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UNIVERSAL FEATURES OF FLUCTUATIONS IN COMPLEX SYSTEMS*

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Properties of fluctuations in many complex systems in particle, nuclear, atomic, condensed matter, and statistical physics are examined with the aim of finding possible universal features among them. Two areas of particular interest are phase transitions and chaotic behaviors. Effective measures are found to quantify the universality.

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1 Introduction

In particle physics a researcher attempts to reduce the system under investigation to as small a unit as possible so that one can focus on the physics at the most fundamental level. In nuclear physics one must take into consideration the nuclear medium, which is, of course, large by the standards of particle physics and is therefore complex. At very high energy there appear many problems of interest that lie in the overlapping region of particle and nuclear physics. J/ψ suppression in heavy-ion collisions is an example of such a problem. Ján Pišút and I have been working on the J/ψ suppression problem in recent years; the union of our different backgrounds provides a wide range of experiences that are useful for our investigation.

Nuclear physics is a good starting point to launch into other problems of complexity. Quark-hadron phase transition is an example of a problem that brings into play another domain that traditionally belongs to statistical physics. Although the subject as a nonperturbative problem in QCD seems to be a special and self-contained field well formulated by lattice gauge theory, the demand to find the appropriate experimental observables of the signature of phase transition opens up the subject to contacts with various other areas of physics, such as percolation, intermittency, the Ising model of magnetic systems, etc. Surprisingly, even laser physics can be relevant. Once fractal behavior is brought into the picture, chaos cannot be far away. All these are problems of complex systems. The aim of this paper is to discuss the universal features that can be found in widely diverse problems, and to develop some measures that can be useful to reveal such universality.

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Whereas simple systems generally involve one scale, a complex system usually has at least two relevant scales. A problem in laser physics must deal with atoms at the microscopic scale as well as the size of the system at the macroscopic scale. Quark-gluon plasma has quark physics at the one end and the large thermal system at the other. Although laser physics and heavy-ion physics may seem to have little in common, the relationships between the microscopic and macroscopic scales in respective problems may share universal features. Such universality may be found in several fields. People who work in quantum Hall effect may not even be aware that by reorienting their experiments only slightly, they may be able to check whether the validity of the universality can be extended to their field. The fragmentation of physics into narrow subfields has been a regrettable trend in recent decades. The study of features that unite them would be a step to reverse that trend.

Due to the limitation of space here, only a general overview can be given. Details on specific points can be found in the references cited. The discussion here is the first time that the various topics are put in the perspective of a unifying framework so that the universal features can be the central theme.

2 Phase Transitions

The critical exponents of continuous phase transitions (PT) are usually endowed with the universality property in that they are independent of the details of the systems undergoing the PT, but depend only on the dimension and symmetry. However, the systems referred to are restricted to condensed matters. Nothing is said about other systems such as quarks, photons, or the primordial universe. Moreover, not all systems can be investigated for their behaviors near the critical temperature and yield some formulas like $|T - T_c|^{-\alpha}$. Temperature is not an experimentally controllable parameter in many systems, so the critical exponent α is not measurable.

To approach the question of universality among systems that span over all fields of physics, and perhaps beyond, it is necessary to adopt a broad view of (a) what constitutes a PT and (b) what can serve as observables. For the former we should not focus on the detail dynamics. Whatever the Hamiltonian of a system may be, it is safe to adopt the view that a continuous PT can be described by the Ginzburg-Landau (GL) theory. As for the latter, the observable has to be experimentally measurable on the one hand, and can be related to the order parameter in the GL theory on the other. To be concrete, let us consider the quark-hadron PT and the photocount fluctuations of a laser as specific examples in one's search for order parameters.

It has been known for nearly thirty years that at the threshold of lasing the production of photons in a laser behaves like a second-order PT [1]. What connection that has with other systems at PT was not studied because there is no temperature to vary in a laser. The pump parameter for a laser can be adjusted such that below the lasing threshold no photons are produced, whereas above the threshold lasing occurs. As the parameter is decreased from above, the photon flux does not smoothly diminish to zero. Photons are emitted sporatically with fluctuations in both rate and lapse intervals. It is well known that at the critical point an extended system exhibits spatial fluctuations of all sizes. In the case of the laser, what is needed is a measure of the fluctuation of photon number that conveys the properties of PT.

The problem of treating the laser is very similar to the problem of finding the appropriate observable for quark-hadron PT. In a heavy-ion collision at very high energy, if the colliding nuclei are large enough, quark-gluon plasma can possible be created. As the plasma expands, its temperature lowers, most effectively at the surface. When the surface temperature reaches T_c, quarks become confined and hadrons are produced. The manner in which the hadrons are emitted is very much like the way the photons are emitted by the laser at threshold. Both are prescribed by the physics of PT. Thus the task common to both is to identify a measure that conveys the same physics, which is the GL description of PT.

In the laser problem it is the fluctuation of photon number in time; in the heavy-ion problem it is the fluctuation of hadron number in rapidity. Mathematically they can be treated similarly. In particle physics the multiplicity fluctuations have for a decade been studied experimentally in terms of factorial moments F_q as functions of the bin size [2]. The same can be done for the photocount time series. One selects a narrow interval δt in time, in which the photon number is counted, waits for a long time interval to lapse to minimize autocorrelation and then counts the photon number again in another δt. The process is repeated many times to determine a distribution of photon number, from which F_q can be calculated [3]. For particle production δt is replaced by δy, the bin size in rapidity, and the repeated measurements are to determine the event-to-event fluctuation.

The density of particles (photons or hadrons) is dn/dt or dn/dy, which may be denoted collectively by ρ. It is this ρ that is to be identified with the order parameter ϕ in the GL theory; more precisely, $\rho = |\phi|^2$. Clearly, ρ is zero on one side of the PT and nonzero on the other, hence ϕ has properties of an order parameter. The fluctuation of ϕ around the minimum of the GL free energy leads to fluctuations in the particle numbers in the window of detection, which in turn give rise to nontrivial values of F_q. The mathematical details of those connections are discussed in [4, 5] and will not be reproduced here. Our emphasis now is to underline the parallelism in the treatments of the two problems. By measuring the same observables, we should be able to show universality in the two widely different systems.

Instead of studying how F_q depend on the window size δt or δy, it is more interesting to exhibit the dependences of F_q on F_2, as δt or δy is varied. It is found that they possess a power-law behavior [3, 4]

$$F_q \propto F_2^{\beta_q} \tag{1}$$

where

$$\beta_q = (q-1)^\nu \quad , \qquad \nu = 1.304 \quad . \tag{2}$$

Thus there exists a universal behavior (1) and a universal constant ν. The value of ν has been determined by the laser experiment [3], and is in agreement with the theoretical prediction of 1.304 [4]. Heavy-ion experiments have not been done at high enough energy to demonstrate uncontroversial evidence for quark-gluon plasma formation, and the appropriate factorial moment analysis has not been carried out. Indeed, the verification of the scaling exponent $\nu = 1.304$ should be able to serve as a signature of quark-hadron PT.

The universality of PT uncovered should not be limited to the production of photons and hadrons. Surely, the ubiquitous electrons should offer a vast arena for probing the extent of the universality. We suggest three areas of research where this type of investigation should be carried out: (a) quantum Hall effect, (b) metal-insulator transition, and (c) superconductor-insulator transition. In (a) it is the change in Hall current from one level to another that seems to be a PT, but the investigators are not certain. Our method of studying the fluctuation of particles produced, which are here the extra Hall electrons measured from a specific quantum level of Hall conductivity, can be applied to the problem and be used to check the validity of (1) and (2) for quantum Hall effect. If the corresponding ν turns out to be 1.304 also, then the result would certify that the transition from one Hall level to another is a continuous PT. Similar statements can be made about (b) MIT and (c) SIT. It is only necessary to do the experiments and measure the fluctuations in electron numbers (i.e. currents) at the transition point.

The ideas discussed above can be applied to problems outside physics also. An example of that is the question of phase transition in evolutionary biology. Minimum complexity is a requirement for such a transition, below which only disconnected subsystems exist, and above which a connected whole emerges. The problem can be made precise by focusing on an evolutionary game as a model for the evolution of cooperative behavior, analogous to the Ising model for ferromagnetism. More specifically, the Prisoners' Dilemma is a game that is particularly pertinent; it acquires interesting characteristics when the game is played in a 2D array [6]. Depending on the payoff parameter, there can be a rich variety of spatial patterns, ranging from dominance of one or the other type of players to chaotically changing ones. At a critical point in the payoff parameter, players with the winning strategy can form clusters of all sizes. It is therefore of interest to examine the possible connection between that state of evolutionary change with phase transition in physics.

The problem has been studied using the method of factorial moments discussed above, and their scaling behavior examined [7]. It is found that when the payoff parameter has the critical value, (1) and (2) are satisfied, and that the value of ν is 1.23 ± 0.15. There is no reason why the evolutionary game should fit into the GL description of PT. However, the value of ν being in agreement with 1.3 within its errors suggests the possibility that biological evolution modeled by the game can be a phase transition as understood by physicists.

3 Chaotic Behaviors

The study of chaos is a broadly interdisciplinary subject. It seems hard to say more about its universality that has not already been said. But there is more. Quantum chaos is still largely unexplored, since the territories investigated are essentially limited extensions of classical chaos. Yang-Mills fields in particle physics satisfy nonlinear equations, yet the chaotic behaviors of only the classical YM fields have been studied at some length [8]. In the quantum reality of particles the strong interaction of quarks and gluons can lead to the proliferation of more quarks and gluons, a phenomenon that is beyond the capability of classical chaos techniques to handle. If a new technique is invented to account for the possibility of the increase of degrees of freedom, would it be applicable to the conventional problems and provide some clues about universality? There are also interesting problems in nonstationary time series, such as electrocardiogram (ECG) and electroencephalogram (EEG) data. Effective measure must be found to describe the nature of chaotic behaviors in those waveforms.

To find a measure of chaos that has broad applicability, broader than such measures as the Lyapunov exponents, one must abandon the usual notion of trajectory. For a system whose degrees of freedom change with time, a trajectory in phase space cannot be defined. Sensitivity to the initial condition is also a concern that is strongly tied to the classical problem. Liberation from that constraint, while being mindful of what constitutes chaos, opens the door to new possibilities. A good point of departure is to generalize the notion of erratic trajectories to fluctuations of spatial patterns. In a sense a finite trajectory is a specific pattern, but a pattern can include other representations of a state besides classical trajectories. For example, the final state of a high-energy collision between two hadrons is a pattern in momentum space. The fluctuation of such patterns from event to event may well contain the effects of chaotic dynamics. Cardiac fibrillation can also be regarded as a change of patterns.

What a pattern is depends on the resolution of its description. At a fixed resolution a pattern should be characterized by at most a few numbers, significantly less than the total number of cells at that resolution. Otherwise, the fluctuation of the patterns would be untractable. Suppose that a well-chosen number F does an acceptable job. Then as the patterns change from event to event, or from one segment of a time series to another, F would change, resulting in a distribution $P(F)$. If $P(F)$ is very narrow, then clearly the patterns do not change very much, and are therefore predictable. If $P(F)$ is wide, then the patterns can change erratically. Thus the width of $P(F)$, or some equivalent quantity, can serve as a measure of the degree of erraticity [9].

By taking into account the entropy content of $P(F)$ and the scaling property as the resolution is changed, an entropy index μ is found to quantify the chaotic behavior of a system giving fluctuating patterns [10]. It is an effective and concise measure of the characteristics of quark and gluon jets, as well as hadronic collisions. However, in order to ascertain that μ is a measure of chaos, it is necessary to apply the analysis to some classical problems known to have chaotic solutions. The best known examples of classical chaos are the logistic map and Lorenz attractor. They give rise to a sequence of numbers between 0 and 1, and a trajectory in 3D space, respectively. If those outputs are represented as discrete sets of numbers recorded at regular time intervals, and then binned in cells of finite size, they can be regarded as spatial patterns. Starting from each distinct initial point within a small neighborhood of one another, a collection of distinct patterns can be generated. From the fluctuation of those patterns, the index μ can be determined. What is remarkable is that, apart from an adjustable overall normalization, the dependence of μ on the control parameters is almost exactly the same as that of the Lyapunov exponent [11]. Thus μ is indeed a characterization of chaos.

Without delving into the details, which can be found in [10, 11], we stress that by finding a new measure of chaos the notion of chaos is enlarged to include dynamical systems that cannot be described by classical methods, such as multiparticle production. Studying the fluctuation of spatial patterns makes possible the treatment of widely different problems in the same framework and the discovery of universal features that relate them.

The entropy index has so far been applied, in addition to what have already been mentioned above, to the Ising model [11], *pp* collisions [12] and ECG waveform of a patient with heart disease [13]. It is expected that the same method can also be applied to the EEG time series of brain activities. In [12] it is found that multiparticle production by soft interaction in hadronic collisions has the same range of values of μ as that in jets by hard interaction. That is an aspect of universality that has hitherto been unsuspected.

Not all μ values are of the same order. For the Ising model they are extremely small even at the critical point [11]. It is well known that near T_c the spatial patterns have clusters of all sizes, and they are certainly not the same from configuration to configuration. However, the factorial moments F_q used to quantify the patterns are not sensitive to the geometrical rearrangement of the clusters. Thus although the patterns appear different, F_q remain approximately the same. Clearly, an alternative method of describing patterns is needed. The use of wavelets is a promising avenue to explore [13].

Despite the smallness of μ for Ising model when F_q is used, there is nevertheless a prominent phenomenon that μ exhibits. In the vicinity of T_c it is found that μ develops a giant peak, relatively speaking, and the maximum occurs at T_c [11]. It implies that the use of F_q to determine μ is sufficiently effective in recognizing pattern fluctuations to allow μ to be a tool to signal PT. Since F_q is experimentally accessible, the combined use of μ and ν (of last section) should provide the best signature available for PT using hadronic probes. Conceptually, the more interesting point to make is that the study of chaos and phase transition can be amalgamated. That is the fruitful result of broadening the scope of investigation in search for universal features among seemingly unrelated fields.

4 Some Remarks

Specialization in science and the narrowing of scientists have been an increasingly more prevalent trend. There are many factors that contribute to it, the most honorable one of which is that the attempt to be deep is justified at the expense of breadth. Yet often what appears to be deep at the outset turns out to be pedantic and not very fundamental at the end. The more injurious factors may well be forces outside science that have more to do with the financial and career aspects of research. Young scientists cannot afford the time to deviate from their specializations; the older ones do not want to jeopardize the continuation of their grants and contracts. The funding agencies bear the greatest responsibility in stifling diversity when they act in preference to projects that are oriented toward familiar goals, and when they fail to value visions broader than the opinions of their reviewers who are provincial and narrow.

The hardest tasks I encountered has been to persuade experimentalists not in my registered field to make only slight deviations in their ongoing experiments to measure fluctuations that may test universality. Many researchers in quantum Hall effect, metal-insulator transition and superconductor-insulator transition all agree that it is interesting to make those measurements; however, they have not so far been carried out, since funded projects undoubtedly demand higher priority.

The interest in chaos theory is fortunately so widespread that the subject is forging the most interdisciplinary study ever in science and beyond. The phenomenon may well signal the emergence of new era when the study of complex systems will be recognized as an exciting venture, instead of being an unrewarding task to be avoided. As more investigators with different backgrounds join the enterprise, the techniques will be improved and scopes broadened. The use of concepts developed in the study of chaotic systems to the investigation of well-established problems, such as phase transition discussed at the end of Sec. 3, is an example of the enrichment of classical problems by fresh viewpoints generated by efforts to interlace across fields of specialization.

It is hoped that the short and qualitative discussions given here offer an alluring view of the study of complex systems and that young and experienced scientists alike can be attracted to the uncharted adventure of searching for universality.

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