# RAMAN AND BRILLOUIN COUPLERS WITH PHASE MISMATCH[1]

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Statistical properties of optical fields in asymmetric nonlinear couplers composed of two waveguides are investigated within the framework of a generalized superposition of coherent fields and quantum noise. Raman or Brillouin processes (with classical pumping) are in operation in the first waveguide. Stokes or/and anti-Stokes modes are connected through the linear interaction with corresponding modes in the second waveguide. Various phase mismatches are assumed. An approach to analytical solution of Heisenberg equations of motion is described. Various regimes for different values of Stokes and anti-Stokes linear coupling constants are discussed. An influence of various phase mismatches on the generation of nonclassical states of light including sub-Poissonian statistics, negative reduced factorial moments and squeezing of quadrature variances is investigated.

## 1. Introduction

Nonlinear couplers are devices composed of two or more waveguides with mutually connected modes by means of evanescent waves. In one or more of these waveguides a nonlinear process takes place. Dynamics of such devices and quantum statistical properties of their optical fields have been investigated. Assumed nonlinear processes vary from second harmonic generation, parametric processes, Kerr nonlinearity to Raman scattering. For details, see Refs. [1–10].

Raman and Brillouin scattering is one of very important and interesting optical processes, having broad spectra of applications from high-resolution spectroscopy to Raman amplifiers and generators.

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In this paper we present an analysis of asymmetric Raman coupler composed of two waveguides. Stimulated Raman or Brillouin scattering with strong classical pumping of laser mode is assumed in first waveguide. This assumption allows us to linearize equations of motion for creation and annihilation operators and find their analytic solution. Stokes or anti-Stokes modes can interact with their counterparts in the second waveguide, which are injected as the inputs. For simplification we set anti-Stokes coupling constant $\kappa_A = 0$ or Stokes coupling constant $\kappa_S = 0$ and analyze influence of Stokes and anti-Stokes coupling separately.

General discussion of such type of couplers has been given in [5]. Authors restrict themselves to the case, when phase matching conditions were fulfilled. In this paper we analyze influence of phase mismatch on the dynamics of coupler. We obtain a strong dependence on the values of various phase mismatches. We also give some further analysis of phase matching case.

## 2. Equations of motion

We consider asymmetric nonlinear coupler schematically illustrated in Fig .1. This coupler can be described with the use of the momentum operator $\hat{G}$ in the form

$$\begin{aligned}\hat{G} &= \sum_{j=S_1,A_1,V_1} \hbar k_j \hat{a}_j^\dagger \hat{a}_j + [\hbar\tilde{g}_{A_1}\hat{a}_{L1}\hat{a}_{V_1}\hat{a}_{A_1}^\dagger + \hbar\tilde{g}_{S_1}\hat{a}_{L1}\hat{a}_{V_1}^\dagger\hat{a}_{S_1}^\dagger + h.c.] \\ &\quad + \sum_{j=S_2,A_2} \hbar k_j \hat{a}_j^\dagger \hat{a}_j + [\hbar\kappa_S\hat{a}_{S_1}\hat{a}_{S_2}^\dagger + \hbar\kappa_A\hat{a}_{A_1}\hat{a}_{A_2}^\dagger + h.c.] \end{aligned} \tag{1}$$

where $\hat{a}_{S1},\ \hat{a}_{A1},\ \hat{a}_{S2},\ \hat{a}_{A2},\ \hat{a}_{L1},\ \hat{a}_{V_1}$ $(\hat{a}_{S1}^\dagger,\ \hat{a}_{A1}^\dagger,\ \hat{a}_{S2}^\dagger,\ \hat{a}_{A2}^\dagger,\ \hat{a}_{L1}^\dagger,\ \hat{a}_{V_1}^\dagger)$ are annihilation (creation) operators of Stokes and anti-Stokes modes in the first and the second waveguides and laser and vibration modes in the first waveguide. Phenomenological constants $\tilde{g}_{S1},\ \tilde{g}_{A1}$ describe nonlinear Stokes and anti-Stokes interaction in the first waveguide and $\kappa_S,\ \kappa_A$ characterize linear coupling of Stokes and anti-Stokes modes in both waveguides.

Equations of motion can be obtained from Heisenberg equation $i\hbar\frac{d\hat{a}}{dz} = [\hat{G},\hat{a}]$ . Symbol [,] represents commutator. Assuming strong classical pumping in laser mode we can replace operator $\hat{a}_{L1}$ with the complex amplitude $\hat{a}_{L1} \to \alpha_{L1}e^{ik_{L1}z}$ and introduce new constants $g_{S1} = \tilde{g}_{S1}\alpha_{L1}$, $g_{A1} = \tilde{g}_{A1}\alpha_{L1}$. We will work in the interaction picture introducing new operators $\hat{A}_j = \hat{a}_j \exp(-ik_jz)$, $j = S_1, A_1, V_1, S_2, A_2$. Equations of motion have the form

$$\begin{aligned}\frac{d\hat{A}_{S_1}}{dz} &= ig_{S_1}e^{i\Delta k_{S_1}z}\hat{A}_{V_1}^\dagger + i\kappa_S^* e^{-i\Delta K_S z}\hat{A}_{S_2}\ , \\ \frac{d\hat{A}_{A_1}}{dz} &= ig_{A_1}e^{i\Delta k_{A_1}z}\hat{A}_{V_1} + i\kappa_A^* e^{-i\Delta K_A z}\hat{A}_{A_2}\ , \\ \frac{d\hat{A}_{V_1}}{dz} &= ig_{S_1}e^{i\Delta k_{S_1}z}\hat{A}_{S_1}^\dagger + ig_{A_1}^* e^{-i\Delta k_{A_1}z}\hat{A}_{A_1}\ ,\end{aligned}$$

$$
\begin{aligned}
\frac{d\hat{A}_{S_2}}{dz} &= i\kappa_S e^{i\Delta K_S z}\hat{A}_{S_1} \ , \\
\frac{d\hat{A}_{A_2}}{dz} &= i\kappa_A e^{i\Delta K_A z}\hat{A}_{A_1} \ ,
\end{aligned}
\tag{2}
$$

where $\Delta k_{S1} = k_{L1} - k_{V1} - k_{S1}$ , $\Delta k_{A1} = k_{L1} + k_{V1} - k_{A1}$ , $\Delta K_S = k_{S_1} - k_{S_2}$ , $\Delta K_A = k_{A_1} - k_{A_2}$ are phase mismatch vectors.

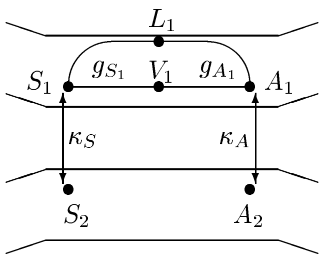


Fig. 1. Scheme of the quantum nonlinear coupler formed from two waveguides. Raman or Brillouin processes take place in the first waveguide. $L_1$, $S_1$, $A_1$, $V_1$ denote laser pump, Stokes, anti-Stokes and vibration modes in the first waveguide. $g_{S_1}$ and $g_{A_1}$ are Stokes and anti-Stokes nonlinear coupling constants. Stokes or (and) anti-Stokes modes are linearly coupled with their counterparts $S_2$, $A_2$ in the second waveguide. $\kappa_S$ and $\kappa_A$ are corresponding linear Stokes and anti-Stokes coupling constants.

We start with the assumption $\kappa_A = 0$, which reduces number of modes from 5 to 4, only Stokes mode S2 in the second waveguide is of importance. Similar analysis can be done in the case $\kappa_S = 0$ so we can restrict description of solution to the above mentioned case. We can transform (2) into the system of linear differential equations with constant coefficients expressing $\hat{A}_j$ by means of new operators

$$
\begin{aligned}
\hat{A}_{S_1}(z) &= \hat{B}_{S_1}(z) e^{\frac{i}{2}(\Delta k_{S_1} + \Delta k_{A_1} - \Delta K_S)z} \ , \\
\hat{A}_{A_1}(z) &= \hat{B}_{A_1}(z) e^{\frac{i}{2}(\Delta k_{S_1} + \Delta k_{A_1} + \Delta K_S)z} \ , \\
\hat{A}_{V_1}(z) &= \hat{B}_{A_1}(z) e^{\frac{i}{2}(\Delta k_{S_1} - \Delta k_{A_1} + \Delta K_S)z} \ , \\
\hat{A}_{S_2}(z) &= \hat{B}_{A_1}(z) e^{\frac{i}{2}(\Delta k_{S_1} + \Delta k_{A_1} + \Delta K_S)z} \ .
\end{aligned}
\tag{3}
$$

Equations for $\hat{B}_j$ written in matrix form are

$$
\frac{d}{dz}\hat{\mathbf{B}} = \mathbf{M}\hat{\mathbf{B}} \ . \tag{4}
$$

The definitions of the matrix $\mathbf{M}$ and the vector $\hat{\mathbf{B}}$ are following

$$
\mathbf{M} = \begin{pmatrix}
\frac{i}{2}\Delta_{S_1,A_1,-S} & 0 & -ig^*_{S_1} & -i\kappa_s \\
0 & -\frac{i}{2}\Delta_{S_1,A_1,S} & ig_{A_1} & 0 \\
ig_{S_1} & ig^*_{A_1} & -\frac{i}{2}\Delta_{S_1,-A_1,S} & 0 \\
-i\kappa^*_S & 0 & 0 & \frac{i}{2}\Delta_{S_1,A_1,S}
\end{pmatrix} , \quad
\hat{\mathbf{B}} = \begin{pmatrix}
\hat{B}^\dagger_{S1} \\ \hat{B}_{A1} \\ \hat{B}_{V1} \\ \hat{B}^\dagger_{S2}
\end{pmatrix} .
$$

Symbols $\Delta_{S_1,\pm A_1,\pm S} = \Delta k_{S_1} \pm \Delta k_{A_1} \pm \Delta K_S$ are used for abbreviation.

We have to find eigenvalues $\lambda_j$ of the matrix $\mathbf{M}$ to solve the system (4). For detailed description of the solution, see [11]. $\mathbf{M}$ is $4 \times 4$ matrix and its characteristic polynom is of the fourth order. We can find its roots $\lambda_j$ analytically using Cardan formulae and therefore we are able (after backward transformation (3)) to write down an analytical solution of (2). This is rather boring and is not explicitly done here.

## 3. Phase matching case

We focus ourselves now to the situation, when all the mismatches are zero. All diagonal elements of $\mathbf{M}$ vanish. Characteristic equation leads to the biquadratic equation.

A) $\kappa_A = 0$. The eigenvalues are

$$\lambda_j = \pm \left[ \frac{1}{2} \left( -(|g_{A_1}|^2 - |g_{S_1}|^2 + |\kappa_S|^2) \pm \sqrt{(|g_{A_1}|^2 - |g_{S_1}|^2 + |\kappa_S|^2)^2 - 4|\kappa_S|^2 |g_{A_1}|^2} \right) \right]^{\frac{1}{2}} . \tag{5}$$

All four combinations of signs are possible. The expression under the symbol of the square root plays a key role. Its sign determines the behaviour of the coupler. There are two possible situations: either all four roots are purely imaginary and we obtain oscillating solution, either they have nonzero real part and we obtain exponentially amplified solution. We find three different regions for different values of $|\kappa_S|$

1. $|\kappa_S| < ||g_{A_1}| - |g_{S_1}||$ – linear coupling is too small to change completely behaviour of scattering, the dynamics of the coupler is similar as for $|\kappa_S| = 0$. We have oscillating solution for $|g_{A_1}| > |g_{S_1}|$ and exponential increase for $|g_{A_1}| < |g_{S_1}|$.
2. $||g_{A_1}| - |g_{S_1}|| < |\kappa_S| < |g_{A_1}| + |g_{S_1}|$ – linear coupling induces exponential increase. The above mentioned expression has negative value in this interval and its square root is purely imaginary. The four eigenvalues have the form $\lambda_j = \pm\lambda_R \pm i\lambda_I$ with nonzero real parts.
3. $|g_{A_1}| + |g_{S_1}| < |\kappa_S|$ – the behaviour is changing again. Strong linear Stokes coupling induces oscillations despite of the rate $|g_{A_1}| : |g_{S_1}|$. All the eigenvalues are purely imaginary.

The same approach can be used in the case $\kappa_S = 0$. We receive the matrix $\mathbf{M}$ in similar form as before but small changes induce strong differences. The eigenvalues can be expressed as

$$\lambda_j = \pm \left[ \frac{1}{2} \left( -(|g_{A_1}|^2 - |g_{S_1}|^2 + |\kappa_A|^2) \pm \sqrt{(|g_{A_1}|^2 - |g_{S_1}|^2 + |\kappa_A|^2)^2 + 4|\kappa_A|^2 |g_{S_1}|^2} \right) \right]^{\frac{1}{2}} . \tag{6}$$

The sign in the expression under the symbol of the square root has changed from $-$ to $+$. This means that two eigenvalues are real (one positive and the other negative) and the remaining two are purely imaginary. The positive real $\lambda_1$ (both signs $+$ in (6)) leads to exponential increase for all nonzero values of $|\kappa_A|$. The limit value for $|\kappa_A| \to \infty$ is $\lambda_1 = |g_{S_1}|$.

Phase mismatches generally reduce corresponding processes, lead to faster oscillations in spatial evolution and can change exponentially increasing character of the solution to oscillating. Their influence will be discussed below in detail.

## 4. Photon statistics, factorial moments and squeezing of vacuum fluctuations

To describe quantum statistical properties of the optical and phonon fields we use normal characteristic function, which is generating function of the normal ordered moments of any order. We work within the framework of the generalized superposition of coherent fields and quantum noise with normal characteristic function in the Gaussian form

$$\begin{aligned} C_{\mathcal{N}}(\{\beta_j\},z) &= \exp\left\{\sum_j\left[-B_j(z)|\beta_j|^2+\left(\frac{1}{2}C_j(z)\beta_j^{*2}+\text{c.c.}\right)\right]\right. \\ &+ \left.\sum_j\sum_{k>j}\left[D_{jk}(z)\beta_j^*\beta_k^*+\bar{D}_{jk}(z)\beta_j\beta_k^*+\text{c.c.}\right]+\sum_j\left[\beta_j\xi_j^*(z)-\text{c.c.}\right]\right\} . \end{aligned} \tag{7}$$

Here $\xi_j(z)=\langle\hat{A}_j(z)\rangle$ are corresponding complex amplitudes and a definition of parameters is

$$\begin{aligned} B_i(z) &= \langle\Delta\hat{A}_i^\dagger\Delta\hat{A}_i\rangle, \qquad D_{ij}(z)=\langle\Delta\hat{A}_i\Delta\hat{A}_j\rangle \\ C_i(z) &= \langle\Delta\hat{A}_i\Delta\hat{A}_i\rangle, \qquad \bar{D}_{ij}(z)=-\langle\Delta\hat{A}_i^\dagger\Delta\hat{A}_j\rangle . \end{aligned} \tag{8}$$

We are interested in the possibility of generation of nonclassical states of light including sub-Poissonian statistics, negative reduced factorial moments or squeezing of vacuum fluctuations. As discussed in [5], these effects do not occur in single modes assuming classical input light. Thus we mostly analyze compound modes, composed of two single modes. Corresponding number operator is $\hat{n}_{ij}=\hat{A}_i^\dagger\hat{A}_i+\hat{A}_j^\dagger\hat{A}_j$ and the integrated intensity $W_{ij}=\langle\{\alpha_k\}|\hat{n}_{ij}|\{\alpha_k\}\rangle$ where $|\{\alpha_k\}\rangle$ is the coherent state.. The equations of motion have the form predicting that for any compound mode $D_{ij}(z)=0$ or $\bar{D}_{ij}(z)=0$ which simplifies our calculations. These special cases are discussed in [4].

We introduce normal generating function and formulae for evaluation of photon probability distribution and factorial moments of k-th order [4]:

$$\begin{aligned} C_{\mathcal{N}}^{(W_{ij})}(\lambda,z) &= \left\langle\prod_{k=1}^{2}\frac{1}{1+\lambda\lambda_k}\exp\left(-\sum_{k=1}^{2}\frac{A_k'}{1+\lambda\lambda_k}\right)\right\rangle , \\ p(n,z) &= \left\langle\exp\left(-\sum_{k=1}^{2}\frac{A_k'}{1+\lambda\lambda_k}\right)\sum_{l=0}^{n}\frac{\lambda_1^l\lambda_2^{n-l}}{l!(n-l)!}(1+\lambda_1)^{-(l+1)}\right. \\ &\times \left.(1+\lambda_2)^{l-(n+1)}\,L_l^0\left(\frac{-A_1'}{\lambda_1(1+\lambda_1)}\right)L_{n-l}^0\left(\frac{-A_2'}{\lambda_2(1+\lambda_2)}\right)\right\rangle , \\ \langle W^k\rangle &= \left\langle\sum_{l=0}^{k}\binom{k}{l}\lambda_1^l\lambda_2^{k-l}L_l^0\left(\frac{-A_1'}{\lambda_1}\right)L_{k-l}^0\left(\frac{-A_2'}{\lambda_2}\right)\right\rangle . \end{aligned} \tag{9}$$

Statistical properties of the fields are described by the superpositions of the coherent signals $A'_k$ and noises $\lambda_k$ (they are completely different from the eigenvalues in previous discussion).

Quadrature components (in the interaction picture) are $\hat{q}_{ij} = \hat{A}_i + \hat{A}_j + \hat{A}_i^\dagger + \hat{A}_j^\dagger$ and $\hat{p}_{ij} = -i(\hat{A}_i + \hat{A}_j - \hat{A}_i^\dagger - \hat{A}_j^\dagger)$. Their variances are [3]

$$\langle(\Delta_{\hat{p}}^{\hat{q}})^2\rangle = 2\left\{1 + B_i(z) + B_j(z) - 2\mathrm{Re}[\bar{D}_{ij}(z)] \pm \mathrm{Re}[C_i(z) + C_j(z) + 2D_{ij}(z)]\right\}$$

and the principal squeezing variance $\lambda$ is

$$\lambda_{ij}(z) = 2\left\{1 + B_i(z) + B_j(z) - 2\mathrm{Re}[\bar{D}_{ij}(z)] - |C_i(z) + C_j(z) + 2D_{ij}(z)|\right\} \ .$$

Squeezing of vacuum fluctuations occurs for $\lambda_{ij} < 2$. We can also define uncertainty product $u_{ij}(z) = \langle(\Delta\hat{p}_{ij})^2\rangle\langle(\Delta\hat{q}_{ij})^2\rangle$ , satisfying the inequality $u_{ij}(z) \geq 4$ (an analogue of the Heisenberg relations of uncertainty).

## 5. Discussion of results

In this part we give a discussion of the influence of various phase mismatches on the generation of nonclassical states of light. We assume stimulated Brillouin (phonon mode in coherent state) or Raman (phonon mode in chaotic state) scattering. We assume coherent state or vacuum state in all input optical fields. Initial phases of the modes are important for generation of nonclassical states of light. If their phase differences are suitably chosen the efficiency of the generation is maximized.

Let us first remark that only oscillating behaviour of the solution is suitable for our purposes. Exponential amplification does not lead to any nonclassical states of light for longer $z$. Therefore the anti-Stokes linear coupling $\kappa_A$ negatively influences such generation and should be eliminated. This can be done with the help of the anti-Stokes linear phase mismatch $\Delta K_A$, which reflects slightly different dispersion in both waveguides.

Stokes linear coupling constant $\kappa_S$ can support generation of nonclassical light as discussed in [5]. Our analytic results in section 3 reveal that this is true only for sufficiently high values of $\kappa_S$. There exists a region of its values causing exponential increase giving no possibility to obtain nonclassical light in the output.

Various phase mismatches generally reduce corresponding interaction, e.g. $\Delta k_{S_1}$ nonlinear Stokes interaction and $\Delta K_S$ linear Stokes coupling between both waveguides. There are generally three regions of their values.

1. Phase mismatch is much smaller than the appropriate coupling constant. The influence of phase mismatch is small, spatial developement is similar to phase matching case, induced oscillations have long period.

2. Phase mismatch has a value compared to appropriate coupling constant. The behaviour of the process is strongly changing. Faster oscillations appear.

3. Phase mismatch is much greater than coupling constant. This leads to effective suppression of influenced process, spatial development is similar to the case when appropriate coupling constant equals zero, and it is modulated with fast oscillations of small amplitude.

Given division is just qualitative and without sharp boundaries. Generally phase mismatches lead to decrease of the nonclassical character of the generated light. Some exceptions however exist. Phase mismatch can compensate wrong phases. From the point of view of equations (2) phase mismatch leads to spatial change of the phase of the corresponding coupling constant. Thus intervals suitable for generation of nonclassical states of light can appear followed with the intervals destroying this character again. Phase mismatches can also suppress effects not suitable for nonclassical light generation such as in the case of non-zero $\kappa_A$.

We restrict our analysis to light modes though statistics of compound photon-phonon modes can also be studied and interesting results can be obtained. Initial statistics of compound modes composed of two Stokes or two anti-Stokes modes are mostly conserved and change from Poissonian to slightly super-Poissonian due to linear coupling between those modes. So we restrict ourselves to modes composed of Stokes and anti-Stokes modes.

*Influence of linear Stokes phase mismatch* $\Delta K_S$

Considering configuration with Stokes linear coupling, nonclassical light with negative reduced factorial moments can be reached in compound modes $(S_1, A_1)$ and $(S_2, A_1)$ for $|g_{A_1}| > |g_{S_1}|$ and Brillouin scattering. Phase mismatch $\Delta K_S$ makes the intervals of negative factorial moments shorter and intervals of higher positive values occur in comparison with phase matching case. This is shown in Fig. 2.

The behaviour of the system changes rapidly with the further increase of $\Delta K_S$. Its high values destroy linear coupling and also the generation of nonclassical light in the above mentioned compound modes. Reduced factorial moments are positive, modulated with fast oscillations as Fig. 3 shows.

*Influence of nonlinear Stokes phase mismatch* $\Delta k_{S_1}$

Nonlinear mismatch reduces Stokes process and decreases an ability of generation of the light with sub-Poissonian statistics. Reduced factorial moments increase but after that they start to decrease and reach zero value and start to increase again. This represents changes from coherent light (Poissonian) to super-Poissonian and back to coherent state. Process repeats periodically, see Fig .4a.

Phase mismatch can support generation of squeezed light in modes $(S_1, A_1)$ and $(S_2, A_1)$. For mode $(S_2, A_1)$ the minimal value of $\lambda$ is 1, which is 50% decrease. Fig. 4b illustrates this situation. Uncertainty product is also plotted there. Its increase and return to starting value corresponds with increase of noise and the following return to the coherent state (compare Figs. 4a and 4b).

*Influence of linear anti-Stokes phase mismatch* $\Delta K_A$

Its possible influence was mentioned before. Anti-Stokes linear coupling suppresses possibility of generation of nonclassical light. Mismatch $\Delta K_A$ can switch off this cou-

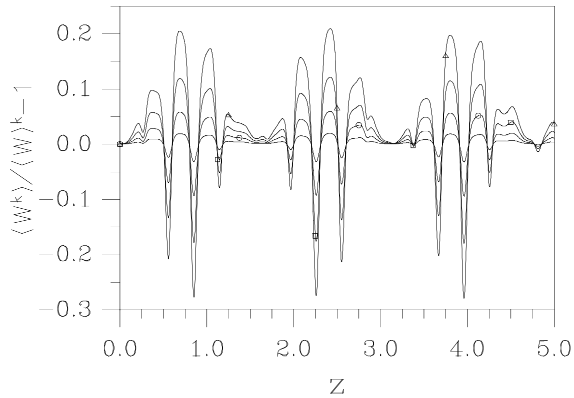


Fig. 2. Reduced factorial moments of the integrated intensity $\langle W^k(z)\rangle/\langle W\rangle^k - 1$ for $k = 2$ $(-)$, $k = 3$ $(\circ)$, $k = 4$ $(\Box)$ and $k = 5$ $(\triangle)$ for mode $(S_2, A_1)$; $g_{S_1} = 1$, $g_{A_1} = 2$, $\kappa_S = -10$, $\xi_{S_1} = 2$, $\xi_{S_2} = 2$, $\xi_{V_1} = 1$, $\Delta K_S = 10$, and the other parameters are zero.

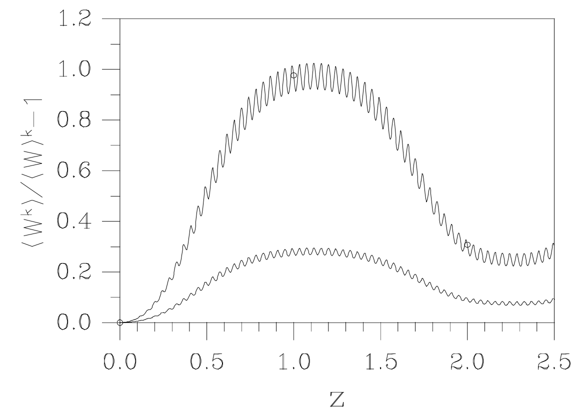


Fig. 3. Second $(-)$ and third $(\circ)$ reduced factorial moments for mode $(S_1, A_1)$; $g_{A_1} = 3$, $\kappa_S = 6i$, $\Delta K_S = 50$ and the other parameters are the same as in Fig. 2.

pling and can improve the conditions for nonclassical light generation. Fig. 5 shows it explicitly.

Here we have to mention that analyzed configuration includes all five modes if $\kappa_S \neq 0$ and $\kappa_A \neq 0$. Transformation (3) can be still used, but now we have five independent variables, matrix $\mathbf{M}$ has 5 lines and columns and we have to find its eigenvalues numerically. In more general configurations including Raman scattering in both waveguides we cannot write down a transformation similar to (3) and we have to solve differential equations with varying coefficients. Generally we have to use numerical calculations

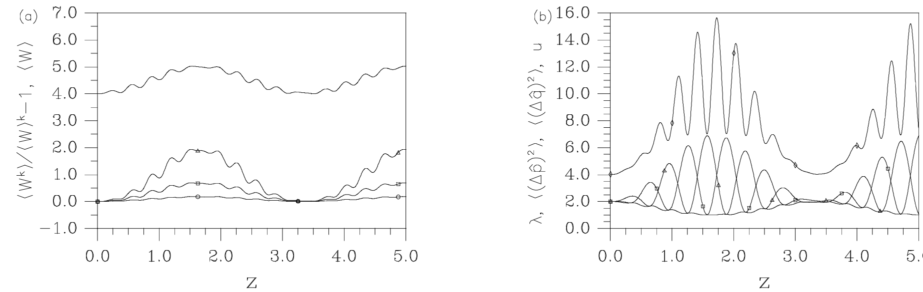


Fig. 4 (a) Reduced moments of the integrated intensity $\langle W^k(z)\rangle/\langle W\rangle^k - 1$ for $k = 2$ ($\circ$), $k = 3$ ($\square$) and $k = 4$ ($\triangle$) and the integrated intensity $W$ ($-$) for mode $(S_2, A_1)$, (b) quadrature variances $\langle(\Delta\hat{p}(z))^2\rangle$ ($\triangle$), $\langle(\Delta\hat{q}(z))^2\rangle$ ($\square$), principal squeeze variance $\lambda(z)$ ($-$) and uncertainty product $u(z)$ ($\diamond$) for mode $(S_2, A_1)$; $\xi_{V_1} = 0$, $n_{V_1} = 0.1$, $\Delta K_S = 0$, $\Delta k_{S_1} = 10$ and the other parameters are the same as in Fig. 2.

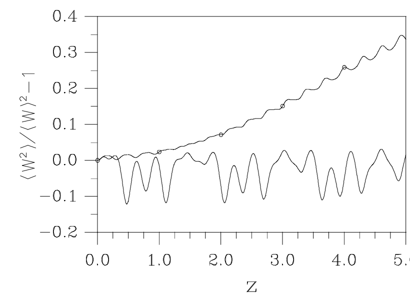


Fig. 5. Second reduced factorial moments for mode $(S_1, A_1)$ for $\Delta K_A = 0$ ($\circ$) and $\Delta K_A = 150$ ($-$); $g_{S_1} = 1$, $g_{A_1} = 2$, $\kappa_S = -10$, $\kappa_A = 10$, $\xi_{S_1} = 1$, $\xi_{S_2} = 1$, $\xi_{V_1} = 1$ and the other parameters are zero.

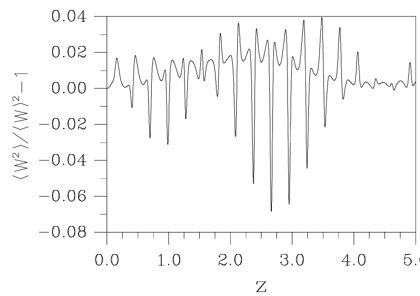


Fig. 6. Second reduced factorial moment for mode $(S_1, A_1)$; $\Delta k_{S_1} = 25$, $\Delta k_{A_1} = -20$, $\Delta K_S = 10$ and the other parameters are the same as in Fig. 2.

but some analytical results can be found. But this is beyond the scope of this paper.

*Influence of nonlinear anti-Stokes phase mismatch* $\Delta k_{A_1}$

Its influence is comparable with Stokes nonlinear mismatch $\Delta k_{S_1}$. With its increase anti-Stokes interaction is strongly suppressed. Statistics of all modes are Poissonian or super-Poissonian and fast oscillations appear.

Stokes and anti-Stokes nonlinear mismatches can partially compensate each other. If condition $\Delta k_{S_1} + \Delta k_{A_1} = 0$ is fulfilled, only one of four diagonal elements of matrix $\mathbf{M}$ is nonzero $(-\frac{i}{2}(\Delta k_{S_1} - \Delta k_{A_1}))$ and remaining three elements are zero. For small absolute values of mismatches this compensation can be effective, but with their increase nonlinear scattering is weakened and initial states of light fields are conserved. An example is given in Fig. 6. Linear Stokes mismatch is also included to demonstrate some general case. Fast oscillations are induced by $\Delta K_S$, $\Delta k_{S_1} + \Delta k_{A_1} = 5$ so the above mentioned compensation takes place.

## 6. Conclusions

We have investigated nonlinear asymmetric coupler composed of two waveguides. Raman scattering is active in the first waveguide and Stokes and anti-Stokes modes of both waveguides are linearly coupled by evanescent waves. We have performed the linearization of equations of motion assuming strong coherent pumping of laser mode in the first waveguide. We have included various phase mismatches. The description of analytic solution in the case $\kappa_A = 0$ or $\kappa_S = 0$ was given. Analysis of influence of Stokes and anti-Stokes linear coupling constants based upon calculation of eigenvalues of matrix $\mathbf{M}$ was performed in phase matching case. Three different regions were obtained for different values of $|\kappa_S|$. Coupling constant $\kappa_A$ supports exponential increase of integrated intensities and suppresses generation of nonclassical light.

General analysis of influence of phase mismatches was performed. They reduce corresponding processes and induce fast oscillations in spatial developement. Quantum statistical properties of compound modes composed of Stokes and anti-Stokes modes were investigated. Strong dependence on values of various mismatches was obtained. Nonlinear Stokes mismatch $\Delta k_{S_1}$ can support generation of squeezed light. Linear anti-Stokes coupling can counteract negative influence of anti-Stokes linear coupling $\kappa_A$ and thus supports generation of nonclassical states of light. Nonlinear Stokes and anti-Stokes phase mismatches can partially compensate each other.

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