acta physica slovaca vol. 49 No. 2, 127 – 136 April 1999

A METHOD FOR TOP QUARK MASS DETERMINATION FROM A DILEPTON $t\bar{t}$ CHANNEL*

J. Antoš[1]
Institute of Experimental Physics SAS, Watsonova 43, Košice, Slovakia
and
Institute of Physics, Academia Sinica, Taiwan

Received 23 September 1998, in final form 18 December 1998, accepted 7 January 1999

A new method of top quark mass determination based on full kinematic event reconstruction is presented. Method was tested on Monte Carlo samples produced by event generator PYTHIA [4] in conditions close to ones expected in experiments at TEVATRON. Resolution of the method in case of 5 events sample is in the range of 5-8 GeV/c^2.

PACS: 14.80.Dq

1 Introduction

After discovery of top quark at TEVATRON collider [1], a next step is to measure its parameters as precisely as possible. Precise determination of top quark mass and W boson mass in the framework of Standard Model constrains possible mass range for Higgs boson. Higgs boson is the last item needed for full triumph of Standard Model. It will be the most important target for the next generation of high energy physics experiments.

Findings of CDF and D0 collaboration support idea that main production mechanism of top quark at TEVATRON collider is a pair production of top (t) and anti-top ($\bar{t}$) quarks. One can take an advantage of this fact and use kinematic constraints in different top quark decay channels for determination of the top quark mass. Consistency of the measurements of top quark mass in different decay channels is an itself important cross check of the basic assumption of Standard Model $t\bar{t}$ production. In this paper we will concentrate on a measurement of top quark mass in dilepton channel.

Production of $t\bar{t}$ in TEVATRON collider and subsequent decay in dilepton decay channel follows the following schema:

* Dedicated to Ján Pišút on the occasion of his 60th birthday.
[1] E-mail address: antos@ns.saske.sk

$$\begin{array}{llllll} p\bar{p} \to & t & \bar{t} & + & X & \\ & & \hookrightarrow & W^-\bar{b} & & \\ & & & \hookrightarrow & l^-\bar{\nu} & \\ & \hookrightarrow & W^+b & & & \\ & & \hookrightarrow & l^+\nu & & \end{array}$$

According to Standard Model t ($\bar{t}$) quark with probability close to 100 % decays into W boson and b-quark. In case of dilepton decay channel events when each W boson decays into lepton (only electron and muon are considered in this case) and neutrino are selected.

In experiment kinematic characteristics of the leptons and jets can be measured as well as transverse components of missing momentum (which in our case represents sum of transverse momentum components of neutrinos). One can easily recognize that even under assumption of the same (known) mass for W bosons involved in the decay and also same (unknown) top mass of both top quarks, there is still one constraint missing to allow full reconstruction of kinematics of the event and consequently also top mass.

There have been different approaches to treat this problem. One solution is based on cascade decay kinematics and tree level standard model dynamics [3]. In this approach relation between average effective mass squared of lepton and b–quark (jet) from the same top quark decay and top mass was derived. Disadvantage of this method is that kinematics of individual events is not taken directly into account. There have been proposed several methods [2] which are based on individual events most probable top mass measurements. In this paper we propose a new method which by extending measured quantities in the event by one added on theoretical grounds enables to fully kinematically reconstruct events and to determine top quark mass.

2 Method

Kinematic constraints in dilepton decay channel of $t\bar{t}$ production can be expressed following way:

$$\begin{array}{lcl} P_{l^+} + P_{\nu} & = & P_{W^+} \\ P_{l^-} + P_{\bar{\nu}} & = & P_{W^-} \\ P_b + P_{W^+} & = & P_t \\ P_{\bar{b}} + P_{W^-} & = & P_{\bar{t}} \\ P_{\nu_x} + P_{\bar{\nu}_x} & = & \not{E}_{T\,x} \\ P_{\nu_y} + P_{\bar{\nu}_y} & = & \not{E}_{T\,y} \\ M_t & = & M_{\bar{t}} \\ M_W & = & 80.4 \\ P_{tz} + P_{\bar{t}z} & = & P_z^{t\bar{t}} \end{array}$$

Where P_i is four momentum of particle i, P_{ix},P_{iy} are transversal components of momentum of particle i and P_{iz} is longitudinal component of momentum of particle i. $\not{E}_{T\,x}$,$\not{E}_{T\,y}$ are transversal components of "missing energy" – amount of transverse momentum needed to balance sum of all measured transverse momenta of particles (jets) produced in interaction. In collider experiment sum of momenta of all produced particles (jets) should be 0. Because neutrinos pass through detector without any significant loss of energy, the only information about their presence in a given event is imbalance of momentum. $\not{E}_T$ therefore corresponds to total transverse energy of neutrinos participated in an event.

It is obvious here that number of equations matches number of independent variables. This was achieved by an artificial inclusion of last equation into a set. In collider experiments for a given channel it is impossible to determine $P_z^{t\bar{t}}$, because this implies determination of longitudinal component of missing energy. Therefore for this equation to have a sense, $P_z^{t\bar{t}}$ should be determined theoretically.

Condition of fixed mass of W boson and top quark mass may seem natural but one should remember that both particles are resonances with mass distributed according to Breit-Wigner distribution. Width of W boson is close to 2 GeV and theoretical width of top quark is close to 1 GeV for top quark mass around 170 GeV/c^2 range. Breit-Wigner distribution is known to have a very long tail. Therefore even in an ideal case of no influence by experimental resolution one does not expect these conditions to be met exactly.

Lifetime of heavy top quark is so short that it cannot create bound state with another (light) quark. But before it decays into W boson and b quark it can radiate gluons and then decay off mass shell. Which means that the top mass reconstructed from decay products will be off mass shell.

Specific problem is that measured quantities are subjects of experimental errors.

Practical method of top mass reconstruction should be robust and insensitive to above mentioned aspects.

Given set of equations for true $t\bar{t}$ production has always two solutions (except in a case when measured quantities are distorted in a way that there is no solution). Proposed method should be able to find efficiently both solutions.

We suggest to apply numerical method solution from beginning. Set of kinematical constrains can be reduced to the following set of three equations. As three variables in terms of which solution will be searched are chosen three components of momentum of one of the neutrinos. To solve this set of nonlinear equations standard numerical procedure – Newton's method can be applied. This method is an iterative procedure which starting from reasonable initial value finds solution in a couple of iterative steps.

$$
\begin{array}{lcl}
(E_{l_1} + E_{\nu_1})^2 - (\vec{P}_{l_1} + \vec{P}_{\nu_1})^2 - M_W^2 & = & 0 \\
(E_{l_2} + E_{\nu_2})^2 - (\vec{P}_{l_2} + \vec{P}_{\nu_2})^2 - M_W^2 & = & 0 \\
(E_{l_1} + E_{\nu_1} + E_{b_1})^2 - (\vec{P}_{\nu_1} + \vec{P}_{l_1} + \vec{P}_{b_1})^2 - & & \\
-(E_{l_2} + E_{\nu_2} + E_{b_2})^2 + (\vec{P}_{\nu_2} + \vec{P}_{l_2} + \vec{P}_{b_2})^2 & = & 0
\end{array}
$$

Three components of the neutrino momentum vector are found this way. Then using kinematic constraints full kinematic chain is reconstructed and one can calculate top quark mass. When one solution is found, one knows that there should be also another one. Other solution $\vec{P}'_{\nu_1}$ can be expressed as:

$$\vec{P}'_{\nu_1} = \vec{P}_{\nu_1} + \vec{X}$$

$\vec{X}$ can be determined by applying Newton's method to following set of equations:

$$\begin{aligned}
\sqrt{M_W^2 + (\vec{P}_{W_1} + \vec{X})^2} - E_{l_1} - \sqrt{(\vec{P}_{\nu_1} + \vec{X})^2} &= 0 \\
\sqrt{M_W^2 + (\vec{P}_{W_2} - \vec{X})^2} - E_{l_2} - \sqrt{(\vec{P}_{\nu_2} - \vec{X})^2} &= 0 \\
\sqrt{(\sqrt{M_W^2 + (\vec{P}_{W_1} + \vec{X})^2} + E_{b_1})^2 - (\vec{P}_{t_1} + \vec{X})^2} - & \\
-\sqrt{(\sqrt{M_W^2 + (\vec{P}_{W_2} - \vec{X})^2} + E_{b_2})^2 - (\vec{P}_{t_2} - \vec{X})^2} &= 0
\end{aligned}$$

3 Test of the method

Decision in iterative process that solution is found was based on three conditions:

- maximal deviation from 0 for any equation is $\leq \epsilon_1$ or
- maximal changes in solution by iteration are $\leq \epsilon_2$ and maximal deviation from 0 for any equation is $\leq 10\epsilon_1$
- if procedure does not converge, condition on equal value of W boson mass for both W bosons is relaxed (difference smaller then 3 GeV/c^2 is accepted) or condition on equal value of top mass for both top quarks is relaxed (top mass difference less than 2 GeV/c^2

There is also parameter which controls that second solution does not coincide with the first one. Minimal distance of two solutions in momentum space for neutrino momenta was set to 10 GeV/c.

Procedure was first tested on ideal cases. Process $p\bar{p} \to t\bar{t} + X \to W^+bW^-\bar{b} + X \to l^+\nu bl^-\bar{\nu}\bar{b} + X$ at 1.8 TeV/c was generated by PYTHIA [4]. As was mentioned in previous section actual reconstructed masses of top quarks and W's are affected by Breit-Wigner shape of these parameters as well as by gluon radiation. As an input for top quark mass reconstruction the four momenta of b–quarks (jets) from t, $\bar{t}$ decay, four momenta of leptons and sum of three momenta of neutrinos from two W decays are used. In case when sign of electric charge of b-jet is not determined in a given experiment, there is twofold ambiguity which pair of lepton and b-jet are associated with the same top quark.

To see influence of initial and final state radiation, results presented in Fig. 1 are based on generation with initial and final state radiation switched off (event generator PYTHIA has this option). Upper row of plots consists of plot of original top quark mass distribution reconstructed from three decay products – lepton, neutrino and b-quark. Next figure in the same raw is difference between current top masses in the event. Second row displays reconstructed top quark mass by above procedure for correct pairing of leptons and jets from the same top quark decay for the first solution and for the second one. Lowest row represents two solutions in case when leptons and jets have been combined from the different top quarks (wrong pairing).

In Fig. 2 same distributions as one on previous figure are presented with exception that in this case initial (ISR) and final state radiation (FSR) is switched on. One can clearly see effect of initial and final state radiation on original top quark mass. Because in reconstruction one assumes that top quark mass and W boson mass have fixed values (which is not the case in reality) reconstruction is not 100 % efficient. The efficiency is 88 % in case of no ISR and FSR

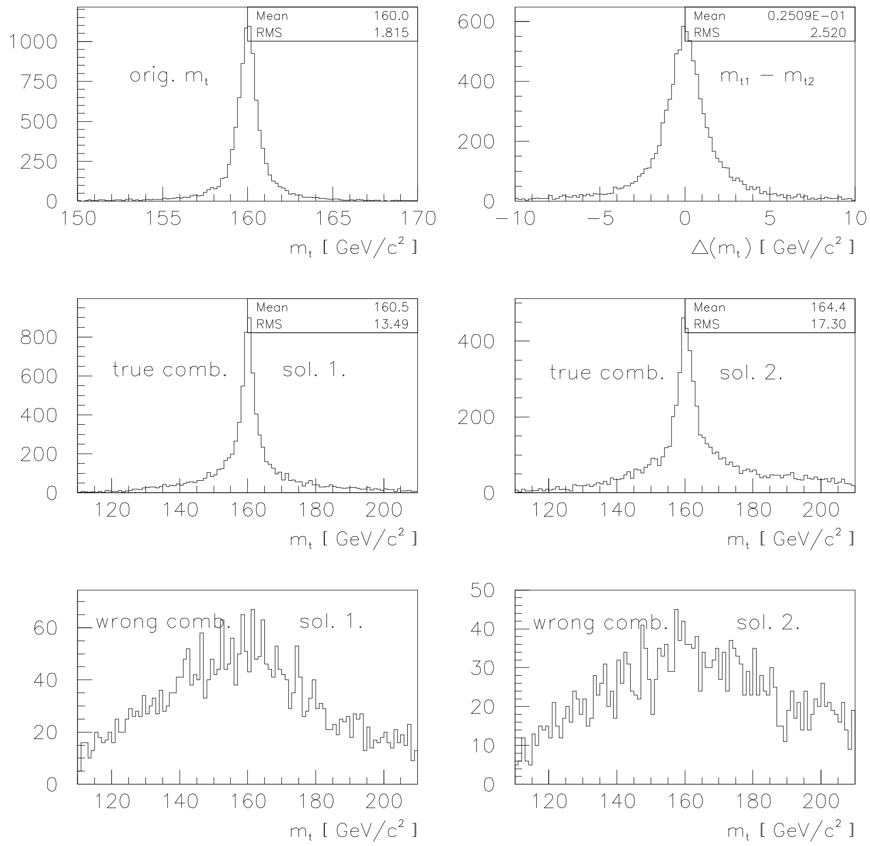

Fig. 1. Initial and final state radiation switched off. Upper row of plots consists of plot of top mass distribution (original) and difference between current top mass in an event. Second row displays reconstructed top mass for correct pairing of leptons and jets from the same top decay for the first solution and for the second solution. Lowest row represents two solutions in case of wrong pairing of leptons and jets.

(in fact only FSR plays a role) and 83 % in case when both are switched on. This statement is virtually independent of the original top mass.

Conclusion one can draw from above figures is that resolution is very drastically influenced by two factors:

- Correct assignment of leptons and jets associated with the same top quark.
- Correct choice of solution.

In real experiment there is a possibility in principle to find correct association of lepton and jet with the same top quark by determination of correct sign of charge of jets and to combine only lepton with jet of different sign of charge.

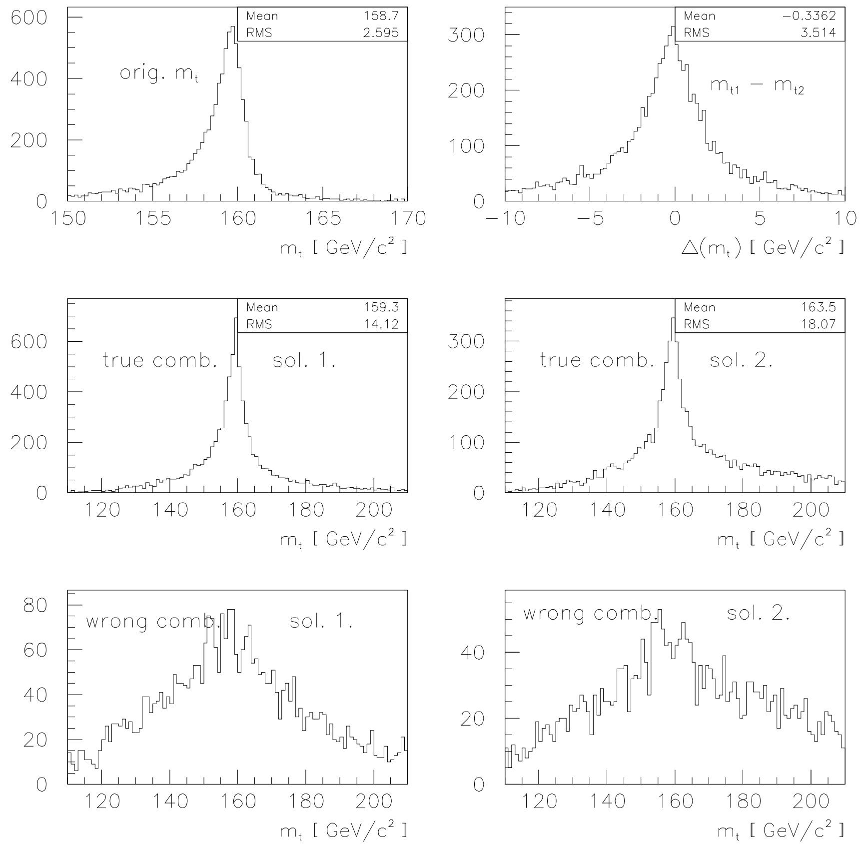

Fig. 2. Same as previous figure except that ISR and FSR is switched on

Two solutions are kinematically equivalent. Standard Model cross section of $t\bar{t}$ production by quark antiquark annihilation mechanism (this is a dominant production mechanism of $t\bar{t}$ at $p\bar{p}$ collisions at 1.8 TeV) is inversely proportional to CMS energy squared ($\hat{s}$) of interacting partons. It follows that from two kinematically equivalent solutions more probable will be solution with smaller $\hat{s}$. Simulation by PYTHIA [4] for reconstructible cases (events which have at least one solution by above procedure) demonstrates that correct solution is chosen in about 80% cases based on minimum of $\hat{s}$ choice. In approximately 50 % cases there does exist also solution for wrong combination of leptons and jets (when lepton and jets from different top quark decay are combined). When minimum $\hat{s}$ choice is applied on sample of right and wrong combination (when it does exist) probability to select correct solution was found close to 70 %.

So far proposed method addressed problems arising even in case when all quantities are measured with absolute precision. Any practical method should deal with problem of resolution. To take into account resolution we apply smearing of input physical quantities according to Gaussian distribution with standard deviation as expected in current experiments at TEVA-

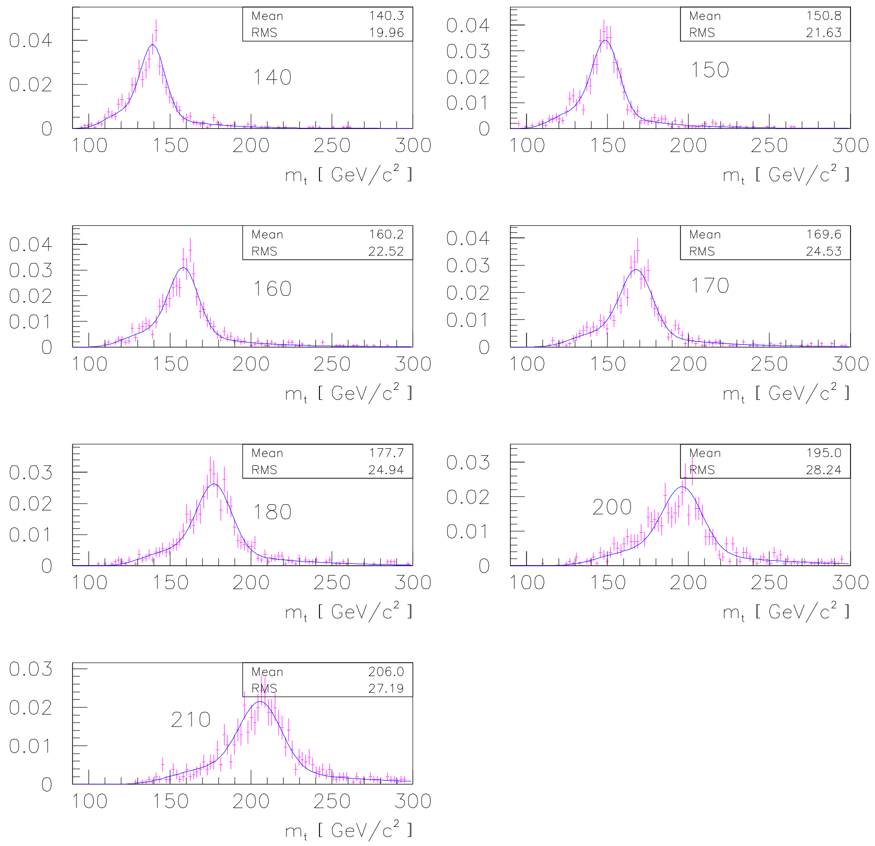

Fig. 3. Normalized distribution of raw reconstructed top mass for different original top mass samples together with fit to parameterization explained in text

TRON [5].

More precisely, error on b-jet transverse energy $\sigma(E_T)$ is assumed to be :

$$\sigma(E_T) = 1.8 + 0.9E_T$$

Error on $\not\!\!E_T$ is assumed to follow relation $\sigma(\not\!\!E_T) = 1.3\sqrt{\not\!\!E_T}$. $P_z^{t\bar{t}}$ is assumed to be distributed according Gaussian with mean zero and σ = 200 GeV/c. Last item is not measured in experiment. It is added as a theoretical input based on the simulation by PYTHIA. Physically this distribution depends on structure function of proton and in top mass range from 140 GeV/c^2 to 200 GeV/c^2 very weakly depends on top mass (σ of the distribution in a given range changes by less than 4 %).

For each event generated by PYTHIA fixed number (1000) of events with smeared characteristics was generated and reconstruction procedure was applied as explained above. The choice of solution was based on minimum of $\hat{s}$ condition. Therefore for each event generated

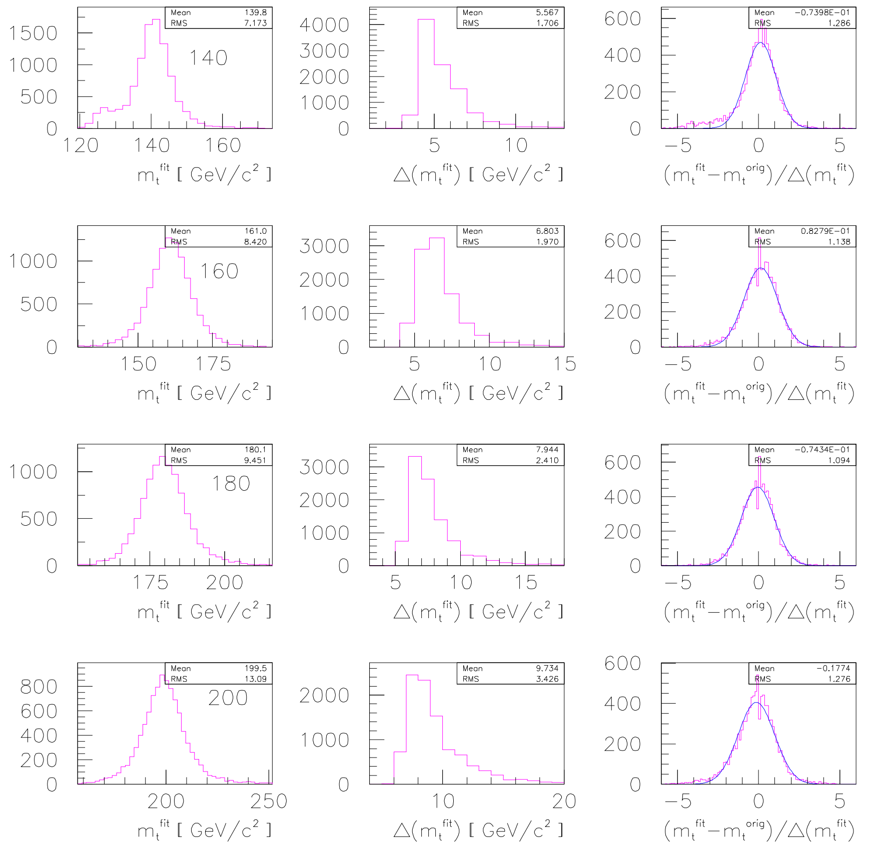

Fig. 4. Distribution of fitted top mass by likelihood fit to a sets of five raw reconstructed top masses, their error and pull distribution

by PYTHIA we have distribution of reconstructed top masses. From this distribution we consider as raw estimate of top mass for a given event a most probable value. To eliminate effect of fluctuations at a finite statistics (1000 smeared events) we apply spline fit to a given distribution and most probable value is taken from spline fit.

Only cases with at least 10% of reconstruction probability are considered. This condition is imposed to have reasonable statistics for estimate of most probable value.

Result of reconstructed raw top masses for range of original top masses from 140 GeV/c^2 to 210 GeV/c^2 is presented in Fig. 3. Rejection rate (based on 10% minimal probability reconstruction cut) for a given range of original top masses was found close to 14 %.

As one can see from Fig. 3 distribution of reconstructed mass is biased and asymmetric. It is necessary to implement procedure which partially or fully eliminates bias and allows to get more accurate estimate of top quark mass from a set of raw top mass measurements. Procedure which follows is similar as one used in e.g. [6].

parameter	a	b
p_1	9.881 ± 4.92	0.83 ± 0.03
p_2	-12.012 ± 5.11	0.178 ± 0.03
p_3	0.37 ± 0.14	0.00027 ± 0.00083
p_4	-3.37 ± 2.16	$0.078 \pm .013$
p_5	6.73 ± 2.06	$0.95 \pm .013$

Tab.1. $p_i = a_i + b_i \times m_t^{orig}$

We parameterize normalized distribution of raw reconstructed top mass for set of chosen original top masses. This will constitute probability density function for definition of likelihood function. The most probable reconstructed top mass from a set of measurements will be estimated by maximum likelihood fit to the templates defined this way.

As a parameterization of templates combination of Landau – like distribution and Gaussian worked remarkably well. Specifically our parameterization was following:

$$\begin{aligned} \lambda_i &= \frac{m_t^{rec} - p_i(m_t^{orig})}{p_{i+1}(m_t^{orig})} \\ f(m_t^{rec}, m_t^{orig}) &= \frac{p_3(m_t^{orig})}{I_1} \exp(-0.5(\lambda_1 + \exp(-\lambda_1)))+ \\ &+ \frac{(1.-p_3(m_t^{orig}))}{I_2} \exp(-0.5\lambda_4^2) \end{aligned}$$

I_1, I_2 are normalization integrals in region $m_t^{rec} \epsilon (90, 300)$ GeV/c^2. Therefore integral of this function in region of $m_t^{rec} \epsilon (90, 300)$ GeV/c^2 is normalized to 1 by definition.

Normalized distribution of reconstructed top mass for original top mass parameter 140, 150, 160, 170, 180, 200, 210 GeV/c^2 together with result of global fit (explained bellow) is presented in Fig. 3. It was observed linear dependence of fitted parameters as a function of m_t^{orig} at least in a range of $m_t^{orig} \epsilon (140, 210)$ GeV/c^2. Therefore we incorporated linear dependence into above parameters and simultaneously (globally) fitted these parameters to all templates. Result is given in tab.1. Quality of fit was characterized by $\chi^2/ND = 0.67$, where ND is number of degrees of freedom for the fit.

Parametrized templates have been used to define likelihood function to determine original top mass by a likelihood fit from a set of raw top mass measurements. Likelihood function is defined as

$$\mathcal{L} = \prod_{i=1}^{n} f({m_t}_i{}^{rec}, m_t^{orig})$$

Where n is number of measurements of top mass in the set. Fitted value of top mass m_t^{fit} determined from a given set of n measurements corresponds to a value of m_t^{orig} at maximum of $\mathcal{L}$.

In Fig. 4 fitted top mass distribution by likelihood fit, estimated error and "pull distribution" based on five events samples of raw top mass measurements is presented. (Pull is defined as $\frac{m_t^{fit}-m_t^{orig}}{\sigma(m_t^{fit})}$). This figure demonstrates full power of the method. Distribution of fitted mass populates narrow region symmetrically around original top mass value. Distribution of estimates of errors of fitted mass correspond to width of fitted mass distribution, therefore prove correctness of error estimates which is in even clearer form demonstrated by pull distribution. Pull distribution is symmetric around 0 with Gaussian shape with σ close to 1. From this figure follows that most probable top mass resolution for top mass in range 140 – 200 GeV/c^2 will be in range 5-8 GeV/c^2.

Analysis is based on 5 events sets because this number is close to present numbers of $t\bar{t}$ candidate events in dilepton channel. Soon (at the beginning of the year 2000) a new series of experiments on TEVATRON (run II) will start. Using this method it would be possible to attain precision of top mass measurement on level 1-2 GeV/c^2 in dilepton channel. To really have this precision experimentalists need to work very hard to beat down systematic errors which will be in this case dominating.

Acknowldgements Discussions with my colleagues and friends in Taiwan and Fermi lab on the subject are kindly acknowledged. Special thanks go to Jan Pišút whose nice anniversary triggered this paper.

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