acta physica slovaca vol. 49 No. 2, 177 – 184 **April 1999**

EVIDENCE FOR A CENTER VORTEX ORIGIN OF THE ADJOINT STRING TENSION*

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Received 30 September 1998, accepted 2 February 1999

Wilson loops in the adjoint representation are evaluated on cooled lattices in SU(2) lattice gauge theory. It is found that the string tension of an adjoint Wilson loop vanishes, if the loop is evaluated in a sub-ensemble of configurations in which no center vortex links the loop. This result supports our recent proposal that the adjoint string tension, in the Casimir-scaling regime, can be attributed to a center vortex mechanism.

PACS: 12.38.Aw, 11.15.Ha

The center vortex theory of quark confinement has received increasing support, in the last year or so, from numerical simulations conducted by our own group [1, 2], by Langfeld et al. [3], and by Kovács and Tomboulis [4] (a brief summary of our own results can be found at the end of ref. [1]). Despite these successes, the existence and approximate Casimir scaling of the adjoint-representation string tension would seem to be problematic for the vortex theory, since adjoint Wilson loops are insensitive to the gauge-group center [5, 6]. This issue was addressed some months ago in ref. [7], where we argued that the adjoint (and higher) representation string tensions can *also* be attributed to a center vortex mechanism. In this paper we present some numerical data in support of that argument.

Let us begin by recalling that, while the quark-antiquark string tension depends on the group representation of the heavy-quark color charges, this dependence is different in two separate regimes. In the **Casimir-Scaling Regime**, extending from the onset of confinement to the onset of color screening, the string tension σ_r for the r-representation seems (from numerical

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experiments [8]) to be very roughly proportional to the eigenvalue of the quadratic Casimir; e.g. for the SU(2) gauge group

$$\frac{\sigma_j}{\sigma_{1/2}} \approx \frac{4}{3} j(j+1). \tag{1}$$

In particular, the SU(2) adjoint string tension is $\approx 8/3$ the fundamental string tension at intermediate distances. However, for sufficiently large quark separation, the quark color charges must be screened by gluons to the lowest representation with the same transformation properties under the Z_N subgroup. This is the **Asymptotic Regime** where, in the SU(2) case,

$$\sigma_j = \begin{cases} \sigma_{1/2} & j = \text{half-integer,} \\ 0 & j = \text{integer.} \end{cases} \tag{2}$$

In the $N \to \infty$ limit, color-screening is suppressed, and Casimir scaling is exact out to infinite quark separations. But even at $N = 2$ there seems to be a Casimir regime of some finite extent.

Explaining the behavior of the string tension in both the asymptotic *and* Casimir scaling regimes is a major challenge for any theory of confinement. In particular, in the case of the vortex theory, creation of a center vortex linking loop C multiplies the value $W_F(C)$ of the fundamental representation Wilson loop, in an SU(N) gauge theory, by an element of the center, i.e. $W_F(C) \to zW_F(C)$, where $z = \exp[2\pi in/N] \in Z_N$. The vortex theory attributes the area-law falloff of $< W_F(C) >$ to fluctuations in the number of vortices linking the loop. However, since $W_A(C) = \left|W_F(C)\right|^2 - 1$, adjoint loops would seem to be unaffected by the presence of center vortices. This insensitivity to vortices leads to the correct asymptotic result that there is no adjoint string tension at large distance scales, but then how does one explain the existence of an adjoint tension in the Casimir regime?

Our proposed answer to this question in ref. [7] (see also related considerations by Cornwall [9]) begins by noting that center vortices are surface-like objects, with a finite thickness (a "core") on the order of magnitude of the confinement scale. Only outside this finite core can the effects of the vortex be simply represented by a discontinuous gauge transformation. The statement that adjoint loops are unaffected by center vortices must, therefore, be qualified: it is only true providing the vortex core nowhere overlaps the loop perimeter. This leads us to ask: What is the effect of vortices on adjoint loops whose minimal area is comparable to, or smaller than, the cross-sectional area of the vortex core? For such loops, the overlap of the vortex core with the loop perimeter cannot be disregarded.

In ref. [7] we studied the effects of the overlap in the context of a simple model. Consider the case of an $R \times T$ loop in SU(2) lattice gauge theory, $T \gg R$, lying in the x-t plane. If the vortex core pierces the minimal area of a loop, but does not overlap the loop perimeter (Fig. 1a), then the effect of the vortex is simply the insertion of a center element ($-I$ in this case) somewhere in the product of the link variables. If we then imagine displacing the vortex so that the cross-section of the core, in the plane of the loop, lies entirely outside the loop (Fig. 1b), then the center element $-I$ is replaced by $+I$. Finally, if the vortex core overlaps the loop perimeter, as shown in Fig. 2, we will assume that its effect on the loop can be represented by insertion of a group element G into the product of link variables

$$G = S \exp\left[i\alpha_R(x)\frac{\sigma_3}{2}\right] S^\dagger \qquad 0 \leq \alpha_R(x) \leq 2\pi \tag{3}$$

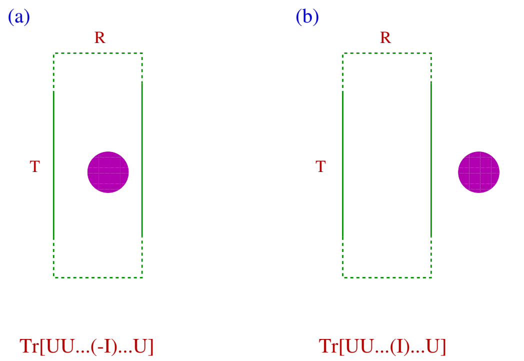

Fig. 1. Vortex core (shaded region) intersecting the plane of an $R \times T$ Wilson loop. With no core/perimeter overlap, the effect of the vortex is the insertion of a center element in the link product.

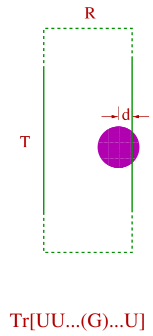

Fig. 2. Overlap of the vortex core with the loop perimeter. The effect of the vortex is represented by insertion of a group element G in the link product.

which interpolates smoothly between the limiting cases $G = \pm I$ at $\alpha_R = 0$ and $\alpha_R = 2\pi$ respectively. Here S is some (randomly distributed) SU(2) group element, and $\alpha_R(x)$ depends on the fraction of the core cross-section contained in the minimal area of the loop. Position x locates the middle of the vortex where it pierces the plane of the loop. The trace of the link product can be taken in any representation. We also define

$$f \equiv \text{probability for the middle of a vortex to pierce a plaquette.} \tag{4}$$

If both the positions x and group orientations S of vortices piercing the plane of the loop are uncorrelated, one finds for the vortex-induced potential between heavy color charges in representation j [7]

$$\begin{aligned} V_j(R) &= -\sum_{n=-\infty}^{\infty} \ln\{(1-f) + f\,\mathcal{G}_j[\alpha_R(x_n)]\}, \\ \mathcal{G}_j[\alpha] &= \frac{1}{2j+1} \sum_{m=-j}^{j} \cos(\alpha m), \end{aligned} \tag{5}$$

where $x_n = (n + \frac{1}{2})a$, with a the lattice spacing, now denotes a (dual) lattice coordinate on the x-axis. For large quark separations, this expression leads to the correct asymptotic form for the string tension

$$\sigma_j = \begin{cases} -\ln(1-2f) & j = \text{half-integer}, \\ 0 & j = \text{integer}, \end{cases} \tag{6}$$

while for small R, where all $\alpha_R(x) \ll 2\pi$, we find

$$V_j(R) = \left\{ \frac{f}{6} \sum_{n=-\infty}^{\infty} \alpha_R^2(x_n) \right\} j(j+1) \tag{7}$$

which is proportional to the SU(2) quadratic Casimir. This derivation of Casimir proportionality at small R generalizes readily from SU(2) to SU(N).

To actually compute $V_j(R)$ at small and intermediate R, even in this simplified model, one needs to know the function $\alpha_R(x)$. Fortunately, this function is constrained to satisfy certain limits, which gives us a good idea of its shape. Take the two static charges to lie at $x = 0$ and $x = R$, and let $d(x)$ be the distance from x to the nearest static charge, taken with positive sign if the middle of the vortex is outside the $R \times T$ loop (i.e. $x < 0$ or $x > R$) and negative otherwise ($x \in [0, R]$). The angular variable $\alpha_R(x)$ can only depend on $d(x)$, and must satisfy the following limits:

1. $\alpha_R(x) \to 0$ as $d(x) \to \infty$ (vortex far outside the loop),
2. $\alpha_R(x) \to 2\pi$ as $d(x) \to -\infty$ (vortex deep inside a large loop),
3. $\alpha_R(x) \to 0$ as $R \to 0$ (small loop).

These conditions are satisfied by, e.g., the simple ansatz

$$\alpha_R(x) = \pi \left[1 - \tanh \left(A\, d(x) + \frac{B}{R} \right) \right]. \tag{8}$$

It then turns out that there does exist a Casimir scaling region, where $V_j(R)$ is both linear, and approximately proportional to the quadratic Casimir.[4] The extent of this Casimir region depends on the choice of parameters A, B. As R increases beyond $1/A$, this Casimir scaling behavior goes smoothly over to the behavior characteristic of the asymptotic regime. The details can be found in ref. [7]. An approach having some similarities to ours (but also differing in important respects, as noted in [1]), was put forward by Cornwall in ref. [9].

While it is very likely that the model described above is an oversimplification of the effects of finite vortex thickness, it is nonetheless useful in showing how the adjoint string tension can, in principle, emerge from a center vortex mechanism. Moreover, quite apart from the specific details of the model, there is one very unambiguous prediction: If an adjoint loop, located at some definite position on the lattice, is evaluated in a subensemble of gauge field configurations in which no vortex core overlaps the loop perimeter, then the string tension of the loop should vanish.

Numerically, since the precise boundary of the vortex core is not sharply defined, it is simpler to study the behavior of the adjoint tension under a somewhat weaker restriction, namely, that the middle of any vortex core, where it crosses the plane of a loop to be evaluated, is exterior to the minimal area of the loop. This choice of cut in the data has been used previously [1, 2] to define "no-vortex" loops in the fundamental representation (see below). As we will see, even this weaker restriction seems sufficient to eliminate the adjoint string tension.

Center vortices are located by a mapping of SU(2) lattice gauge fields onto Z_2 gauge fields, as explained in refs. [1, 2]. One begins by fixing to the "maximal center gauge," defined as the gauge which maximizes

$$R = \sum_{x,\mu} \mathrm{Tr}[U_\mu(x)]^2 \tag{9}$$

[4] We find, however, that the deviation from exact SU(2) Casimir scaling at intermediate distances tends to increase with j.

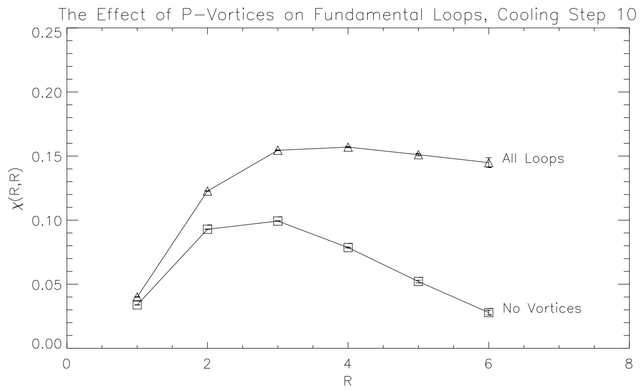

Fig. 3. Fundamental representation Creutz ratios on the cooled lattice, at $\beta = 2.3$. Results are shown for the full data set (triangles), and for the no-vortex loops (squares).

leaving a residual Z_2 symmetry. This gauge brings the link variables as close as possible, on average, to center elements $\pm I$. "Center projection" is the mapping

$$U_\mu(x) \to Z_\mu(x) = \text{sign}\,\text{Tr}[U_\mu(x)] \tag{10}$$

where the $Z_\mu(x)$ transform like Z_2 gauge fields under the residual gauge symmetry. It is found that the "thin" (1-plaquette thick) center vortices of the projected configurations – the "P-vortices" – locate a surface in the middle of the "thick" center vortices of the full, unprojected configurations [1, 2]. We can then numerically evaluate Wilson loops in the unprojected configurations, subject to the constraint that a given loop on the lattice is evaluated if and only if a definite number n of plaquettes, lying in the minimal area of the loop, are pierced by P-vortices. This procedure defines the "vortex-limited Wilson loops" $W^{(n)}(C)$, and in this way we can study the effects of vortices on unprojected Wilson loops in various representations.

For the fundamental representation, it was found in refs. [1, 2] that the zero-vortex Creutz ratios $\chi_F^0(R,R)$, extracted from zero-vortex Wilson loops $W_F^0(C)$, drop to zero at sufficiently large R. This is one of the pieces of evidence in favor of the center vortex theory. Our proposed vortex mechanism for higher representations leads us to predict similar results for adjoint-representation, zero-vortex Creutz ratios in the Casimir regime, where the full adjoint quark potential is roughly linear.

Testing this prediction is very straightforward in principle. The practical problem is that the VEVs of higher-representation Wilson loops are far smaller than VEVs of the corresponding loops in the fundamental representation, and this means that some reduction in noise due to short-range fluctuations is essential. A number of noise-reduction methods exist; we have chosen to use the constrained cooling procedure of ref. [10]. Our strategy, as explained in ref. [1], is to locate the P-vortices via center-projection on an uncooled lattice, count the number of P-vortices piercing the minimal area of each loop, and then evaluate the loops in the corresponding cooled, unprojected lattice. Cooling tends to thicken the core of center vortices [1];

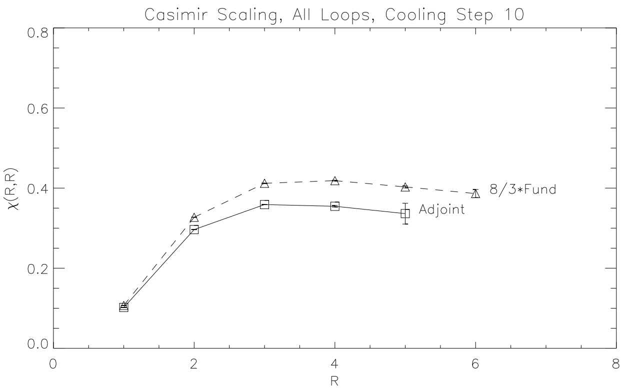

Fig. 4. All-loop adjoint Creutz ratios (squares) compared to the corresponding Casimir-rescaled (i.e. $\times 8/3$) data for the all-loop fundamental Creutz ratios (triangles).

according to our model this effect should simply extend Casimir scaling out to larger distances.

Figure 3 shows our results for Creutz ratios in the fundamental representation after 10 constrained cooling steps. Data for this and the subsequent figures was taken from 760 configurations separated by 100 update sweeps, on a 16^4 lattice at $\beta = 2.3$, with loops evaluated after 10 constrained cooling sweeps of each configuration. The upper line in Fig. 3 joins data points for the standard Creutz ratio $\chi_F(R,R)$, extracted from all fundamental Wilson loops of the appropriate sizes, while the lower line joins data points for $\chi_F^0(R,R)$, extracted from only the zero-vortex fundamental Wilson loops $W_F^0(C)$. Very similar data, up to $R = 5$, was reported previously in ref. [1].

Figure 3 shows quite clearly, on a cooled lattice, what has also been found on uncooled lattices; namely, the no-vortex restriction effectively eliminates the asymptotic string tension of Wilson loops [1, 2]. The full Creutz ratio varies only a little, in the range from $R = 3$ to $R = 6$, and we expect it to converge to the nearby value of the asymptotic string tension (quoted as $\sigma = 0.136(2)$ in ref. [11]) as $R \to \infty$. In sharp constrast, the zero-vortex ratio drops drastically in the same range, and is evidently tending to zero.

Figure 4 is an illustration of Casimir scaling on the cooled lattice. The solid line connects Creutz ratios $\chi_A(R,R)$ (squares) extracted from all loops in the adjoint-representation of appropriate size; for comparison we also display the corresponding data $\frac{8}{3}\chi_F(R,R)$ (triangles) for fundamental Creutz ratios rescaled by a factor of $8/3$. If Casimir scaling were exact, these two sets of data points would coincide. In fact, we find that $\chi_A(R,R)/\chi_F(R,R) \approx 2.25$ at $R = 3,4,5$, rather than 2.66. Because the signal is so much smaller for adjoint as compared to fundamental loops, we have only obtained meaningful data for the adjoint ratios up to $R = 5$. However, as far as we can tell from this limited data set, the all-loop adjoint Creutz ratios have stabilized around $R = 3$ in parallel with the corresponding all-loop fundamental ratios. The existence and (approximate) Casimir scaling of the adjoint string tension have also been seen in many

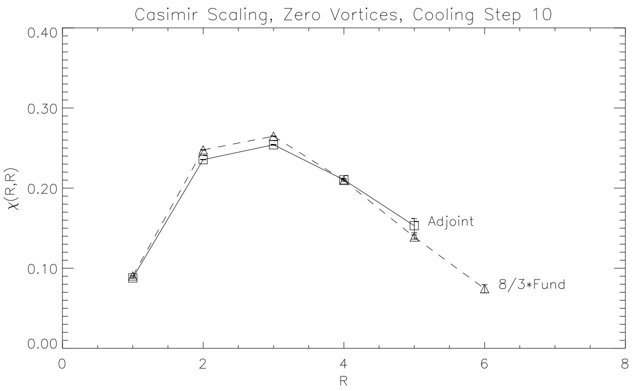

Fig. 5. Zero-vortex adjoint Creutz ratios (squares) compared to the corresponding Casimir-rescaled (i.e. $\times 8/3$) data for the zero-vortex fundamental Creutz ratios (triangles).

previous studies [8].

The final (and crucial) figure is Fig. 5. Here we show the data for the zero-vortex adjoint Creutz ratios $\chi_A^0(R,R)$ (squares) as compared to the zero-vortex fundamental Creutz ratios $\chi_F^0(R,R)$ (triangles), again rescaled by the factor of $8/3$. There are two rather striking features of this data: The first is that Casimir scaling of the zero-vortex data is nearly exact. The second is that the zero-vortex data for the adjoint representation $\chi_A^0(R,R)$, like the zero-vortex data for the fundamental representation, is falling rapidly towards zero. It would be nice to have data at still larger R, particularly for the adjoint zero-vortex loops, but in the data that we do have there is not the slightest indication of the zero-vortex ratios stabilizing at a finite value, which would be required for the existence of a non-zero string tension in the Casimir regime.

The conclusion is that if we make a "cut" in the Monte-Carlo data so that only zero-vortex loops, as defined above, are evaluated, then there is no sign of a finite string tension for either fundamental *or* adjoint loops in the Casimir-scaling regime. The simplest interpretation of this data is that center vortices give rise, not only to the fundamental, but also to the adjoint string tension in the Casimir regime. This effect, in our opinion, is due to the mechanism proposed in ref. [7].

Acknowledgements We would like to thank J. Ambjørn for helpful discussions and comments, and also for providing access to a fast workstation.

This work was supported in part by Fonds zur Förderung der Wissenschaftlichen Forschung P11387–PHY (M.F.), the U.S. Department of Energy under Grant No. DE–FG03–92ER40711 and Carlsbergfondet (J.G.), the "Action Austria–Slovak Republic: Cooperation in Science and Education" (Project No. 18s41) and the Slovak Grant Agency for Science, Grant No. 2/4111/97 (Š.O.).

Š.O. would like to express his gratitude to Ján Pišút for the opportunity and privilege to be first his student, then collaborator, and friend for many years.

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