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# LANDAU DAMPING OF SELF-INTERACTING SCALAR FIELDS IN THE BROKEN PHASE*

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An effective kinetic theory is constructed representing the action of high frequency fluctuations on the low frequency modes. Specifically, the classical statistical expression of the modified mass term in the wave operator of these modes is studied and the Landau damping coefficient is extracted in the broken phase. The domain of validity of the classical approach is found from comparing it with the result of a 1-loop quantum calculation.

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## 1 Introduction

Relevance of combined classical field theory and kinetic theory to the high temperature linear response theory of quantum gauge fields has been demonstrated by several authors [1, 2]. For gauge fields the central object which allows explicit construction of the kinetic Boltzmann equation is the Wong-equation [3], since it determines the force exerted on non-Abelian charged particles by external fields. The calculation of the appropriate damping coefficients proceeds by relating the external fields with the corresponding induced currents, via the polarisation tensor. The imaginary part of the polarisation tensor determines the damping rate of the field.

For selfinteracting scalar fields the notion of the external field and of the force felt by the modes with low wave number is not so obvious. The physical idea based on the renormalisation group is to separate the high wave number fluctuations in form of a gas of massive particles from the low spatial frequency modes, and study the effect of the gas on the dynamics of the slow modes. The derivation of the Boltzmann equation for the gas in the low wave number background field was presented for the scalar fields only very recently [5]. The authors have found the effective action from which the high temperature behaviour of the thermal Green functions of the scalar theory can be derived.

In Ref. [5] the kinetic theory for the gas corresponding to the high wave number modes was developed from a non-relativistic potential picture. One should check whether this assumption

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is essential or the same result is received if a fully relativistic treatment of the Lagrangian dynamics of the effective particle system is applied [6].

The authors also have assumed positive curvature for the potential felt by the effective particles at the origin. There are known certain scalar models with specific internal symmetries where in the high temperature phase the symmetry is not restored and the question of the derivation of a kinetic equation is still of interest [4]. If the scalar theory is thought to be part of a Higgs+gauge system, non-perturbative treatments [7] of the coupled one-point and two-point Schwinger-Dyson equations leads to a high temperature solution with non-vanishing vacuum expectation value for the scalar field. Therefore it is of interest to explore the physical consequences of the presence of a constant nonvanishing background from the point of view of the kinetic behavior of the effective gas.

In this paper we concentrate our discussion on the Landau damping in scalar field theory in the broken phase (with non-zero average background). In section 2 we give a physically transparent simplified rederivation of the force exerted on an effective particle representing the fast modes in the background of low-frequency field configurations in the non-relativistic approximation and making use of the ususal wave-particle correspondence. In section 3 a fully relativistic rederivation of the results of Ref. [5] is presented for the kinetic theory of the fields with large spatial momenta. The emerging correction to the Bose-Einstein phase-space distribution function is used in Section 4 to compute corrections to the classical field equations of the low momentum modes. In particular the terms responsible for the generation of the Debye mass and of the damping coefficient are evaluated explicitly. In the concluding section 5 we outline the strategy of the extension of our results to phenomenologically more appealing theories.

## 2 Dynamics of the fast modes of the self-interacting scalar field

The lagrangian of the $\Phi^4$ theory is

$$\mathcal{L} = \frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi - \frac{1}{2}m^2\Phi^2 - \frac{\lambda}{4!}\Phi^4, \tag{1}$$

which leads to the following field equation:

$$(\Box - m^2)\Phi(x) = j(x), \quad \text{where} \quad j(x) = \frac{\lambda}{3!}\Phi^3(x), \quad \Box = -\partial_\mu\partial^\mu = \nabla^2 - \partial_0^2. \tag{2}$$

We would like to construct a wave equation for the slow (low-$k$) modes of the field. It will contain an additional source term, which describes the action of the fast (high-$k$) modes. For this reason we split the $\Phi$-field into two terms:

$$\begin{aligned} \Phi(x) &= \int \frac{d^4k}{(2\pi)^4}\Phi(k)e^{-ik\cdot x} = \int_0^\Lambda \frac{d^4k}{(2\pi)^4}\Phi(k)e^{-ik\cdot x} + \int_\Lambda^\infty \frac{d^4k}{(2\pi)^4}\Phi(k)e^{-ik\cdot x} \\ &= \phi(x,\Lambda) + \varphi(x,\Lambda), \end{aligned} \tag{3}$$

We assume a non-zero constant average for the slow modes, therefore we shift the right hand side of the above expression by $\bar\phi$

$$\Phi(x) = \bar\phi + \phi(x) + \varphi(x). \tag{4}$$

The wave equation for $\phi(x)$ will be modified by an induced source density, determined as a statistical average over the phase space distribution of the gas particles representing the fast modes. In this section we give a simple argument, which reproduces the expression of the force felt by the effective particles representing the high-$k$ modes in a background. For this we write down the approximate expression of the lagrangian quadratic in the fast mode $\varphi$

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2(\bar{\phi}+\phi)^2 + \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{1}{2}m^2\varphi^2 \\ &\quad -\frac{\lambda}{4!}(\bar{\phi}+\phi)^4 - \frac{\lambda}{4}(\bar{\phi}+\phi)^2\varphi^2.\end{aligned} \tag{5}$$

The linearised equation of motion for $\varphi$ looks like:

$$\Box\varphi = (m^2_{\text{eff}}[\bar{\phi}] + \lambda V[\bar{\phi},\phi])\varphi, \tag{6}$$

$$m_{\text{eff}}[\bar{\phi}] = \sqrt{m^2 + \frac{\lambda}{2}\bar{\phi}^2}, \qquad V[\bar{\phi},\phi] = \bar{\phi}\phi + \frac{1}{2}\phi^2. \tag{7}$$

Using the local dispersion equation suggested by (7)

$$E^2 - \mathbf{P}^2 = m^2 + \frac{\lambda}{2}\bar{\phi}^2 + \lambda V[\bar{\phi},\phi] \tag{8}$$

we might say on the basis of wave-particle duality, that the fast modes can be represented by particles of local $\phi$-dependent mass

$$M_{\text{loc}}[\phi,\bar{\phi}] = \sqrt{m^2 + \frac{\lambda}{2}(\bar{\phi}+\phi)^2}. \tag{9}$$

On the average the true mass of the particles is

$$m_{\text{eff}}[\bar{\phi}] = \sqrt{m^2 + \frac{\lambda}{2}\bar{\phi}^2}. \tag{10}$$

From the non-relativistic approximation to the energy

$$\begin{aligned}E &= \left(\mathbf{P}^2 + m^2_{\mathit{eff}}[\bar{\phi}] + \lambda V[\bar{\phi},\phi]\right)^{1/2} = m_{\text{eff}}[\bar{\phi}]\left(1 + \frac{\mathbf{P}^2 + \lambda V[\bar{\phi},\phi]}{m^2_{\text{eff}}[\bar{\phi}]}\right)^{1/2} \\ &\approx m_{\text{eff}}[\bar{\phi}] + \frac{\lambda V[\bar{\phi},\phi]}{2m_{\text{eff}}[\bar{\phi}]} + \frac{\mathbf{P}^2}{2m_{\text{eff}}[\bar{\phi}]},\end{aligned} \tag{11}$$

one can read off the $\phi$-dependent potential felt by the particle locally:

$$U[\bar{\phi},\phi] := \frac{\lambda}{2m_{\text{eff}}[\bar{\phi}]}V[\bar{\phi},\phi]. \tag{12}$$

From here the non-relativistic expression of the force exerted on the particle by the inhomogenous $\phi$-configurations is

$$\mathbf{F} = -\text{grad}_\phi U[\bar{\phi},\phi] = -\frac{\lambda}{2m_{\text{eff}}[\bar{\phi}]}\nabla V[\bar{\phi},\phi]. \tag{13}$$

It is easy to suggest intuitively a relativistic generalisation of this expression. Instead, in the next section we shall restart with the effective action based on the local mass and find the relativistic expression for $dp/d\tau$ applying the rules of analytical mechanics.

### 3 Kinetic theory of particles with background field dependent mass

Our starting Lagrangian for the classical point particles is designed to mimic the mass generation by the symmetry breaking background field:

$$S = -\int_{\sigma_1}^{\sigma_2} M_{\rm loc}[\bar\phi, \phi] d\tau, \tag{14}$$

where $\bar\phi$ is the constant background value of the scalar field, and $\phi$ is the long wavelength fluctuation amplitude on the top of the average field. The local mass is determined by the field values along the path $\xi(\tau)$ of the particle:

$$M^2_{\rm loc}[\bar\phi, \phi(\xi(\tau))] = m^2 + \frac{\lambda}{2}(\bar\phi + \phi(\xi(\tau))^2, \tag{15}$$

where $m^2$ is the squared mass parameter of the theory, which can be negative. This local mass is different from the "average" or "effective" mass:

$$m^2_{\rm eff} = m^2 + \frac{\lambda}{2}\bar\phi^2 \tag{16}$$

which defines the kinetic momentum of the particle through the relation

$$p_\mu = m_{\rm eff}\frac{d\xi_\mu}{d\tau}. \tag{17}$$

We introduce also the canonical momentum

$$P_\mu = M_{\rm loc}\dot\xi(\tau) = \frac{M_{\rm loc}}{m_{\rm eff}} p_\mu. \tag{18}$$

The canonical equation of motion can be derived by the usual variational procedure [6] giving

$$\frac{dP_\mu}{d\tau} = \frac{dM_{\rm loc}}{dV[\bar\phi, \phi]}\partial_\mu V[\bar\phi, \phi], \tag{19}$$

with $V[\bar\phi, \phi]$ given in (7). Now one applies the chain rule to Eq. (18) for computing the $\tau$-derivative of the canonical momentum. Combining this with Eq. (19) one obtains the proper time derivative of the kinetic momentum:

$$\frac{dp_\mu}{d\tau} = \frac{m_{\rm eff}}{M_{\rm loc}}\frac{dM_{\rm loc}}{dV[\bar\phi, \phi]}\left[\partial_\mu V[\bar\phi, \phi] - \frac{p_\mu}{m^2_{\rm eff}}(p\cdot\partial)V[\bar\phi, \phi]\right]. \tag{20}$$

The right hand side of this equation defines the relativistic generalisation of the force acting on the point particle. If the explicit expression of $M_{\rm loc}$ is used in it, one arrives at

$$\frac{dp_\mu}{d\tau} = \frac{\lambda}{2m_{\rm eff}}\frac{1}{1 + \frac{\lambda V[\bar\phi, \phi]}{m^2_{\rm eff}}}\left[\partial_\mu V[\bar\phi, \phi] - \frac{p_\mu}{m^2_{\rm eff}}(p\cdot\partial)V[\bar\phi, \phi]\right], \tag{21}$$

which almost agrees with Eq. (17) of [5], when the relation $p^2 = m^2_{\rm eff}$ is taken into account and the correspondence $m^2 \leftrightarrow m^2_{\rm eff}$ is made. There is a factor of 2 difference in the coefficient of

$\lambda V[\bar{\phi},\phi]/m_{\rm eff}^2$ in the denominator of the second factor of the previous equation. This can be traced back to the fact that Ref. [5] expands the local mass into powers of $\phi$, when apply non-relativistic approximation. Steps involving the non-relativistic approximation as was suggested by the Schrödinger-interpretation of the wave equation for the large wave number modes in [5] are not necessary in the derivation of the Boltzmann-equation.

The collisionless Boltzmann-equation arises by the application of the chain rule to the proper-time derivative of the one-particle phase space distribution:

$$\frac{df(x_\mu,p_\mu)}{d\tau}=\left[p\cdot\partial+\frac{dp_\mu}{d\tau}\frac{\partial}{\partial p_\mu}\right]f(x,p)=0, \tag{22}$$

and exploiting Eq. (20).

Here one follows the usual perturbative procedure to find corrections to the equilibrium Bose-Einstein distribution and finds results which are essentially identical to those of [5]:

$$f=f^0+\lambda f^1+..., \quad f^0(p^0)=\frac{1}{e^{\beta p^0}-1}, \tag{23}$$

$$f^1(x,p)=-\frac{1}{2}\left[\frac{1}{(p\cdot\partial)}(\bar{\phi}+\phi)\partial_\mu\phi-\frac{p_\mu}{p^2}V[\bar{\phi},\phi]\right]\frac{df_0}{dp_\mu}. \tag{24}$$

In the next section we shall calculate corrections to the linearized field equation for the long wavelength fluctuations arising from the statistically averaged source term emerging through the $\phi$-dependence of the particle action (14).

## 4 The wave equation of the slow modes

The modified wave equation of the slow modes

$$\left(\Box-m^2\right)\phi(x)-C=j(x)+\frac{\lambda}{2}\int\left[\frac{1}{M_{\rm loc}[\bar{\phi},\phi]}\frac{dV[\bar{\phi},\phi]}{d\phi}\right]\delta^{(4)}(x-\xi(\tau))d\tau, \tag{25}$$

contains the source term arising from the fast effective particles through the $\phi$-dependence of their local mass. One has to average over the phase space around the point $x$. We include into the wave equation also a constant source $C$ which should cancel any constant piece arising from the statistical averaging of the second term on the right hand side.

Using the identity

$$\int\delta^{(4)}(x-\xi(\tau))d\tau=\frac{1}{\gamma}\delta^{(3)}(\underline{x}-\underline{\xi}(t)) \tag{26}$$

one can rewrite the modified wave equation as

$$\left(\Box-m^2\right)\phi(x)-C=j(x)+\frac{\lambda}{2}\frac{m_{\rm eff}}{M_{\rm loc}}\frac{dV[\bar{\phi},\phi]}{d\phi}\frac{\delta^{(3)}(\underline{x}-\underline{\xi}(\tau))}{p_0}. \tag{27}$$

Here we have exploited the following kinematical equalities:

$$\gamma^{-1}=\sqrt{1-v^2}=\frac{m_{\rm eff}}{p_0}=\frac{M_{\rm loc}}{P_0}. \tag{28}$$

For the investigation of the damping phenomenon, it is sufficient to retain on the right hand side of equation (25) only terms which are at most linear in $\phi$. For this, we shall expand $m_{\rm eff}[\bar\phi]/M_{\rm loc}[\bar\phi,\phi]$ in powers of $\phi$ and only the first term of the expansion is kept:

$$\frac{m_{\rm eff}}{M_{\rm loc}}=\left(1+\frac{\lambda V[\bar\phi,\phi]}{m_{\rm eff}^2}\right)^{-1/2}\approx 1-\frac{\lambda V[\bar\phi,\phi]}{2m_{\rm eff}^2}. \tag{29}$$

Then the result of the statistical averaging is the following:

$$j_{\rm ind}(x)=\frac{\lambda}{2}\left(\bar\phi(x)+\phi(x)\right)\left(1-\frac{\lambda V[\bar\phi,\phi]}{2m_{\rm eff}^2}\right)\int\frac{d^3\underline{p}}{p_0}f(x,p). \tag{30}$$

Again keeping only terms up to first power in $\phi$ and the lowest power in $\lambda$ we have

$$j_{\rm ind}(x)=\frac{\lambda}{2}\left(\bar\phi(x)+\phi(x)\right)\int\frac{d^3\underline{p}}{p_0}f(x,p). \tag{31}$$

Using the perturbative solution of the Boltzmann-equation presented in the previous section one finds for the leading terms of the induced source the following expression (the indices 0 and 1 refer to the contributions of the $f^0$ and $f^1$ pieces of the phase space distribution, respectively):

$$\begin{aligned} j_{\rm ind}(x) &= j_0(x)+j_1(x) \\ &= \left(\bar\phi(x)+\phi(x)\right)\left[\frac{\lambda}{2}\int\frac{d^3\underline{p}}{p_0}f^0(p_0)+\frac{\lambda^2}{2}\int\frac{d^3\underline{p}}{p_0}f^1(\phi(x),p)\right]. \end{aligned} \tag{32}$$

Clearly, there is a non-zero constant term, proportional to the average field $\bar\phi$, which is canceled by the constant $C$ in the wave equation. In the remaining terms one has to perform the momentum integration with the integration measure $d^3\underline{p}/p_0,\quad p_0=\sqrt{|\underline{p}|^2+m_{\rm eff}^2}$. It can be rewritten in a relativistically invariant form through the equality

$$\begin{aligned} \int dp &= \int\frac{d^3\underline{p}dp_0}{(2\pi)^3}2\Theta(p^0)\delta(p_0^2-|\underline{p}|^2-m^2-\frac{\lambda}{2}\bar\phi^2) \\ &= \int\frac{dp_0}{2p_0}\delta(p_0-\sqrt{|\underline{p}|^2+m_{\rm eff}^2})2\Theta(p_0)\frac{d^3\underline{p}}{(2\pi)^3} \\ &= \int\limits_{p_0=\sqrt{|\underline{p}|^2+m_{\rm eff}^2}}\frac{d^3\underline{p}}{(2\pi)^3p_0}. \end{aligned}$$

For the explicit expression of the current the following standard integrals were performed:

$$I_0 = \int dp\, f^0(p_0), \tag{33}$$

$$I_1 = \int dp\,\frac{\omega}{p\cdot k}\,\frac{df^0}{dp_0}, \tag{34}$$

$$L_1 = \int dp\,\frac{p_0}{p^2}\,\frac{df^0}{dp_0}. \tag{35}$$

With their help the induced source terms which are at most linear in $\phi$ look like

$$j_0(x) = \frac{\lambda}{2}\bar{\phi}(x)I_0 + \frac{\lambda}{2}\phi(x)I_0, \tag{36}$$

$$j_1(x) = \frac{\lambda^2}{4}\bar{\phi}(x)\phi(x)(L_1 - I_1). \tag{37}$$

Here we also have assumed that the $\phi$-field consists solely of a single mode, characterised by the momentum 4-vector $(\omega, \mathbf{k})$.

The $I_0$ integral is easy to evaluate in the high temperature limit. In dimensionless variables it can be rewritten as

$$\begin{aligned} I_0 &= \frac{1}{2\pi^2}\int d|\underline{p}|\frac{|\underline{p}|^2}{\sqrt{|\underline{p}|^2+m_{\text{eff}}^2}}\left.\frac{1}{e^{\beta p_0}-1}\right|_{p_0=\sqrt{|\underline{p}|^2+m_{\text{eff}}^2}} \\ &= \frac{m_{\text{eff}}^2}{2\pi^2}\int_1^\infty dy\frac{\sqrt{y^2-1}}{e^{\beta m_{\text{eff}} y}-1}. \end{aligned} \tag{38}$$

Making use of a high temperature ($\beta m_{\text{eff}} << 1$) expansion in the above integral, namely

$$F(u) = \int_1^\infty dx\frac{\sqrt{x^2-1}}{e^{ux}-1} = \frac{2\pi^2}{u^2}\left(\frac{1}{12} - \frac{u}{4\pi} + O(u^2\ln u)\right), \tag{39}$$

we obtain

$$I_0(x) = \frac{1}{12}\frac{1}{\beta^2}, \tag{40}$$

which reproduces the thermal mass expression for the low-$k$ modes correctly.

Since we are interested in the Landau damping of the slow modes it is sufficient to calculate the imaginary part of $I_1$ integral. $L_1$ contributes only to the real part of the linear response function thus it presents no interest for our following discussion.

$$\begin{aligned} I_1 &= \frac{1}{4\pi^2}\frac{\omega}{|\underline{k}|}\int d|\underline{p}|dp_0dy|\underline{p}|^2\delta\left(p_0^2-|\underline{p}|^2-m_{\text{eff}}^2\right)2\Theta(p_0)\frac{1}{p_0\frac{\omega}{|\underline{k}|}-|\underline{p}|y}\frac{d\,f^0}{dp^0} \\ &= \frac{1}{4\pi^2}\frac{\omega}{|\underline{k}|}\int d|\underline{p}|dy\,\frac{|\underline{p}|^2}{(|\underline{p}|^2+m_{\text{eff}}^2)^{\frac{1}{2}}}\,\frac{1}{(|\underline{p}|^2+m_{\text{eff}}^2)^{\frac{1}{2}}\,\frac{\omega}{|\underline{k}|}-|\underline{p}|y}\left.\frac{d\,f^0}{dp^0}\right|_{(|\underline{p}|^2+m_{\text{eff}}^2)^{\frac{1}{2}}} \\ &= \frac{1}{4\pi^2}\frac{\omega}{|\underline{k}|}\int_1^\infty dt\int dy\,\sqrt{t^2-1}\,\frac{1}{\omega/|\underline{k}|\,t-\sqrt{t^2-1}\,y}\,\frac{d\hat{f}^0}{dt}, \end{aligned} \tag{41}$$

where in the last line we used the substitution $t = \sqrt{|\underline{p}|^2+m_{\text{eff}}^2}/m_{\text{eff}}$ and $\hat{f}^0(t) = 1/(e^{\beta t m_{\text{eff}}} - 1)$.

Using the principal value theorem when the $\epsilon$-prescription of Landau is applied to the denominator of (41)

$$\frac{1}{z+i\epsilon}=\mathcal{P}\frac{1}{z}-i\pi\delta(z), \tag{42}$$

one finds

$$\begin{aligned}\Im I_1 &= -\frac{1}{4\pi}\frac{\omega}{|\underline{k}|}\int_1^\infty dt\int dy\,\sqrt{t^2-1}\;\delta\Big(\Big(\frac{\omega}{|\underline{k}|}t-\sqrt{t^2-1}\,y\Big)\;\frac{d\hat{f}^0}{dt}\\ &= -\frac{1}{4\pi}\frac{\omega}{|\underline{k}|}\int_1^\infty dt\int_{-1}^1 dy\,\delta\left(y-\frac{\omega}{|\underline{k}|}\frac{t}{\sqrt{t^2-1}}\right)\frac{d\hat{f}^0}{dt}\\ &= -\frac{1}{4\pi}\frac{\omega}{|\underline{k}|}\int_{\frac{1}{\sqrt{1-(\omega/|\underline{k}|)^2}}}^\infty dt\frac{d\hat{f}^0}{dt}=\frac{1}{4\pi}\frac{\omega}{|\underline{k}|}\;\frac{1}{e^{\beta m_{\rm eff}/\sqrt{1-(\omega/|\underline{k}|)^2}}-1}.\end{aligned} \tag{43}$$

Substituting this into (37) we arrive at the central result of our investigation:

$$\mathrm{Im}\Sigma=\frac{\lambda^2\bar{\varphi}^2}{16\pi}\frac{1}{e^{\beta m_{\rm eff}/\sqrt{1-\omega^2/|\underline{k}|^2}}-1}\cdot\frac{\omega}{|\underline{k}|}\Theta(|\underline{k}|^2-\omega^2). \tag{44}$$

For the assessment of its domain of validity we compare (44) to the result of a complete one-loop quantum theoretical calculation of Boyanovsky *et al.* [8]. Using the real time finite temperature quantum theory they have derived the following expression for the damping coefficient in the spacelike $|\underline{k}|^2>\omega^2$ region:

$$\begin{aligned}\mathrm{Im}\Sigma_{\rm Landau} &= \tfrac{\lambda^2\bar{\varphi}^2T}{16\pi|\underline{k}|}\ln\tfrac{1-e^{-\beta\omega_k^+}}{1-e^{-\beta\omega_k^-}}\Theta(|\underline{k}|^2-\omega^2),\\ \omega^\pm &= \left|\tfrac{\omega}{2}\pm\tfrac{|\underline{k}|}{2}\sqrt{1-\tfrac{4M^2}{\omega^2-|\underline{k}|^2}}\right|.\end{aligned} \tag{45}$$

Here $M$ is the temperature dependent mass parameter in the finite temperature quantum theory. With explicit calculation one may check that in the limit $\beta|\underline{k}|<<1$ and $\beta\omega<<1$ the expression of the damping rate calculated from (45) goes over into (44), under the assumption that $m_{\rm eff}$ of the classical theory is identified with $M$. In this way the classical theory provides a convincing argument clearly demonstrating that Landau damping is the correct interpretation of the result derived from quantum theory.

## 5 Conclusion

In this paper we have constructed a classical effective kinetic theory of the Landau damping of self-interacting scalar fields in the broken phase. We have shown that in the low-$\omega$ region the quantum description of the phenomenon coincides with our description. The generalisation of the treatment to the case of $n$-component scalar fields is rather simple. The next step is the investigation of the coupled gauge+Higgs system. Especially important is to treat the gauge fixing properly. Technically it is most convenient to use a gauge, where the Lagrangian quadratic in the fast modes is maximally diagonalised. We believe that the classical kinetic theory of the linear response in the standard particle physics model will provide interesting contribution to the characterisation of the electroweak phase transition.

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