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TOWARDS DYNAMICAL MASS CALCULATIONS*

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$\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ electroweak gauge model without Higgs sector is extended by a new vector field C^μ interacting with leptons and quarks of both chiralities. This interaction is treated under a dynamical assumption in a self-consistent approximation. Fermion masses are calculated in terms of new Abelian hypercharges, and the intermediate boson masses are calculated in terms of the fermion masses by sum rules. Self-consistency requires a rich sector of heavy spin-1 collective fermion-antifermion excitations.

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Experimentally observed mass spectrum of leptons and quarks ranges from the vanishing or the vanishingly small $(m_\nu < O(1\mathrm{eV}))$ neutrino masses over the MeV range of the electron, and u,d quark masses up to 175 GeV of the recently observed top-quark mass. Theoretical understanding of such a sparse, wide and irregular fermion mass spectrum is completely missing. Its successful phenomenological description in terms of the independently renormalized Yukawa couplings of fermions to a condensing elementary Higgs field provides, however, a hope [1] for the existence of an underlying microscopic dynamics in which the fermion mass ratios are the calculable numbers. Alternatively, the problem of understanding the fermion mass spectrum is shifted towards the very high mass scales in which the Higgs sector is theoretically likely [2].

We suggest to extend the standard $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ gauge-invariant Lagrangian of three massless families f of leptons and quarks (neutrino singlets ν_{fR} are added for completeness) by a vector boson C^μ with the mass m_C interacting with all fermions of both chiralities with the coupling constant h [3].

With a dynamical assumption on the nonperturbative behavior of the running "Abelian" charge $h^2(p)$ we calculate the huge fermion mass ratios naturally in terms of the new hypercharges associated with different fermions. It is then suggestive to call these hypercharges the heaviness. Being pure numbers, all of the same order of magnitude they can be fixed (quantized) by embedding the model into a GUT group at a scale $M \approx 10^{15}$ GeV. This we assume but do not specify the particular group structure. The intermediate boson masses m_W and m_Z are calculated in terms of the fermion masses by sum rules as a consequence of the SU(2)× U(1) Ward identities.

*Dedicated to Ján Pišút on the occasion of his 60th birthday.
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The dynamical assumption is that the C-boson-fermion interaction above a critical strength generates specific spin 1 fermion-antifermion composites (collective excitations) with calculable couplings to the C-boson. These new interactions then modify the behavior of the running "Abelian" charge $h^2(p)$ to the phenomenologically desirable "walking" form.

Replacement of the Higgs sector by a simple interaction characterized above brings, however, a serious problem. It is the problem of unwanted global symmetries. When the fermion masses are dynamically generated, most of these symmetries are spontaneously broken and there is nobody except the weakly coupled W and Z bosons ready to "eat" the corresponding massless fermion-antifermion Nambu-Goldstone (NG) bosons. The most economic solution to this problem is to assume that the verified NG eaters i.e., the massless spin 1 fermion-antifermion composites are generated by the same nonperturbative dynamics which generates the NG bosons. Hence, the role of the assumed dynamically generated spin 1 composites [4,6] is in fact 3-fold: (i) They absorb the unwanted NG bosons and acquire the masses $m_V^2 \sim \alpha^{-1}\pi m_W^2$. (ii) They contribute to the C-boson vacuum polarization and justify a posteriori the dynamical assumption on the behavior of the running $h^2(p)$. (iii) They provide a distinctive phenomenological signature of the model soon experimentally testable.

Understanding of the behavior of strongly interacting (Lorentz-invariant) systems of the quantum fields is largely nonexisting at present. What are the relevant degrees of freedom (particle-like excitations) in a sensible perturbative framework is a priori not known, and an underlying effective field theory appropriate for the considered energy range must look upon them humbly in accordance with the experimental data. The best example which we have found of what we suggest here is the Hubbard model [5]: The (nonrelativistic) system of electrons strongly interacting on the lattice by a fourfermion interaction can be converted in a number of elaborate mean field solutions to various phases having the excitations with quite unexpected properties. In one case, for example, the spin and the charge are carried separately by the spinons (electrically neutral fermions) and the holons (electrically charged bosons) at the mean field level, but there exist important dynamically generated gauge-field fluctuations.

1. Dynamical generation of the fermion masses by chirally invariant interactions is a genuinely nonperturbative phenomenon by definition: In perturbation theory the chiral symmetry forces fermions to stay massless order by order. Operationally the generation of the fermion mass $m_f = \Sigma_f(m_f^2)$ amounts to finding a finite solution (chiral symmetry does not tolerate the fermion mass counter-terms) of the Schwinger-Dyson equation [7] for the fermion proper self-energy $\Sigma_f(p^2)$:

$$\Sigma_f(p^2) = \frac{3}{4} y(f_L) y(f_R) h^2 \int \frac{d^4k}{(2\pi)^4} \frac{c[(p-k)^2]}{(p-k)^2 + m_C^2} \frac{\Sigma_f(k^2)}{k^2 + \Sigma_f^2(k^2)} \tag{1}$$

In Eq. 1 the function $c(p^2)$ specifies [8] the behavior of the momentum-dependent charge in the whole momentum range. In practice it is known explicitly only in a restricted momentum range where the perturbation theory is justified. Our assumption of a GUT unification at $p^2 = M^2$ is important: $c(p^2) \sim \ln^{-1}(p^2/M^2)$ at $p^2 \to \infty$ (asymptotic freedom = AF) implies [9] a finite $\Sigma_f(p^2)$, since $\Sigma_f(p^2) \sim p^{-2}(\ln p^2)^{C_{GUT}}$ at $p^2 \to \infty$.

Numerical analyses of Eq. 1 reveal that nontrivial solutions Σ start to exist only above certain critical value [10] of the coupling, $h_{cr}^2/4\pi^2$ rather large (~ 1). Consequently, perturbative form of the Abelian $c(p^2)$ running to the Landau pole is highly unlikely. It is truly remarkable that if $c(p^2)$ stays essentially constant from 0 to M^2 and then falls to zero according to AF, the Eq. 1 is extremely sensitive [11] to the details of $c(p^2)$, and provides a hope for true calculations of the fermion masses in terms of heaviness. For an illustration we present the approximate solution of (1) found for such a case by Akiba and Yanagida [12]:

$$m_f \sim \Sigma_f(0) \sim M \exp[-\pi/(\frac{3}{4}y(f_L)y(f_R)\frac{h^2}{4\pi^2} - \frac{h_{cr}^2}{4\pi^2})^{1/2}] \tag{2}$$

Sensitivity to details of $c(p^2)$ together with the mass formula (2) lead us to the conclusion that the sparse, wide and irregular fermion mass spectrum might be understood provided the prescribed behavior of $c(p^2)$ is theoretically justified. Unbearable lightness of calculating the fermion masses in terms of heaviness using Eq. 2 is schematically illustrated by taking $M = 10^{15}$ GeV, $h^2/4\pi^2 = 4/3$, $h_{cr}^2/4\pi^2 = 1$ and $Y_f \equiv \frac{1}{\pi}[y(f_L)y(f_{R)} - 1]^{1/2}\ln 10$ i.e., $m_f(1/Y_f) = M\exp(-1/Y_f)$: $m_{"\nu"}(24) = 1$ eV, $m_{"e"}(18) = 1$ MeV, $m_{"d"}(17) = m_{"u"}(17) = 10$ MeV, $m_{"\mu"}(16) = m_{"s"}(16) = 100$MeV, $m_{"\tau"}(15) = m_{"c"}(15) = 1$ GeV, $m_{"b"}(14) = 10$GeV, $m_{"t"}(13) = 100$ GeV. We believe these results justify the dynamical assumptions we make.

The arguments in favor of the prescribed form of $c(p^2)$ are the following. First, unconventional behavior of the momentum-dependent charge $c(p^2)$ can only be due to additional (nonperturbative) contributions to the C-boson vacuum polarization tensor. The standard (perturbative) one-loop contributions of the elementary fermions drive $c(p^2)$ to the Landau pole. Second, the new contributions cannot be due to anything but to the loops of appropriate fermion-antifermion composites (collective excitations). Third, of a limited arsenal of the field theory the only known particles the loops of which can stop the running $c(p^2)$ to the Landau pole are the effectively non-Abelian spin 1 fermion-antifermion composites. Hence, we assume in the following that such massless spin 1 composites are dynamically generated. We will specify this new sector below in relation to another reason for its existence. The spin 1 fermion-antifermion composites in a model similar to ours were discussed by Lindner and Ross, see Ref.3.

2. That the strongly coupled "Abelian" dynamics in the nonperturbative regime defined by the prescribed form of $c(p^2)$ generates the bosonic collective excitations is theoretically acceptable and in some cases phenomenologically most welcome: There is an important instance in our program where the particular composites are guaranteed by a theorem: If $\Sigma_f(p^2)$ are found by solving Eq. 1 the global chiral symmetries of the original Lagrangian are spontaneously broken down to unbroken [13] $U(1)_B \times U(1)_{L_e} \times U(1)_{L_\mu} \times U(1)_{L_\tau} \times U(1)_{em}$. Since the original global symmetry is $(U(2)_L)^3 \times (U(1)_R)^3 \times (U(1)_R)^3$ for quarks and the same global symmetry for leptons, there are 31 massless fermion-antifermion composite Nambu-Goldstone bosons coupled to fermions.

Three of them are needed as we now show. One $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ subgroup of the large global symmetry is in fact gauged, i.e., there are 4 perturbatively massless gauge bosons weakly coupled to the fermions. Coupled to the fermions are, however, also the massless NG bosons. For the gauged part $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ we will employ the corresponding Ward-Takahashi identities. Being the consequence of the symmetry of the Lagrangian rather than of the symmetry of the ground state, they should remain valid regardless of whether the fermion masses had spontaneously broken the chiral symmetry or not.

The full fermion-W vertex (and analogously the fermion-Z vertex) allows to calculate the effective W-NG boson vertex (see, e.g. Ref. 3):

$$iq^{\alpha}\frac{g}{2\sqrt{2}}N^{1/2} = \sum_{\text{fermions}}\int\frac{d^4p}{(2\pi)^4} \tag{3}$$
$$\times\, Tr\frac{g}{2\sqrt{2}}\gamma^{\alpha}(1-\gamma_5)S_u(p)\frac{1}{N^{1/2}}[(1-\gamma_5)m_d-(1+\gamma_5)m_u]S_d(p+q)$$

In evaluating the finite integral (3) with $S_f(p) = (\not p\ + \Sigma_f(p^2))/(p^2-m_f^2)$ ($f = u, d$ where $u \equiv$ upper, $d \equiv$ down fermion in a doublet) we use for definiteness and a first orientation the Pagels-Stokar model [14] of $\Sigma_f(p^2) = m_f M^2/(p^2+M^2)$ consistent within the log accuracy with $\Sigma_f(p^2) \sim p^{-2}(\ln p^2)^{C_{GUT}}$ dictated by the prescribed form of $c(p^2)$ in (1). As a result, the vertex (3) gives rise to an important new tree-level contribution to the longitudinal part of the vacuum polarization $\Pi^W_{\mu\nu}(q)$ of the W boson (analogously for the Z boson). Residue at the massless pole [15] of $\Pi^W_{\mu\nu}$ equals m_W^2 (analogously for m_Z^2):

$$m_W^2 = \frac{1}{4}g^2\sum n_c[m_u^2 I_{u;d} + m_d^2 I_{d;u}] \tag{4}$$

$$m_Z^2 = \frac{1}{4}(g^2+g'^2)\sum n_c[m_u^2 I_{u;u} + m_d^2 I_{d;d}] \tag{5}$$

In (4,5) $n_c = 1$ for leptons and $n_c = 3$ for quarks, and the quantities I are given as

$$I_{u;d} = \frac{1}{2\pi^2}\int_0^1 x\ln\frac{(M^2-m_d^2)x+m_d^2}{(m_u^2-m_d^2)x+m_d^2}dx$$

The sum rules (4,5) are saturated essentially by the top-quark mass. In contrast to the canonical Higgs case, there is no genuine weak-interaction mass scale in this approach. As the formulas (4,5) suggest, the eaten NG bosons are to be viewed as the particular combinations of many components:

$$\begin{aligned}
\mid\pi^+> &= \frac{1}{N^{1/2}}\sum[m_u I_{u;d}^{1/2}\mid\bar{u}(1-\gamma_5)d> -m_d I_{d;u}^{1/2}\mid\bar{u}(1+\gamma_5)d>]\\
\mid\pi^-> &= \mid\pi^+>^+\\
\mid\pi^0> &= \frac{1}{N^{1/2}}\sum[m_u I_{u;u}^{1/2}\mid\bar{u}\gamma_5 u> -m_d I_{d;d}^{1/2}\mid\bar{d}\gamma_5 d>]
\end{aligned} \tag{6}$$

3. The combinations of states orthogonal to (6) correspond to 28 unwanted NG bosons of the spontaneous symmetry breaking pattern $[U(2)]^6\times[U(1)]^{12}/[U(1)]^5$. The couplings of these composite NG bosons to the fermions are dictated by the Goldstone theorem. If the model pretends not to be in a serious conflict with the experimental facts, these states must not appear in its physical spectrum. For this reason we specify the dynamical assumption already made: the same UV stable dynamics which produces the NG bosons produces in the same channels also the massless spin 1 fermion-antifermion composites [5] V^{μ} (i.e., there is a hidden gauge symmetry [4,16]).

Since the origin of the masses of V^μ is the same as of m_W and m_Z, we take the formulas (4,5) and rescale g, g' by the large effective V-fermion couplings $f_V^2/4\pi^2 \cong 1$. This then leads to a rough estimate that $m_V^2 \cong \alpha^{-1}\pi m_W^2$, and it clearly amounts to "carrying the own skin to the market": The present experimental limits on the masses of the extra W and Z vector bosons are almost in the same range: $m_{W'} > 720$ GeV [17] and $m_{Z'} > 505$ GeV [18].

4. The effectively non-Abelian dynamically generated vector composites V^μ make the assumption on the behavior of $c(p^2)$ self-consistent: The (finite) fermion-loop diagrams proportional to and vanishing with the dynamically generated fermion masses give rise (with some adjustment) to the vertices corresponding to the interaction Lagrangian

$$\begin{aligned} \mathcal{L}_{\rm int}^{\rm eff} \quad = \quad & -ihC_h[\partial_\mu V_\nu^+(C^\nu V^\mu - C^\mu V^\nu) - \partial_\mu V_\nu(C^\nu V^{\mu+} - C^\mu V^{\nu+})] + \\ & (hC_h)^2[C_\mu C^\mu V_\nu^+ V^\nu - C_\mu V^\mu C^\nu V_\nu^+] - \\ & ihC_h(\partial_\mu C_\nu - \partial_\nu C_\mu)V^{\mu+}V^\nu \end{aligned} \tag{7}$$

where C_h is a calculable factor. The divergent fermion loops not proportional to the fermion masses give rise to the V-boson kinetic term by virtue of imposing the compositeness condition $Z = 0$ [4,6] upon the V-boson wave function renormalization.

Since the interaction (7) is known to produce an antiscreening [19] contribution to the C-boson vacuum polarization, the resulting $c(p^2)$ should walk [20]. For C_h fixed as to cancel the fermion contribution and without GUT unification $c(p^2)$ would go to a non-trivial fixed point [21].

Within an effective field theory we have attempted at identifying the basic scenario of the dynamics underlying the extremely successful Higgs phenomenology. Although the model is defined by the Lagrangian suggested by the present author already in Ref.3, its dynamical content is made much more explicit here. First, we have suggested a particular realization of the assumption that an "Abelian" interaction can become ultraviolet stable. Second, we have addressed an important question of the fate of spontaneously broken global symmetries of the model ignored in Ref.3, and provided a possible answer. Third, the sector of new heavy spin-1 fermion-antifermion composites the existence of which is a result of our dynamical assumptions represents a clear prediction of the model. Concerning the generation of lepton, quark, and intermediate-boson masses the present model can be characterized as follows: (i) It allows for calculating the mass ratios. (ii) It is economic in the sense of the number of free parameters in the primary Lagrangian. (iii) It does not shift the solution of the problem of the mass generation to the far future. In fact the fermion masses are put on the same footing with the fermion electric charges, both being fixed by their corresponding Abelian hypercharges. (iv) It provides distinctive experimental predictions in the form of the new vector bosons V with masses in the $\mathcal{O}$(10 TeV) range. These collective excitations having the couplings to quarks should manifest themselves by their propagator effects if not as resonances both at the Tevatron and HERA.
(v) Its detailed description clearly demands further elaboration.

Field-theoretically, there is no a priori way of knowing whether our dynamical assumption on the nonperturbative behavior of the Abelian β-function is justified or not [22]. We also cannot refrain from pointing out that the concept of the dynamically generated gauge particles which is crucial in our approach is seriously considered in the effective field theory description of the high-T_c superconductors [5]. This analogy also nicely illustrates an uneasy and ambiguous task of finding the correct microscopic dynamics to a given (say, Higgs) phenomenological one: While both the standard and high-T_c superconductors are equally well parametrized by the same Ginzburg- Landau-Higgs phenomenological theory, their microscopic dynamics are apparently quite distinct: While the standard superconductors are definitely governed by the microscopic BCS dynamics, understanding of the microscopic dynamics of the high-T_c superconductors is still far from complete.

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References

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