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A SIMPLE MODEL FOR TRANSVERSE ENERGY DISTRIBUTION IN HEAVY ION COLLISIONS*

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A simple geometrical model (often quoted in literature as the Glauber model) of heavy ion collisions is recapitulated. It is shown that the transverse energy distribution of heavy ion collisions follow the geometry of the collision. An extension of the model to include rapidity and transverse mass particle spectra is discussed.

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1 Introduction

Detailed studies of nucleus-nucleus interactions require the use of detailed event generators such as QGSM, RQMD or VENUS. These models incorporate many features, and it can be difficult to understand the origin of relations between variables. In the following, we describe a simple geometrical model relating the number of collisions to the cross section.

We will assume a Glauber model. This model is widely used in heavy ion community [1]-[10]. Present estimates of the collision centrality by the WA97 and NA50 collaborations are based on this kind of model. The physical ideas behind the model are found in [11].

2 Model

A good introduction to the model is given in [12]. Here we recapitulate the basic assumptions of the model. The nucleus-nucleus collision is described in terms of nucleon-nucleon collisions. The nucleons of the nucleus with mass number A are distributed in nucleus with probability density $\rho(\vec{b}, z)$ normalised to unity. It is convenient to introduce the thickness function for a nucleus A as

$$T_A(\vec{b}) = \int \rho(\vec{b}, z) dz$$

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If the nucleus A collide with a nucleus B at impact parameter b the probability of having at least one inelastic nucleon-nucleon collision is

$$p = \sigma_{in} \int T_A(\vec{s}) T_B(\vec{b} - \vec{s}) d^2\vec{s}$$

where σ_{in} is the total inelastic nucleon-nucleon cross section. The probability for the occurrence of n inelastic baryon-baryon collisions in the A-B collision is then

$$P(n, \vec{b}) = \left(\begin{array}{c} AB \\ n \end{array} \right) p^n (1-p)^{AB-n}$$

It is easy to represent above formulas by a Monte-Carlo program:

- the nucleons in nucleus A are generated according the probability density $\rho_A(\vec{b}, z)$
- the nucleons in nucleus B are generated according the probability density $\rho_B(\vec{b}, z)$
- nucleons from nucleus A are "collided" with nucleons from nucleus B. The collision of nucleon i in nucleus A at coordinates $(\vec{b}^i_A, z^i_A)$ with nucleon of nucleus B at coordinates $(\vec{b}^j_B, z^j_B)$ takes place if $|\vec{b}^i_A - \vec{b}^j_B| < \sqrt{(\sigma_{in}/\pi)}$

The number of participants N_{part} (sometimes they are called "wounded nucleons") is defined as the number of nucleons in the target and projectile which have at least one inelastic collision. The number of participants at impact parameter $\vec{b}$ is

$$N_{part}(b) = A \int d^2\vec{s} T_A(\vec{s}) \left(1 - (1 - \sigma_{in} T_B(\vec{s} - \vec{b}))^B\right) +$$

$$B \int d^2\vec{s} T_B(\vec{s} - \vec{b}) \left(1 - (1 - \sigma_{in} T_A(\vec{s}))^A\right). \quad (1)$$

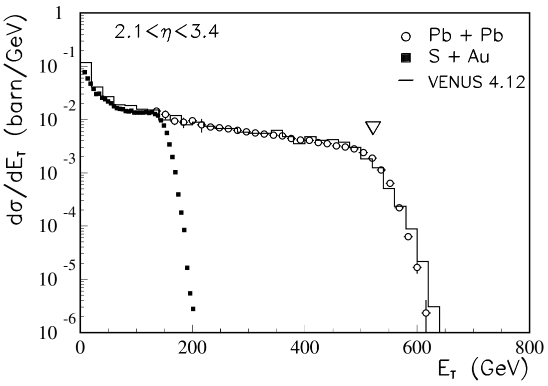
The number of collisions is the number of binary nucleon-nucleon collisions. The number of collisions at impact parameter $\vec{b}$ is

$$N_c(b) = AB\sigma_{in} \int T_A(\vec{s}) T_B(\vec{b} - \vec{s}) d^2\vec{s} \quad (2)$$

Both the number of participants and the number of collisions are easily calculated using a Monte-Carlo code.

3 Relation with Transverse Energy

It is not possible to relate the geometrical model described above to physically observable quantities without further assumptions. To proceed we shall make two assumptions both based on empirical observations:



Figl 1. Differential cross section of the transverse energy produced in Pb-Pb and S-Au at central rapidity as measured by NA49 coll. [17]

1. The number of produced particles in a given phase space window is proportional to the number of participants

2. The transverse energy produced in a given phase space window is proportional to the number of participants.

The NA35 collaboration has studied multiplicity distributions in SS collisions using detailed simulations in terms of the FRITIOF and VENUS models [13]. In both models they find that the average charged multiplicity per participant is approximately constant as a function of the number of participants. The simple proportionality of $< n_- >$ and $< N_p >$ was also assumed by Białas in the wounded nucleon model [1] and a similar result is obtained in the Dual Parton Model [14]. Comparison of several detailed models (FRITIOF, DTNUC 1.02, VENUS 4.12) with data is done in [15]. The proportionality between the transverse energy and number of produced particles was discussed in [16].

As an example of measured transverse energy distribution, Fig.1 shows the differential cross section of the transverse energy produced in Pb-Pb and S-Au collisions at central rapidity as measured by NA49 collaboration [17].

The differential cross section of the number of participants for the Pb-Pb and S-Au collisions calculated in our model is shown in Fig. 2. The density of the nucleons in nucleus is taken to be uniform in a sphere of radius $R = 1.2A^{1/3}$. The inelastic cross section is $\sigma_{in} = 30$ mbarn. The cross section is calculated by Monte-Carlo program according to formula:

$$\frac{\triangle\sigma}{\triangle N_{part}} = \frac{S_{gen}}{N_{gen}}\frac{\triangle N_{gen}}{\triangle N_{part}} \qquad (3)$$

where N_{gen} is number of generated nucleus-nucleus collisions, S_{gen} is the area of the circle in which impact parameters of incoming nucleus are generated, $\triangle N_{gen}$ is the number of nucleus-nucleus collisions (out of N_{gen}) which results in number of participants between N_{part} and

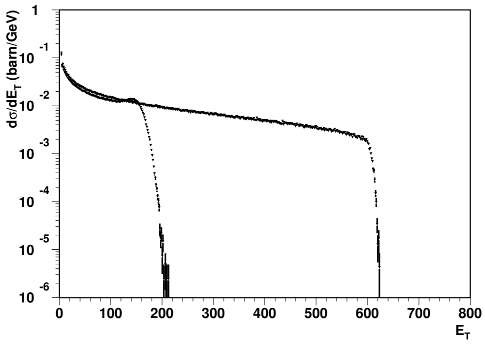


Fig 2. Differential cross section of the number of participants in Pb-Pb collisions (crosses) and S-Au collisions (histogram)

$N_{part} + \triangle N_{part}$. In generation in Fig. 2 the values of these quantities are $N_{gen} = 10^6$, $S_{gen} = 6.35ba$ for Pb-Pb and $S_{gen} = 3.66ba$ for S-Au collisions.

The maximum transverse energy in Fig. 1 is $600GeV$ and the maximum number of participants in Pb-Pb collision is about 400, so the transverse energy per participant is $600GeV/400 = 1.5GeV$. The differential cross section in E_T is then

$$\frac{d\sigma}{dE_T} = \frac{1}{1.5GeV}\frac{d\sigma}{dN_{part}}.$$

Comparing Fig. 1 and Fig. 2 we see that model is able to reproduced data.

We also present a variation of this model based on [7, 10]. We assume that particles are produced in each inelastic nucleon-nucleon collision. When traversing the nucleus, a nucleon loses its energy. We assume a simple law according to which the momentum of nucleon in the CMS nucleon-nucleon system is reduced by a factor 1.7 ($p_{old}/p_{new} = 1.7$) in each collision. This corresponds to the lost of about 0.5 units of rapidity per collision. The number of charged particles or the transverse energy (normalised to 1. at SPS beam energy $\sqrt(s) = 20.$) emitted in each collision is then calculated according to the empirical rule:

$$n(\sqrt(s)) = \ln(\sqrt(s)/2.3)/\ln(8.7). \tag{4}$$

$\sqrt(s)$ is the energy of the nucleons in their CMS system. Note that $\sqrt(s) = 20GeV$ is the SPS heavy ion energy for sulphur-sulphur collision and $\sqrt(s) = 17GeV$ is SPS energy for lead-lead collision.

The total number of particles or the total transverse energy produced in nucleus-nucleus collision is then proportional to

$$w = \sum_{i=1}^{N_c} n(s_i) \tag{5}$$

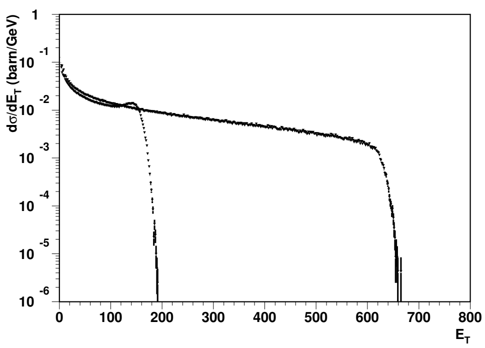


Fig. 3. Differential cross section of the transverse energy in modified model, for explanation see text.

N_c is the number of nucleon-nucleon collsions in nucleus-nucleus collision as defined by (2). s_i is the CMS energy of the colliding nucleons in their CMS system before the i-th collision. So if there is no degradation of energy in subsequent nucleon-nucleon collisions $s_i = s_0$ for all i, therefore $n(s_0) = 1$ and the quantity w is according to (5) equal to the number of collisions. The differential cross section of the quantity w was calculated analogically to (3) as

$$\frac{\triangle\sigma}{\triangle w} = \frac{S_{gen}}{N_{gen}}\frac{\triangle N_{gen}}{\triangle w} \tag{6}$$

and

$$\frac{d\sigma}{dE_T} = \frac{1}{1.5GeV}\frac{d\sigma}{dw}. \tag{7}$$

The cross section calculated with this formula is plotted on Fig. 3. Comparison of Fig. 3 and Fig. 1 shows that the modified model is also able to reproduce the data. It is to be noted that a factor $p_{old}/p_{new} = 1.7$ and a factor 1.5 in the cross section (7) are in the present approach free parameters of the model and have been tuned to get a reasonable description of the data.

The description of momentum spectra in nucleus-nucleus collision might be achieved in a generalisation of the model. It is possible to generate the full momentum spectrum in each nucleon-nucleon collision and thus the superposition of the nucleon-nucleon spectra would result in momentum spectrum of particles produced in nucleus-nucleus collision.

4 Conclusions

We show that the transverse energy distributions of heavy ion collisions are determined by the geometry of nuclei collisions. Assuming that the number of produced particles and the transverse energy are proportional to the number of participants the model is able to describe the data for different heavy ion systems.

This model allows us to estimate the number of participants and impact parameters corresponding to different centrality triggers [18]. Models based on the same principle were used to quantify the centrality of the collision in the WA97 [19] and NA50 [20] collaborations. (The NA49 collaboration uses a different approach as they are able to measure the number of participants using either a zero degree calorimeter or the number of measured protons [21].

The phenomenological observation that the number of produced particles is proportional to the number of participants can be obtained in a modified model where

1. nucleons loose their energy in each collision when traversing the nucleus
2. particles are produced in each nucleon-nucleon collision in amount dependent on the CMS energy of the nucleon-nucleon collision.

The extension of the modified model to include the rapidity and transverse momentum spectra of produced particles is proposed.

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