$F_{2val}(x) = \frac{8}{9}xu_V(x) + \frac{1}{9}xd_V(x), \quad \int u_V(x)dx = \int d_V(x)dx = 1. \tag{194}$$

Apparently, the agreement can be considered good. Nevertheless, let us note, a part of the difference between both the curves can be due to our simplification: $u_V \approx d_V$. On the other hand, the distributions of the $u$- and $d$-valence quarks are not equal and enter the functions $g_1$ and $F_{2val}$ with different weights, see Eqs. (171, 194). At the same time, our prediction for $F_{2val}$ is based on the combination valid for $g_1$

$$a\frac{8}{9}u_V(x) + (1-2a)\frac{1}{9}d_V(x) \approx \frac{1+6a}{9}q_V(x), \tag{195}$$

which is related to $F_{2val}$

$$F_{2val}(x) \approx \frac{8}{9}xq_V(x) + \frac{1}{9}xq_V(x) \quad = \quad xq_V \approx \frac{9}{1+6a}\left(\frac{8}{9}xu_V(x) + \frac{1}{9}xd_V(x)\right) \tag{196}$$

$$= \quad \frac{16}{15}xu_V(x) - \frac{1}{15}xd_V(x) \tag{197}$$

for our value $a = 2/3$. The last combination is also shown in Fig. 9 and apparently gives a slightly better agreement with our simplified $F_{2val}$. In any case, one can note, that the two different procedures, the standard one (uses input on $F_2, F_3^{\nu N}$ + QCD) and our (uses input on $F_2, g_1$ + our model) obtain a very similar picture of the function $F_{2val}(x)$, which is not directly measurable.

## 6 Discussion

Let us make a few comments on the obtained results. First of all, it should be pointed out, that our structure functions in Figs. 6-9 are calculated on the basis of very simple parameterization of the unknown distributions $\mu(x_0)$ and $V_0(x,x_0)$, but on the other hand it is essential, that the contributions from the individual components of the quark distribution correctly take into account the intrinsic quark motion, which is particularly important for the spin structure function. The effect of this motion on $g_1$ is demonstrated in Fig. 3 and the fact, that we succeeded to achieve a good agreement with the data also in Fig. 6 is just thanks to this effect. For a better insight, how our structure functions are generated, in the Fig. 10 we have displayed the initial distribution function $V_0(x,x_0)$ drawn for a few values $x_0$, together with the corresponding structure functions $F_2, g_1, xg_2$. The complete structure functions are their superpositions - weighted in the corresponding way with the use of the distribution $\mu_V(x_0)$.

Further, also some other assumptions of the model are possibly oversimplified, for a more precise calculation, at least some of them could be rightly modified - but at a price of introducing the additional free parameters. For example, the constant $w_{sea}$ [see Eq. (162)] should take into account some suppression of the $s-$ quarks [27] and probably should allow a weak dependence on $x_0$. Also for the constant $c_q$ [see Eq. (154)] some $x_0-$dependence should be allowed. Concerning this constant, let us make one more comment. The standard global fit [27] suggests (at $Q^2 = 10$ GeV$^2$) the quarks carry $\simeq 56\%$ of the nucleon energy and our fitted value $c_q$ from the Eq. (154) is roughly 43%. This difference is mainly due to the different relations between the distribution and structure function in both the approaches, see Eqs. (67, 91). The second relation (valid for a subset of quarks with effective mass $x_0$), multiplied by $x^2$ and then integrated by parts gives

$$\int_{x_0^2}^{1} xF(x,x_0)dx = \frac{1}{4}\int_{x_0^2}^{1} F_2(x,x_0)\left(3+\frac{x_0^2}{x^2}\right)dx, \tag{198}$$

which for the static quarks [$F(x,x_0) \simeq F_2(x,x_0)/x \simeq \delta(x-x_0)$, see discussion after Eq. (91)] coincides with the standard relation. Nevertheless, generally both the relations imply different rate of the nucleon energy carried by quarks. One can check numerically that for our $F_2(x,x_0)$ in a dominant region of $x_0$ the term $(x/x_0)^2$ in the integral (198) plays a minor role (see also Fig. 10, positions of the maxima of $F_2$'s are above the corresponding $x_0$, in particular for lower $x_0$), so as a result we get $3/4$ of the standard estimation of the quark contribution to the nucleon energy. This ratio agrees with the ratio obtained from the corresponding fits: $3/4 \simeq 43\% \,/\, 56\%$.

In the previous section we mentioned the effect of different shape of the $u-$ and $d-$quark distributions. A proper accounting for this difference into the model should enable to calculate consistently in a better approximation not only the proton and neutron structure functions $F_2, g_1, g_2$, but also the neutrino structure functions. Apparently then one could make a

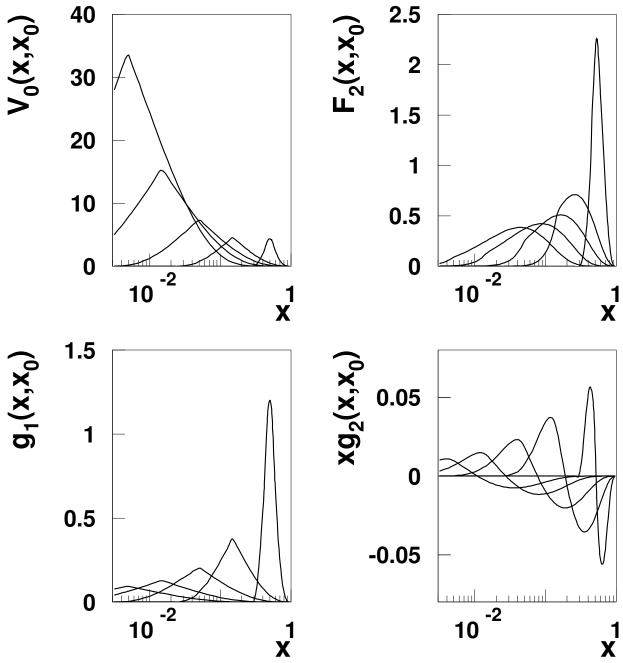


Fig. 10. Distribution functions $V_0(x,x_0)$ drawn for $x_0 = 0.005, 0.015, 0.05, 0.15, 0.5$ and the structure functions generated correspondingly.

'super-global' fit covering the both unpolarized and polarized DIS data. As a result, the flavor-dependent quark distributions $V_0(x,x_0)$ [or equivalently $\rho(\varepsilon,x_0)$] together with the corresponding effective mass distributions and the parameter $a$ controlling the relative spin contribution of the $u-$ and $d-$quarks, could be obtained.

Finally, let us point out, inclusion the spin structure function into the fit in our model enables to obtain some information about the distribution of the quark effective masses. Within our approach there are two distributions, $\mu_V$ and $\mu$, relevant for the description of the quark effective masses in the nucleon. The extrapolation of our parameterization for the $\mu$ distribution with the use of the relations $\alpha \approx m/\langle E_{kin}\rangle$ and (158, 181, 187, 193) give for $x_0 \to 0$ :

$$\mu(x_0) \sim \frac{\mu(x_0)}{\overline{\varepsilon}(x_0)} \to \frac{x_0^r}{Mx_0 + \langle E_{kin}\rangle} \to \frac{x_0^{-1.49}}{|\ln x_0|^{1.5}}, \tag{199}$$

which implies the extrapolated $\mu$ is not integrable in this limit. On the other hand, the basic distribution $\mu_V$, parameterized by Eq. (181) with the $r,s$ from the set (193) and with the use of the known relation $z\Gamma(z) = \Gamma(z+1)$ can give an estimate of the mean value:

$$\langle x_0 \rangle_V = \frac{r+1}{r+s+2} \simeq 0.064, \tag{200}$$

i.e. $\langle m \rangle \simeq 60$ MeV for $Q^2 = 10$ GeV$^2$. The corresponding kinetic term calculated as

$$\langle E_{kin} \rangle_V = \int \mu_V(x_0)(\overline{\varepsilon}(x_0) - Mx_0)dx_0 \tag{201}$$

gives a similar number ($\simeq 60$ MeV). Let us recall, these numbers are related to the valence quark term. The $Q^2$-dependence is involved only in the distribution $\mu_V(x_0, Q^2)$, i.e. in our parameterization (181) only via the powers $r(Q^2), s(Q^2)$. It follows, the structure functions, which enhance in a low$-x$ region for increasing $Q^2$, must be generated by the distribution $\mu_V(x_0, Q^2)$ in which the mean effective mass $\langle x_0 \rangle_V$ drops for increasing $Q^2$ - in the qualitative agreement with an intuitive expectation.

## 7 Summary

In the present paper we proposed an alternative covariant formulation of the QPM. The initial postulates of the standard and our approach are basically the same, despite of that the relations between the structure and distribution functions obtained in both the approaches are not identical. It is due to the fact, that in the standard approach the intrinsic quark motion is effectively suppressed by the use of the approximation $p_\mu = xP_\mu$. On the other hand, we have shown the master equations can be solved without the use of the this approximation, so in the corresponding solution the quark intrinsic motion is consistently taken into account. On the basis of the obtained relations (a priori valid for the version of naive QPM - with non-static quarks on mass shell) we propose the model, in which the distributions $(\mu_V, V_0)$ reflecting the parton dynamics are introduced with some free parameters. With the use of this model we calculated simultaneously the proton structure functions $F_2, F_{2val}, g_1, g_2$, assuming only the valence quarks term contributes to the proton spin. Then by a comparison with the data ($F_2, g_1; Q^2 = 10$ GeV$^2$) we fixed the free parameters. We found out:

1) Both the unpolarized structure functions are well reproduced by the model. The comparison is done with the data on $F_2$ and with the $F_{2val}$ obtained from the standard global analysis data.

2) At the same time, the model well agrees with the data on $g_1$. The calculated $g_2$ does not contradict the existing experimental data as well, but since in the data there are still rather big statistical errors, it is hard to say more.

3) Analysis of the fixed parameters within our approach suggests:
*i*) The quarks carry less the proton energy (almost by the factor 3/4), than estimated from the standard analysis.
*ii*) The average effective mass related to the valence quark term can be roughly 60 MeV and a similar energy can be ascribed to the corresponding motion.

So on the end we can underline the proposed model offers apart of the other results also the consistent explanation, why the experimentally measured proton spin function $g_1$ is less, than it is predicted in the standard approach based on the QCD improved QPM.

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