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ARE THE PRODUCTION AND DECAY OF A RESONANCE ALWAYS INDEPENDENT?*

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The widely accepted assumption that the decay of a resonance proceeds independently of its production, quantitatively expressed as a factorizing formula for the differential cross section in the invariant mass of unpolarized resonance debris, is put under scrutiny. It is shown that the factorization is always valid for scalar and pseudoscalar resonances, although the usual version of the formula is not entirely correct. For resonances with nonzero spins the factorization does not generally take place. We deal in more detail with the spin–one case, where we show a condition on the decay matrix element that ensures the validity of the same factorizing formula as in the spinless case. This condition is satisfied for $\rho \to \pi\pi$ but not, e.g., for $K^* \to \pi K$ or $a_1 \to \pi\rho$. The formalism is applied also to the case when the resonance is produced not in two–body collisions but in the decay of a heavier particle or resonance (chain decay). Application of our formulas to the e^+e^- production in one–photon approximation agrees with what is known from quantum electrodynamics and thus provides another test of their soundness.

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1 Introduction

It has been conjectured from the early sixties (see, e.g., [1, 2]) that a process in which some of the final state particles, let us say a and b, appear as decay products of a resonance R can be viewed as a sequence of two independent steps: (i) a reaction in which the resonance (treated as a stable particle with mass M) together with other final–state particles is produced; (ii) the decay of the resonance, $R \to a + b$.[3] This notion has been expressed quantitatively by factorizing the cross section for producing the final system that includes resonance decay products

* Dedicated to Ján Pišút on the occasion of his 60th birthday.

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[3] We will consider only two–body decay modes to keep the notation transparent. Our conclusions are valid for any decay mode.

(integrated over the angles in their common rest frame) with the invariant mass from interval $(M, M+dM)$ as the product of the cross section for producing the resonance with a (generally "off–shell") mass M and a function describing its decay [3]

$$d\sigma_{ab} = d\sigma_R(M) \left[\frac{\pi^{-1} M_0 \Gamma_{R\to a+b}(M)}{\left(M_0^2 - M^2\right)^2 + M_0^2 \Gamma^2(M)} \right] dM^2 \; . \tag{1}$$

Here, M_0 is the nominal mass of the resonance, $\Gamma_{R\to a+b}(M)$ is the partial decay rate, and $\Gamma(M)$ is the mass dependent total decay width.

We will argue in what follows that this formula is a little incorrect. The correct formula, which will be derived in Sec. 2, contains M instead of M_0 in the numerator.

For the one–decay–mode resonances, the numerator and denominator in (1) contain the same $\Gamma(M)$. The latter quantity was treated differently by different authors. Jackson [1] related it to the decay rate of the resonance, which can be expressed in terms of the S–matrix element and evaluated if the interaction Lagrangian among the resonance and its decay products is postulated. On the contrary, Pišút and Roos [2], who dealt with the $\rho(770)$ resonance, considered $\Gamma(M)$ a phenomenological object which can and should be determined by fitting the experimental data.

The idea of the cross section factorization arose in the isobaric nucleon model [4, 5]. Although Eq. (1) has never been proven generally, it has widely been accepted and heavily used because it matches the natural physical expectation. As far as we know, it was first introduced in its one-mode version in [6], as a generalization based on the cross sections for several reactions evaluated in the lowest order of the perturbation expansion using simple Lagrangians [7, 8, 9].

An attempt to prove Eq. (1) was done in the textbook [3]. Unfortunately, as we show below, in a not fully correct way and, because of the choice of the propagator, only for spinless resonances.

In this paper we show that the factorization of the cross section in the sense of Eq. (1) is not as trivial as the naive physics intuition suggests and that it is not valid generally. We prove that for the spinless resonances the cross section really factorizes, although the correct formula differs a little from (1). For spin–one resonances we get the factorizing formula only if an additional assumption is made about the matrix element for the $R \to a + b$ decay. This assumption represents a severe constraint that excludes many well–known resonances (K^* and a_1, for example).

The motivation for this study was twofold, experimental and theoretical. On the experimental side, there is a longstanding unresolved discrepancy between the parameters of the a_1 resonance determined from the τ–lepton decay on one side and from hadronic production processes on the other, see [10], p. 380. It is true that the experimentally observed invariant mass spectrum of the decay products may be influenced by the resonance mass dependence of the production cross section already in the case when the factorization takes place. But the discrepancy among the observed widths of a_1 is too big, which casts doubts on the validity of the general assumption that the decay of a resonance is independent of the way the resonance has been produced.

Also the mass spectrum of the $K\pi$ system resulting from the K^* decay was found to be process dependent (see [11] and literature cited there). But if the factorization in the sense of (1) is correct and the smooth factor $d\sigma_R(M)$ is properly parametrized this dependence should not affect the values of resonance parameters M_0 and $\Gamma(M_0)$ determined by fitting to the experimental data. It is the very essence of the factorization hypothesis that the resonance part of the cross section formula is process independent, and all the properties of the resonance production mechanism are hidden in $d\sigma_R(M)$.

On the theoretical side, many papers have appeared that calculated the cross sections and decay rates of the processes in which a meson resonance emerges as one of the final state particles. What follows are a few examples rather than the complete list: $\phi \to \rho\pi$ [12, 13], $D(1285),\ E(1420) \to \rho\pi\pi$, $D \to a_1\pi$, $K_1 \to K^*\pi\pi$ [12], $\eta' \to \rho\gamma$ [14], $\pi\pi \to \rho\gamma$ [15], $a_1 \to \rho\pi$ [16, 17, 18], $f_2 \to \rho\gamma$, $\rho\pi\pi$ [19], $K_1 \to K\rho$, $b_1 \to \pi\omega$ [18]. In these calculations, the outgoing resonances were treated as stable particles with sharp masses. It is not clear to what extent such an approximation is justified. It is obvious that it cannot give reliable results in cases when the available phase space is small due to a close proximity to the kinematic threshold. A typical example is $K_1 \to K\rho$, where the available energy is less than 10 MeV, much smaller than the widths of participating resonances. There is also at least one case in which the sharp resonance mass approach is not applicable at all, namely the $K_1(1270) \to K\omega$ decay. This decay has a relatively big branching ratio $(11.0 \pm 2.0)\%$ [10] although the nominal mass of the K_1 resonance is less than the sum of the K and ω masses.

In simple processes that do not suffer from severe phase space limitation there is no obvious reason for disregarding the sharp resonance mass method. The question of how well the method works under those circumstances naturally arises. Answering it by comparing with experimental data is usually impossible not only because of nonexistent or statistically limited data but also because additional theoretical assumptions are usually made simultaneously. Thus only one way remains–to compare the sharp resonance mass method with a more complete calculation that involves also the last step–the conversion of the produced resonance to the hadrons that are really observed experimentally. Of course, the calculation involving this step is more difficult. But if the factorization takes place there is no need to go through the complete procedure for each process considered. We only need to know the decay rate or cross section for producing the resonance with arbitrary mass in the sharp mass approximation. The conversion of the resonance to its decay products is then described by a simple universal (independent of the dynamics of the production process) function. This simplifies the evaluation of some of the processes mentioned above greatly.

The paper is organized as follows: In Sec. 2 we present the derivation of the factorizing relation for the cross section in the spinless case and show a sufficient condition under which it is valid also for unit spin resonances. Processes in which a resonance itself appears as a decay product are dealt with in Sec. 3. Sec. 4 is devoted to the connection of our formalism with methods used in quantum electrodynamics. In Sec. 5 we investigate how is the factorization condition for spin-one resonances fulfilled in some important cases. Our main conclusions are summarized and commented upon in Sec. 6.

2 Derivation of the cross section formula

2.1 Generalities

Let us consider a two–body reaction (see Fig. 2.1)

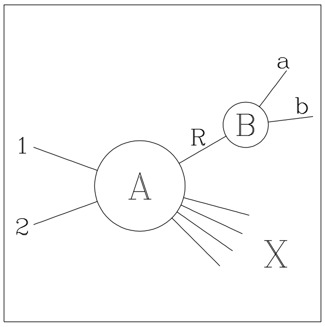

Fig. 2.1. Feynman diagram of the reaction $1 + 2 \to a + b + X$ under the assumptions stated in Sec. 2.1. Symbols A and B are used in Sec. 2.3.

$$1 + 2 \to a + b + X \tag{2}$$

with particles a and b originating from the decay of a resonance R and X representing the system of n other outgoing particles. We will assume that none of the latter is identical with either a or b. We will be interested in the cross section for producing the (a, b) system with fixed invariant mass M, $M^2 = (p_a + p_b)^2$, and fixed three–momentum $\mathbf{P} = \mathbf{p}_a + \mathbf{p}_b$. For a and b the sum over the spin states is assumed together with integration over the momenta in their common rest frame.

Concerning the particles from the X–system, we will write our formulas for the cross section integrated over their momenta and summed over their spin states. But our conclusions would remain unchanged if we opted for a more detailed differential cross section. For simplicity, we will not display the dependence of matrix elements on momenta and polarizations of particles from the X–system and will use the simplified notation λ_X for the complete set of their polarization indices. For reader's convenience we start with the general cross section formula

$$d\sigma = \frac{(2\pi)^4}{4F} \, |\mathcal{M}|^2 \, \delta^{(4)}(p_1 + p_2 - \sum_i p_i) \prod_i \frac{d^3 p_i}{(2\pi)^3 2E_i} \, , \tag{3}$$

where the sum and product run over the final state particles and $F = E_1 E_2 \, |\mathbf{v}_1 - \mathbf{v}_2|$.[4] Using (3) and

$$2E \, \delta(M^2 - (p_a + p_b)^2) \, \delta(\mathbf{P} - \mathbf{p}_a - \mathbf{p}_b) = \delta^{(4)}(P - p_a - p_b) \tag{4}$$

it is easy to show that the cross section of reaction (2) we are dealing with is given by

$$E \frac{d\sigma_{ab}}{dM^2 d^3P} \quad = \quad \frac{1}{32\pi^2 F} \int \sum_{\lambda_X, \lambda_a, \lambda_b} |\mathcal{M}_{ab}|^2 \, \delta^{(4)}(p_1 + p_2 - P_X - P) \tag{5}$$

[4]This formula is more general than that given in [10]. The latter applies only to head–on collisions [20], in which $F = p_{1cm}\sqrt{s} = m_2 p_{1lab}$.

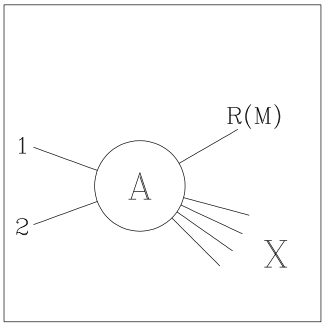

Fig. 2.2. Feynman diagram of the reaction $1 + 2 \to R(M) + X$. Symbol A is used in Sec. 2.3.

$$\times \quad \delta^{(4)}(P - p_a - p_b)\, \frac{d^3 p_a}{2E_a} \frac{d^3 p_b}{2E_b}\, d\Phi_X \ ,$$

where

$$d\Phi_X = \prod_{i \in X} \frac{d^3 p_i}{(2\pi)^3 2E_i} \ . \tag{6}$$

For further considerations it is necessary to introduce the sharp mass approximation. We define a particle $R(M)$ with the quantum numbers identical to those of the resonance R, but with a fixed mass M, which may be different from the nominal resonance mass M_0. The cross section of the reaction

$$1 + 2 \to R(M) + X \ , \tag{7}$$

in which the unpolarized $R(M)$ with momentum P is produced (see Fig. 2.2), is given by

$$E \frac{d\sigma_R(M)}{d^3 P} = \frac{\pi}{4F} \int \sum_{\lambda_X, \lambda_R} |\mathcal{M}_R|^2\, \delta^{(4)}(p_1 + p_2 - P_X - P) d\Phi_X \tag{8}$$

Now, let us turn to the decay

$$R(M) \to a + b \ , \tag{9}$$

depicted in Fig. 2.3. The general formula for the differential decay rate of a particle with mass M into n daughter particles reads [10]

$$d\Gamma = \frac{(2\pi)^4}{2M} |\mathcal{M}|^2\, \delta^{(4)}(P - \sum_{i=1}^{n} p_i) \prod_{i=1}^{n} \frac{d^3 p_i}{(2\pi)^3 2E_i} \ . \tag{10}$$

On the basis of it we can write the decay rate of (9) averaged over the spin states of $R(M)$ and summed over the final state ones as

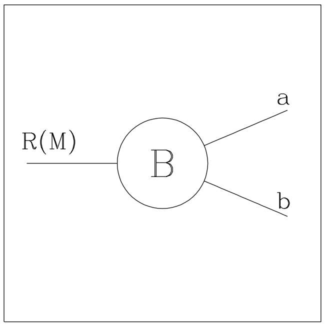

Fig. 2.3. Feynman diagram of the decay $R(M) \to a + b$. Symbol B is used in Sec. 2.3.

$$\Gamma_{R\to a+b}(M) = \frac{1}{8(2J_R+1)M\pi^2}\int \sum_{\lambda_R,\lambda_a,\lambda_b} |\mathcal{M}_D|^2 \times \delta^{(4)}(P-p_a-p_b)\frac{d^3p_a}{2E_a}\frac{d^3p_b}{2E_b} \,. \tag{11}$$

Here, J_R denotes the spin of the resonance and $\mathcal{M}_D$ is the corresponding matrix element.

2.2 Formula for spinless resonances

For a scalar or pseudoscalar resonance the matrix element for reaction (2) is given by

$$\mathcal{M}_{ab} = \mathcal{M}_R \mathcal{P}(P)\mathcal{M}_D \,, \tag{12}$$

where the propagator of the spin–zero resonance is written in the form[5]

$$\mathcal{P}(P) = \frac{i}{P^2 - M_0^2 + iM_0\Gamma(P^2)} \tag{13}$$

to which the bare propagator of spinless particle develops after convoluting with one-particle-irreducible bubbles and summing over all such diagrams. Substitution of (12) into (5) gives

$$E\frac{d\sigma_{ab}}{dM^2 d^3P} = \frac{|\mathcal{P}(P)|^2}{32\pi^2 F}\int \sum_{\lambda_X} |\mathcal{M}_R|^2\, \delta^{(4)}(p_1+p_2-P_X-P)d\Phi_X \times \int \sum_{\lambda_a,\lambda_b} |\mathcal{M}_D|^2\, \delta^{(4)}(P-p_a-p_b)\frac{d^3p_a}{2E_a}\frac{d^3p_b}{2E_b}$$

[5]We will change notation of $\Gamma(P^2)$ to $\Gamma(M)$ later on.

The integrals can be expressed in terms of observable quantities using (8) and (11) with $J_R = 0$ and no summing over λ_R. The result is

$$E\frac{d\sigma_{ab}}{dM^2 d^3P} = E\frac{d\sigma_R(M)}{d^3P}\left[\frac{\pi^{-1} M\Gamma_{R\to a+b}(M)}{\left(M_0^2 - M^2\right)^2 + M_0^2\Gamma^2(M)}\right] . \tag{14}$$

Our formula (14) differs from the formula (1) used in analysing the resonance production experiments up to now. The fixed M_0 in numerator is replaced by variable M. To localize the source of discrepancy we went through the derivation in textbook [3]. We suspect that the factor m_d in the right–hand side of Eq. (8.7) on p. 166 should be replaced by $s_d^{1/2}$ (there is no other m_d, either explicit or hidden, in that equation). After this modification, Eq. (1) agrees with (14).

2.3 Case of spin–one resonances

The matrix element of the decay (9) can be written as

$$\mathcal{M}_D(P, \lambda_R) = B_\nu(P)\epsilon^\nu(P, \lambda_R) , \tag{15}$$

where P is the four–momentum of the resonance $R(M)$ and $\epsilon(P, \lambda_R)$ is its polarization vector. The four–vector B reflects the dynamics of the Rab interaction. We suppressed the momenta and possible polarization indices of particles a and b. The sum of the matrix elements squared over possible spin states of the resonance, which enters the formula for the decay rate of unpolarized $R(M)$'s, yields

$$\sum_{\lambda_R} |\mathcal{M}_D(P, \lambda_R)|^2 = -\left(g^{\nu\nu'} - \frac{P^\nu P^{\nu'}}{M^2}\right) B_\nu(P) B^*_{\nu'}(P) . \tag{16}$$

With its help we can cast the decay rate formula (11) into

$$\Gamma_{R\to a+b}(M) = \frac{1}{24M\pi^2}\left(g^{\nu\nu'} - \frac{P^\nu P^{\nu'}}{M^2}\right) T_{\nu\nu'}(P) , \tag{17}$$

where we have defined the tensor

$$T_{\nu\nu'}(P) = \int \sum_{\lambda_a,\lambda_b} \mathcal{B}_{\nu\nu'}(P)\, \delta^{(4)}(P - p_a - p_b)\, \frac{d^3p_a}{2E_a}\frac{d^3p_b}{2E_b} \tag{18}$$

with

$$\mathcal{B}_{\nu\nu'} = -\sum_{\lambda_a,\lambda_b} B_\nu B^*_{\nu'} . \tag{19}$$

The tensor (18) cannot depend on anything else but four-vector P because p_a and p_b are integrated out and the sum over spin states is performed.

Similarly, writing the matrix element for the reaction (7) in the form

$$\mathcal{M}_R(P, \lambda_R) = A_\mu(P)\epsilon^\mu(P, \lambda_R) , \tag{20}$$

we arrive at the following cross section formula

$$E\frac{d\sigma_R(M)}{d^3P} = \frac{\pi}{4F}\left(g^{\mu\mu'} - \frac{P^\mu P^{\mu'}}{M^2}\right) \times \int \sum_{\lambda_X} \left(-A_\mu A^*_{\mu'}\right) \delta^{(4)}(p_1 + p_2 - P_X - P)\, d\Phi_X \tag{21}$$

Now we are ready to attack the formula for the cross section of the reaction (2). The matrix element of the latter is given by

$$\mathcal{M}_{ab} = A_\mu \mathcal{P}^{\mu\nu}(P) B_\nu \ , \tag{22}$$

where $\mathcal{P}^{\mu\nu}$ is the propagator of the spin–one resonance. Following [21] we write it near the resonance mass ($P^2 \approx M_0^2$) as

$$\mathcal{P}^{\mu\nu}(P) = -i\frac{g^{\mu\nu} - w(P^2)P^\mu P^\nu/P^2}{P^2 - M_0^2 + iM_0\Gamma(P^2)} \ . \tag{23}$$

The scalar function $w(P^2)$ reflects the properties of the one–particle–irreducible bubble. Fortunately, as we will see, under the assumption that enables us to write formula (14) also in the spin–one case, the term containing it will not contribute.

Making use of (22) we may rewrite (5) into the form

$$E\frac{d\sigma_{ab}}{dM^2 d^3P} = \frac{1}{32\pi^2 F}\mathcal{P}^{\mu\nu}(P)\mathcal{P}^{*\mu'\nu'}\, T_{\nu\nu'} \tag{24}$$

$$\times \int \sum_{\lambda_X} \left(-A_\mu A^*_{\mu'}\right) \delta^{(4)}(p_1 + p_2 - P_X - P) d\Phi_X \tag{25}$$

Without further assumptions, the right–hand side cannot be converted into a simple product of two factors, one describing the production of a resonance, the other its decay.

To proceed further let us assume that, as a result of a special dynamics of the $R \to a + b$ transition, the tensor $T_{\nu\nu'}(P)$, defined in (18), satisfies the relations

$$P^\nu T_{\nu\nu'}(P) = T_{\nu\nu'}(P)P^{\nu'} = 0\ . \tag{26}$$

These relations may be connected with conservation of some quantities, but their physical meaning varies from case to case. For example, in quantum electrodynamics they are a consequence of the gauge invariance and hence also the charge conservation. As an immediate consequence of our assumption we can simplify (24) to

$$E\frac{d\sigma_{ab}}{dM^2 d^3P} = \frac{1}{32\pi^2 F}\left|\mathcal{P}(P)\right|^2 T^{\mu\mu'} \times \int \sum_{\lambda_X} \left(-A_\mu A^*_{\mu'}\right) \delta^{(4)}(p_1 + p_2 - P_X - P)\, d\Phi_X \ , \tag{27}$$

where $\mathcal{P}(P)$ is given by Eq. (13). Condition (26) also signifies that the tensor $T^{\mu\mu'}$ can be represented as

$$T^{\mu\mu'}(P) = \left(g^{\mu\mu'} - \frac{P^\mu P^{\mu'}}{M^2}\right) T(M^2)\ , \tag{28}$$

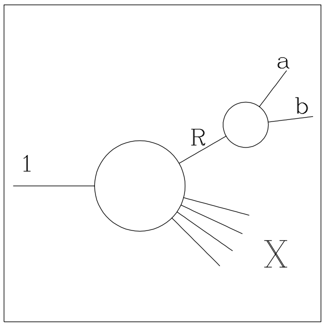

Fig. 3.1. Feynman diagram of the decay $1 \to a + b + X$.

where T is a scalar function of $M^2 = P^2$. It can be determined from

$$T(M^2) = \frac{1}{3} g^{\nu\nu'} T_{\nu\nu'}(P) \; . \tag{29}$$

The result is

$$T(M^2) = 8M\pi^2 \Gamma_{R\to a+b}(M) \; . \tag{30}$$

After inserting (28) with (30) into (27) and using (21) we complete the proof that our central formula (14) is valid also for spin-one resonances if condition (26) is satisfied.

For practical calculations it is good to notice that if the condition (26) is fulfilled then the decay rate can be calculated from a simpler formula following from (17), namely

$$\Gamma_{R\to a+b}(M) = \frac{1}{24M\pi^2} \int g^{\nu\nu'} \mathcal{B}_{\nu\nu'} \; \delta^{(4)}(P - p_a - p_b) \frac{d^3p_a}{2E_a} \frac{d^3p_b}{2E_b} \; . \tag{31}$$

Let us also note that it follows from (26) that

$$P^\nu P^{\nu'} T_{\nu\nu'} = 0 \; . \tag{32}$$

This formula thus represents the necessary condition for (26) be fulfilled.

3 Resonances as decay products

In this section we deal with the case when a resonance R appears itself as one of the decay products of a heavier particle or resonance and then converts to stable particles (see Fig. 3.1). We again call a and b the debris of the resonance R. On the basis of (10) and (4) we write the following formula for the differential decay rate

$$\begin{aligned} E\frac{d\Gamma_{1\to a+b+X}}{dM^2 d^3P} &= \frac{1}{16M\pi^2} \int \sum_{\lambda_X,\lambda_a,\lambda_b} |\mathcal{M}_{ab}|^2 \, \delta^{(4)}(p_1 - P_X - P) \\ &\times \; \delta^{(4)}(P - p_a - p_b) \, \frac{d^3p_a}{2E_a} \frac{d^3p_b}{2E_b} \, d\Phi_X \; , \end{aligned} \tag{33}$$

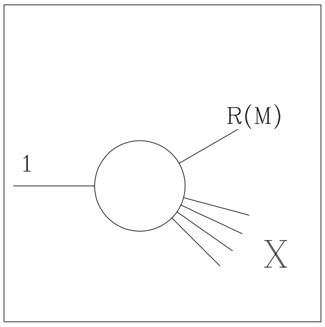

Fig. 3.2. Feynman diagram of the decay $1 \to R(M) + X$.

which is a decay analogue of Eq. (5). The rate of the decay in which the unpolarized sharp-mass resonance $R(M)$ is produced, see Fig. 3.2, reads as

$$E\frac{d\Gamma_{1\to R+X}(M)}{d^3P} = \frac{\pi}{2M}\int \sum_{\lambda_X,\lambda_R} |\mathcal{M}_R|^2\, \delta^{(4)}(p_1 - P_X - P)d\Phi_X \tag{34}$$

Repeating steps performed in Sec. 2 we can easily check that the following relation between the decay rates of $1 \to a + b + X$ (Fig. 3.1) and $1 \to R(M) + X$ (Fig. 3.2) holds

$$E\frac{d\Gamma_{1\to a+b+X}}{dM^2 d^3P} = E\frac{d\Gamma_{1\to R+X}(M)}{d^3P}\left[\frac{\pi^{-1}M\Gamma_{R\to a+b}(M)}{(M_0^2 - M^2)^2 + M_0^2\Gamma^2(M)}\right]. \tag{35}$$

This relation is valid for all scalar and pseudoscalar resonances and for such decays of intermediate spin-one resonances that satisfy condition (26). Again, spins of the parent particle 1, of the particles from the X system, and of particles a and b are not important, but the sum over the a and b polarizations must be performed.

4 Application to quantum electrodynamics

Up to now we have spoken about short–lived hadronic resonances in intermediate states of reactions and decays. But in the derivation of our key formulas (14,35) what was important was the structure of Feynman diagrams, not the type of interactions that cause the production and decay to happen. Our results can be applied to all strong, electromagnetic and weak processes with the same pattern of Feynman diagrams.

In this section we are going to show that the so called invariant integration method, which was often used for the cross section calculations in quantum electrodynamics [22], follows from our approach as a special case.

Let us consider a reaction in which a pair of oppositely charged leptons is produced in one–photon approximation. See Fig. 2.1 with $R \to \gamma, a \to e^+, b \to e^-$. The reaction can be viewed

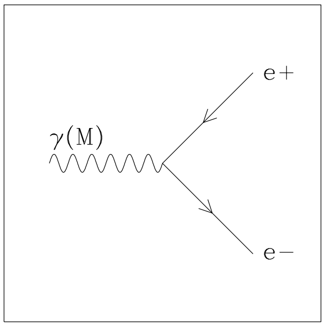

Fig. 4. Feynman diagram of the “massive” photon decay to an e^+e^- pair.

as a two step process: (i) the production of a massive photon with mass M, which is different from its “nominal” mass $M_0 = 0$; (ii) the decay of that massive photon, which is described by Feynman diagram depicted in Fig. 3. The decay matrix element is given by Eq. (15) with

$$B_\nu = e\overline{u}_{\lambda_b}(p_b)\gamma_\nu v_{\lambda_a}(p_a) \ . \tag{36}$$

Quantity (19) now becomes (m is the electron mass)

$$\mathcal{B}_{\nu\nu'} = 2e^2\left(m^2 g_{\nu\nu'} - 2p_{a\nu}p_{b\nu'} - 2p_{b\nu}p_{a\nu'}\right) \tag{37}$$

and vanishes when contracted with P^ν or $P^{\nu'}$. This guarantees that the condition (26) is fulfilled and we may use Eqs. (14), (35), and (31). The contraction of (37) with metric tensor yields the constant $4(M^2 + 2m^2)$. What remains to calculate in (31) is the well–known two–particle phase–space integral

$$\int \frac{d^3p_a}{2E_a}\frac{d^3p_b}{2E_b}\,\delta^{(4)}(P - p_a - p_b) = \frac{\pi}{2}\sqrt{1 - \frac{4m^2}{M^2}} \ . \tag{38}$$

Putting everything together we are getting the following decay rate of the massive photon to an e^+e^- pair

$$\Gamma_{\gamma(M)\to e^+e^-} = \frac{\alpha M}{12}\left(1 + \frac{2m^2}{M^2}\right)\sqrt{1 - \frac{4m^2}{M^2}} \ , \tag{39}$$

where $\alpha = e^2/4\pi$ is the fine structure constant. If we now explore our factorizing formula with the photon mass $M_0 = 0$ we are getting the well–known QED relation [22]

$$E\frac{d\sigma_{e^+e^-}}{dM^2d^3P} = E\frac{d\sigma_{\gamma(M)}}{d^3P}\,\frac{\alpha}{3\pi M^2}\left(1 + \frac{2m^2}{M^2}\right)\sqrt{1 - \frac{4m^2}{M^2}} \ . \tag{40}$$

Let us note that if we had used the usual factorizing formula (1) instead of our Eq. (14) we would not have obtained (40), but zero.

5 Spin-one resonances and their decays

In this section we are going to investigate vector and axial-vector mesons and their main decay modes to see whether they satisfy the condition that ensures the factorization of the cross section (14) and decay rate (35).

5.1 Vector resonance decaying to two pseudoscalar mesons

The charge-invariant interaction among a vector field and two pseudoscalar fields is described by the Lagrangian density

$$\mathcal{L}_{VPP} = \frac{ig}{2}\mathrm{Tr}\left(V^{\mu}\phi_1^{\dagger}\overleftrightarrow{\partial}_{\mu}\phi_2\right) + \mathrm{h.c.}, \quad (41)$$

where V is the matrix in isospin space of vector field operators and ϕ_1 and ϕ_2 are those of pseudoscalar fields. Evaluation of the decay matrix element provides it in the form (15) with

$$B_{\nu} = c\left(p_{a\nu} - p_{b\nu}\right)\ , \quad (42)$$

where p_a and p_b are the four–momenta of outgoing mesons and c includes the coupling constant g and possible constants arising from the isospin structure of (41). Contraction of (42) with the four–momentum of the parent vector meson gives

$$P^{\nu}B_{\nu} = c\left(m_a^2 - m_b^2\right)\ . \quad (43)$$

For final–state mesons with equal masses we have $P^{\nu}B_{\nu} = 0$, what immediately implies that the factorization condition (26) is satisfied. For unequal masses we proceed further and get

$$P^{\nu}P^{\nu'}T_{\nu\nu'}(P) = -c^2\pi\left|\mathbf{p}_a^*\right|\left(m_a^2 - m_b^2\right)^2\ . \quad (44)$$

The momentum in the decay rest frame was marked by asterisk. It is clear from (44) that the factorization condition (26) cannot be fulfilled for unequal masses of pseudoscalar mesons.

We have just shown that the factorization formula can be used for the $\rho \to \pi\pi$ decay, but not for decays of vector mesons to a pair of unequally heavy pseudoscalar mesons, e.g., $K^* \to K\pi$.

5.2 Axial-vector resonance decaying to vector and pseudoscalar mesons

The most general Lagrangian for interaction among the axial-vector (R), vector (a), and pseudoscalar (b) mesons consists of three parts. For simplicity and because some authors used simpler Lagrangians consisting of one of those terms only, we will study these parts individually. For simplicity, we consider A, V, and ϕ as field operators of individual particles. We also introduce a shorthand notation for the polarization vector of the vector particle $\epsilon_a = \epsilon(p_a, \lambda_a)$.

In one of the first papers about the a_1 resonance in the τ-lepton decay [23] the following Lagrangian was used:

$$\mathcal{L} = gA_{\mu}^{\dagger}V_{\mu} + \mathrm{h.c.} \quad (45)$$

Four-vector B, defined by (15), is now proportional to the polarization vector of the vector meson

$$B_{\nu} = g\epsilon_{a\nu}\ . \quad (46)$$

After a little algebra we obtain

$$P^{\nu}P^{\nu'}\mathcal{B}_{\nu\nu'} = \frac{g^2}{4m_a^2}\lambda(M^2, m_a^2, m_b^2)\ , \tag{47}$$

where we have introduced the triangle function $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$. This result implies that the expression $P^{\nu}P^{\nu'}T_{\nu\nu'}$ must acquire a nonvanishing constant value, what leads to violation of (32). Condition (26) cannot therefore be satisfied.

Another possibility of a simple Lagrangian, which corresponds to the vertex factor used, e.g., in [16], is

$$\mathcal{L} = gA_{\alpha}^{\dagger}\ \left(\frac{\partial V^{\alpha}}{\partial x_{\beta}} - \frac{\partial V^{\beta}}{\partial x_{\alpha}}\right)\frac{\partial \phi}{\partial x^{\beta}}\ + \text{h.c.} \tag{48}$$

It leads to

$$B_{\nu} = g\left[(p_a p_b)\,\epsilon_{a\nu} - (p_b \epsilon_{a\nu})\,p_{a\nu}\right]\ . \tag{49}$$

Because the quantity

$$P^{\nu}P^{\nu'}\mathcal{B}_{\nu\nu'} = \frac{g^2}{4}\lambda(M^2, m_a^2, m_b^2) \tag{50}$$

comes out nonvanishing, we conclude, as in the previous case, that the factorization cannot take place.

Finally, from the Lagrangian

$$\mathcal{L} = g\frac{\partial A_{\alpha}^{\dagger}}{\partial x^{\beta}}\left(\frac{\partial V^{\alpha}}{\partial x_{\beta}} - \frac{\partial V^{\beta}}{\partial x_{\alpha}}\right)\phi\ + \text{h.c.} \tag{51}$$

it follows that

$$B_{\nu} = g\left[(p_a P)\,\epsilon_{a\nu} - (P\epsilon_a)\,p_{a\nu}\right]\ . \tag{52}$$

Now we have $P^{\nu}B_{\nu} = 0$ and the condition (26) for formula (14) being valid is satisfied.

Lagrangian (51), which is the only Lagrangian leading to factorizing cross section for the production of an axial-vector resonance decaying to a vector meson and a pseudoscalar meson, was utilized, for example, in meson exchange model for $\pi\rho$ scattering [24].

Unfortunately, there is strong theoretical [12, 25] and phenomenological [26] evidence that the correct Lagrangian for the $a_1\rho\pi$ system must contain at least two terms of those we have studied. This hampers the factorization of the cross section and makes the use of formulas (14) and (35) impossible.

6 Comments and conclusions

We have shown that the usual phenomenological way of describing the invariant mass spectrum of the resonance decay products is not fully correct in two respects.

Firstly, the field theory derivation leads to a factorizing formula for the cross section that is a little different from that used so far. The difference may not be very important from the pragmatic point of view because in the close vicinity of the resonant mass the corrections are small. For masses far from the resonant mass both formulas lose validity anyhow because so do the expressions for resonance propagators [Eqs.(13) and (23)]. The validity of our formula has been confirmed independently by applying it to a class of electromagnetic processes in which a pair of oppositely charged leptons (or quarks) is produced.

The second aspect is more important. We have argued that in some cases one cannot use a factorizing formula at all. A sufficient condition for the factorization taking place was shown (26) for vector and axial–vector resonances. This condition is not satisfied, *e.g.*, for $K^* \to K + \pi$ and $a_1 \to \rho + \pi$ decays.

In this connection a more general question arises how to define the mass and width of a resonance in the situation when a factorizing formula cannot be used. The usual procedure of choosing the resonance production cross section as a few–parameter smooth formula and fitting the mass and width entering the resonance decay part of (14) is not applicable any longer. We must use a more complete model for the matrix element of the whole process spanning from the initial state to the final state containing stable hadrons which are directly observed in the experiment. The mass and width of a resonance thus enter the final mass spectrum formula in a way that is both different from Eq. (14) and model dependent. In simple cases, when partial wave amplitudes can be determined, one may certainly turn for help to analytic methods and define the resonance parameters by means of the position of a pole in the complex $s = M^2$ plane. But this can be done very rarely due to the complexity of final states in contemporary high energy experiments.

In addition to these two problems, which we addressed in this paper in some detail, there is another one, more general, but also ignored very often. It is the problem of the interference of diagrams when a particle is produced both in the resonance decay and by other mechanisms. Its effect may be disastrous and it would be very useful to study the justification of the resonance formula in such an environment at least in some simple examples. One study of this kind was done a long time ago [5] and showed that the interference modified radically the mass spectrum in isobaric nucleon model.

All this puts forward a question whether it has sense to define the mass and width of some resonances (a_1, for example) as unique parameters, independently of the process the resonance takes part in. We suspect that the parameters of the resonance which does not satisfy the factorizing condition have a well-defined meaning only if accompanied by the specification of the production process in which they were measured and by the formula which was used to fit the experimental data.

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