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THE TRANSPORT COEFFICIENTS FOR FUSION-FISSION REACTIONS*

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The collective friction and mass tensors are computed for the composite systems formed in the fusion of ^{18}O+^{208}Pb, ^{20}Ne+^{249}Cf and ^{48}Ca+^{238}U (for the temperatures $0 \leq T \leq 2$ MeV) in the framework of the linear response theory and locally harmonic approximation. All components of friction and inertia tensors considered as functions of deformation show the peaked structure caused by avoided crossing of single-particle levels as well as by shell and pairing effects. The numerical values of friction and mass parameters differ considerably from the ones obtained with the wall formula for the friction and the Werner-Wheeler method for the collective mass.

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1 Introduction

The synthesis of the superheavy elements is one of the most challenging problems of nuclear physics. Intensive experiments in this direction are carried out at JINR, Dubna and GSI, Darmstadt. A theoretical description of fusion-fission reaction is often based on the Langevin equation for collective variables Q_μ which parameterize the shape of the composite system. To solve the Langevin equation for $Q_\mu(t)$ one first of all needs to define the potential energy, friction $\gamma_{\mu\nu}$, mass $M_{\mu\nu}$ and diffusion $D_{\mu\nu}$ tensors.

The potential energy is commonly calculated within the microscopic-macroscopic shell correction method [2] which describes the collective energy in the quasi-static picture very accurately. The diffusion tensor is often related to the friction tensor by the Einstein relation: $D_{\mu\nu} = T\gamma_{\mu\nu}$ and the tensors of friction $\gamma_{\mu\nu}$ and mass $M_{\mu\nu}$ are computed within the macroscopic approaches (the wall and window formula for friction [3] and the Werner-Wheeler (WW) method for inertia [4]). These methods provide rather simple expressions for the collective friction and mass coefficients. However the deformation and temperature dependence of macroscopic transport coefficients $\gamma_{\mu\nu}$ and $M_{\mu\nu}$ is not very reliable, at least at small excitation energies. For example, the observed [5] increase of the damping parameter $\eta = \gamma/2\sqrt{M|C|}$ with the excitation

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energy is difficult to explain either within the wall formula value for friction (where the friction parameter practically does not depend on the temperature T or excitation energy) or within the hydrodynamical model (friction parameter decreases as $1/T^2$). Also, the shell and pairing effects which are very important at low excitation energies are completely ignored in macroscopic models. Thus the necessity of having a microscopic theory for the collective transport coefficients is quite evident.

The possible solution of the problem can be based on an application of linear response theory (for review see [1]). The results of calculations presented below are obtained within an approach developed in [1] as well as in publications [6, 7]. Because of limited space only few main quantities are defined below. For details, see [1, 6–8].

2 Collective dynamics in locally harmonic approximation

It is supposed that the nuclear many-body Hamiltonian can be approximated by [1]

$$\hat{H}(\hat{x}_i,\hat{p}_i,Q_\mu) = \hat{H}_{rmf}(\hat{x}_i,\hat{p}_i,Q_\mu) + \hat{V}^{(2)}_{res}(\hat{x}_i,\hat{p}_i), \tag{1}$$

where renormalized mean field Hamiltonian $\hat{H}_{rmf}$ depends explicitly on a few collective variables Q_μ and the residual two body interaction is assumed to be *independent* of Q_μ.

Considering the variation of Hamiltonian (1) due to the time dependence of $Q_\mu(t)$ as a small perturbation one can show that the slow collective motion in Q_μ can be described locally in terms of the so-called causal response function $\tilde{\chi}_{\mu\nu}(t-s)$

$$\tilde{\chi}_{\mu\nu}(t-s) = \Theta(t-s)\frac{i}{\hbar}\mathrm{tr}\,\left(\hat{\rho}_{\mathrm{qs}}(Q_0,T_0)[\hat{F}^I_\mu(t),\hat{F}^I_\nu(s)]\right) \equiv 2i\Theta(t-s)\chi''_{\mu\nu}(t-s). \tag{2}$$

Here, $\hat{F}^I_\mu(t)$ is the interaction representation of the derivatives of the Hamiltonian (1) with respect to Q_μ taken at an arbitrary fixed point Q^0_μ,

$$\hat{F}_\mu \equiv \frac{\partial\hat{H}(\hat{x}_i,\hat{p}_i,Q_\mu)}{\partial Q_\mu}\Big|_{Q_\mu=Q^0_\mu} \equiv \frac{\partial\hat{H}_{rmf}(\hat{x}_i,\hat{p}_i,Q_\mu)}{\partial Q_\mu}\Big|_{Q_\mu=Q^0_\mu}. \tag{3}$$

The $\hat{\rho}_{\mathrm{qs}}$ represents the thermal equilibrium at Q^0_μ and is given by the canonical distribution $\rho_{\mathrm{qs}}(Q^0_\mu,T) \propto \exp(-\hat{H}(Q^0_\mu)/T)$. To account for the pairing interaction the renormalized mean field Hamiltonian is chosen as that of independent quasiparticles (see [8]) $\hat{H}_{rmf} = U_0 + \sum_k E_k(\alpha^+_k\alpha_k + \alpha^+_{\bar{k}}\alpha_{\bar{k}})$, where U_0 is the ground state energy, E_k are the quasiparticle energies and α^+_k and α_k are creation and annihilation operators for quasiparticles.

The quantity of a special interest in the linear response theory is the Fourier transform $\chi_{\mu\nu}(\omega)$ of the response function (2). This can be found directly from (2)

$$\begin{aligned}\chi_{\mu\nu}(\omega) = &\sum_{kj}{}'(n^T_k - n^T_j)\xi^2_{kj}\frac{2(E_k-E_j)^2}{(\hbar\omega+i0)^2-(E_k-E_j)^2}F^{kj}_\mu F^{jk}_\nu \\ &+\sum_{kj}(n^T_k+n^T_j-1)\eta^2_{kj}\frac{2(E_k+E_j)^2}{(\hbar\omega+i0)^2-(E_k+E_j)^2}F^{kj}_\mu F^{jk}_\nu,\end{aligned} \tag{4}$$

where $n^T_k = 1/(1+\exp(E_k/T))$, $\eta_{kj} \equiv u_k v_j + v_k u_j$, $\xi_{kj} \equiv u_k u_j - v_k v_j$ and u_k, v_k are coefficients of the Bogolyubov-Valatin transformation. The summation in (4) is over the single-particle states $|k\rangle$ and $|j\rangle$ and F^μ_{kj} are the matrix elements of $\hat{F}^\mu$ on these states.

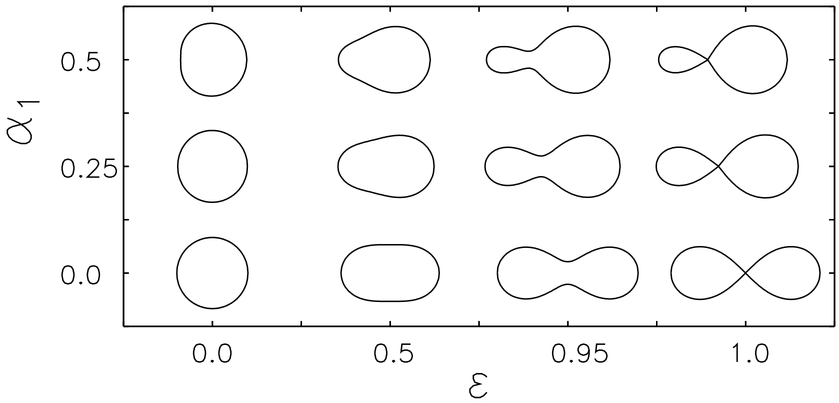

Fig. 1. The two-dimensional family of shapes based on the parameterization of the nuclear surface in terms of Cassinian ovaloids (see [9]).

To account for the residual interaction, $\hat{V}^{(2)}_{res}$, we first express the response function (4) in terms of the Green function $\mathcal{G}^{(0)}_{ks}(\omega \pm i0)$ for independent quasiparticles and then modify $\mathcal{G}^{(0)}_{ks}(\omega \pm i0)$ by introducing a spreading of the quasiparticle states. This is achieved by expressing the Green function in terms of the real and imaginary parts of the self-energy $\Sigma(\omega \pm i0, \Delta, T) = \Sigma'(\omega, \Delta, T) \mp i\Gamma(\omega, \Delta, T)/2\mathrm{i}$,

$$\mathcal{G}^{(0)}_{ks}(\omega \pm i0) \to \mathcal{G}_{ks}(\omega \pm i0) = \frac{1}{\hbar\omega - E_{ks} - \Sigma'(\omega) \pm i\Gamma(\omega)/2}. \tag{5}$$

The calculation of Σ which includes the pairing is discussed in detail in [8].

The simplest expressions for the friction $\gamma_{\mu\nu}$ and mass $M_{\mu\nu}$ tensors within the linear response theory are offered by the so called zero frequency approximation,

$$\gamma_{\mu\nu}(0) = -i\frac{\partial\chi_{\mu\nu}(\omega)}{\partial\omega}\Big|_{\omega=0} \quad \text{and} \quad \mathrm{M}_{\mu\nu}(0) = \frac{1}{2}\frac{\partial^2\chi_{\mu\nu}(\omega)}{\partial\omega^2}\Big|_{\omega=0} \tag{6}$$

which express the friction and mass tensors in terms of the derivatives of response function with respect to the frequency taken at $\omega = 0$.

3 Numerical results

In the present work the friction and mass tensors (6) are computed for the composite systems formed in the fusion reactions ^{18}O+^{208}Pb, ^{20}Ne+^{249}Cf and ^{48}Ca+^{238}U which were investigated recently at JINR, Dubna.

Like in [7], the independent particle Hamiltonian based on the deformed Woods-Saxon potential was used. The nuclear shape was parameterized in terms of Cassini ovaloids by two parameters ε and α_1 which describe the overall elongation and left-right asymmetry of the nucleus (see [9] and Fig. 1).

The most interesting $\varepsilon\varepsilon$-component of the friction tensor (6), which corresponds to the elongation mode, is shown in Fig. 2. To remove the geometrical factors the friction coefficient $\gamma_{\varepsilon\varepsilon}$ was

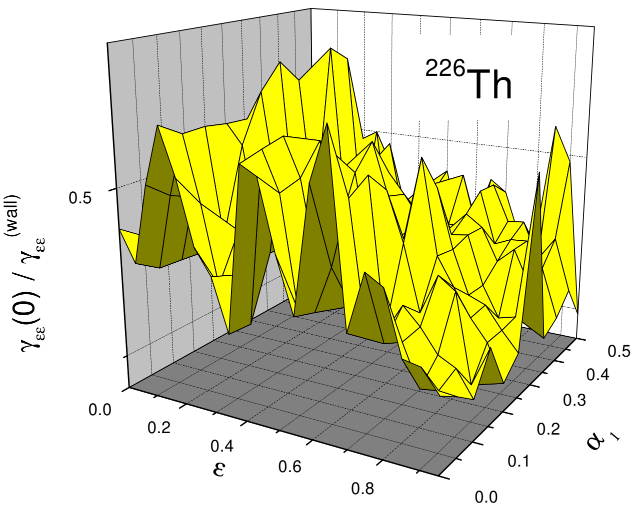

Fig. 2. The εε-component of the friction tensor (6) (in units of wall formula value) for the ^{226}Th nucleus at the temperature $T = 1$ MeV.

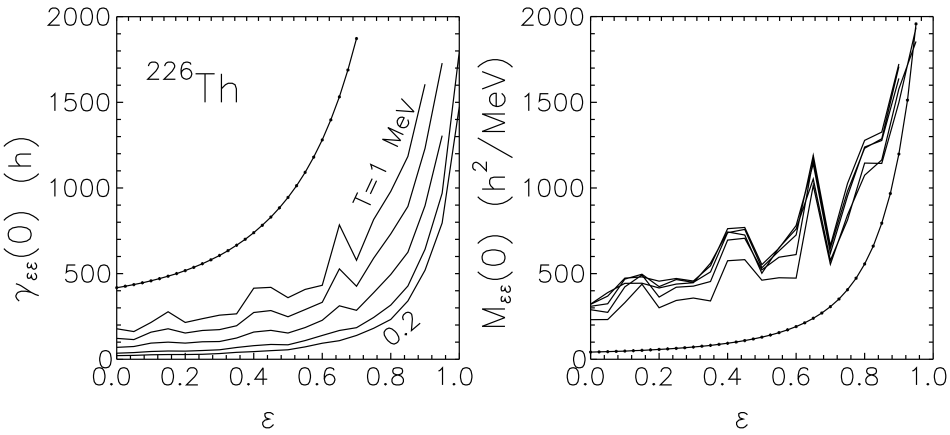

Fig. 3. The εε-components of friction (left) and mass (right) tensors (6) for ^{226}Th nucleus versus the elongation parameter ε for temperatures $T = 0.2$, 0.4, 0.6, 0.8, 1.0 MeV. The lines with dots mark the wall formula friction value and the Werner-Wheeler mass parameter.

divided by the wall formula value [3]. First of all the rapid fluctuations against the deformation attract our attention. The analysis shows that the sharp peaks are mainly caused by the avoided crossings of single-particle levels. The fluctuations due to shell effects are much smoother.

An example of the temperature dependence of the εε-component of friction and mass tensors (6) is shown in Fig. 3. It is seen that the friction parameter is a growing function of temperature.

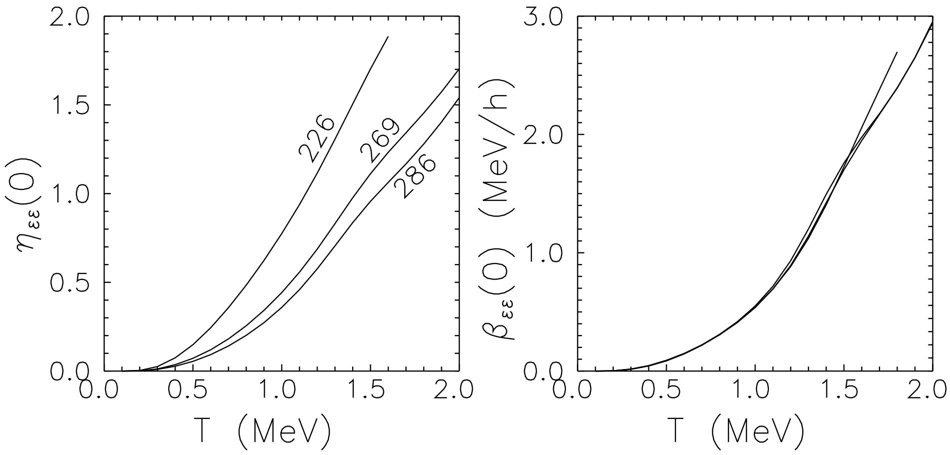

Fig. 4. The damping factor $\eta_{\varepsilon\varepsilon} = \gamma_{\varepsilon\varepsilon}/2\sqrt{|C_{\varepsilon\varepsilon}|M_{\varepsilon\varepsilon}}$ (left) and the reduced friction coefficient $\beta_{\varepsilon\varepsilon} = \gamma_{\varepsilon\varepsilon}/M_{\varepsilon\varepsilon}$ (right) versus temperature. The numbers in Figure mark different mass numbers.

At $T = 1$ MeV $\gamma_{\varepsilon\varepsilon}(0)$ is smaller than the wall formula value still by a factor 2 approximately. As it was shown in [7] the wall formula value is reached at $T = 3-4$ MeV only.

The $\varepsilon\varepsilon$-component of the mass tensor is not very sensitive to the temperature below $T =$ 1 MeV. At the same time $M_{\varepsilon\varepsilon}(0)$ is substantially larger than $M_{\varepsilon\varepsilon}^{WW}$ computed within the WW method. The same relation between microscopic and macroscopic results for the friction and inertia holds for heavier systems, too.

Finally, Fig. 4 shows the damping factor $\eta_{\varepsilon\varepsilon} = \gamma_{\varepsilon\varepsilon}/2\sqrt{M_{\varepsilon\varepsilon}|C_{\varepsilon\varepsilon}|}$ and reduced friction coefficient $\beta_{\varepsilon\varepsilon} = \gamma_{\varepsilon\varepsilon}/M_{\varepsilon\varepsilon}$ at the saddle point as a function of the temperature. The damping factor reveals whether the collective motion is underdamped ($\eta \ll 1$) or overdamped ($\eta \gg 1$). As Fig. 4 shows, the collective motion changes from the underdamped to overdamped one at $T \approx 1-1.5$ MeV.

The damping factor can be deduced from the experimental data since it appears in the well known Kramers' formula of the decay rate: $\mathrm{R_K} = (\varpi_m/2\pi)\left(\sqrt{1+\eta^2}-\eta\right)\,\exp(-B/T)$. Here B is the fission barrier and ϖ_m determines the frequency of collective vibrations around the minimum of the potential.

Both η and β shown in Fig. 4 increase with the temperature. This behaviour is in qualitative agreement with the one found in [5] for ^{224}Th.

In conclusion, it is possible to affirm that the friction parameter is very small at low excitation energies. Both friction and mass tensors show complicated fluctuating structure caused by the avoided crossings of single-particle levels as well as by shell effects, and differ substantially from the ones obtained within macroscopic models in the $0.2 \leq T \leq 2$ MeV temperature range. Thus the results of dynamical computations based on the macroscopic friction and inertia parameters are not reliable at low excitation energies.

The derived transport coefficients will be used in the dynamical code for the description of fusion-fission reactions. The preliminary results of similar computations were reported in [10].

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