SECOND-ORDER COLLAPSES AND REVIVALS[1]

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Second-order collapses and revivals caused by the photon-number mechanism in the Jaynes-Cummings model with a nonlinear Kerr medium and those in the Dicke model related to the collective mechanism are reviewed. The revival period of the oscillations is dependent on whether the initial mean photon-number $\bar{n}$ is integer or non-integer in the first case, and strongly related to the parity of either n (the photon number) if $n < A$ or A (the number of atoms) if $A < n$ in the latter.

1. Introduction

The Jaynes-Cummings model (JCM), describing the interaction of a two-level atom with a single-mode field, is one of the most intensively studied models in quantum optics. The rotating-wave approximation Hamiltonian of this exactly solvable model reads ($\hbar = 1$)

$$H = \omega a^\dagger a + \omega_{\text{at}} S_z + g\left(a^\dagger S_- + a S_+\right) . \tag{1}$$

S_+, S_- and S_z are pseudo-spin raising, lowering and inversion operators, respectively and g is the atom-field coupling. The symbols $a^\dagger$ and a are the photon creation and annihilation operators and ω denotes the frequency of the field mode while ω_{at} is the atomic transition frequency.

In the case of exact resonance ($\omega = \omega_{\text{at}}$) and an initially excited atom, the Rabi frequency of the oscillations of the JCM is: $\Omega_n = 2g\sqrt{n+1}$. The spectrum of the Rabi frequencies is nonlinear in n. Let us treat this frequency as a continuous quantity and expand the dispersion curve Ω_n around the point $\bar{n}$

$$\Omega_n = \Omega_{\bar{n}} + \Omega_{\bar{n}}^{(1)}(n-\bar{n}) + \Omega_{\bar{n}}^{(2)}(n-\bar{n})^2 + \ldots, \quad \Omega_{\bar{n}}^{(r)} = \frac{1}{k!}\, d^k\Omega_n/dn^k\big|_{n=\bar{n}} . \tag{2}$$

The first term of the above expansion is responsible for rapid oscillations of the model while the remaining terms are responsible for their envelope. In general, if only the first-order derivative of such an expansion were different from zero, the collapses and revivals

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of the oscillations would be perfectly periodic (linear or harmonic approximation). Such a situation takes place in the generalized model proposed by Buck and Sukumar[1]. If higher-order terms in Eq. (2) are nonzero, but the nonlinearity of the frequency spectrum is slight, they spread the revivals arising from the linear expansion and, in particular, lead to their incompleteness, overlapping and a ringing structure [2, 3]. They are also the source of well pronounced fractional revivals as in the JCM coupled to a sub-Poissonian field [4].

Although the harmonic approximation

$$\Omega_n = \Omega_{\bar{n}} + \Omega_{\bar{n}}^{(1)} (n - \bar{n}) \tag{3}$$

does not describe revivals in quantitative detail, i.e., their structure, nonetheless the revival period estimated from this approximation is satisfactorily good. This approximation is also sufficient to describe correctly the initial collapse.

With respect to the linear dependence of the frequency (3) on the photon number, there are no terms oscillating with the same frequency during the whole evolution of the model. The revival of the oscillations occurs if, at least the terms oscillating with the greatest weights acquire a phase difference of 2π. For a coherent field these terms correspond to $n=\bar{n}$ and $n=\bar{n}\pm 1$

$$T_R = 2\pi/(\Omega_{\bar{n}+1} - \Omega_{\bar{n}}) = 2\pi/(\Omega_{\bar{n}} - \Omega_{\bar{n}-1}) = 2\pi/\Omega_{\bar{n}}^{(1)} = (2\pi/g)\sqrt{\bar{n}+1}\,. \tag{4}$$

In fact, in the pure linear approximation all terms become phased at this time instant. This revival period is valid for integer as well as non-integer initial mean photon numbers.

The on-resonant Rabi frequency is but slightly nonlinear if a given photon number distribution P_n is localized beyond the region of small n's. For a coherent field this takes place for sufficiently "large" mean photon numbers [5] and a sequence of collapses and revivals of the oscillations has been revealed [6, 7]. The same situation occurs for initially squeezed coherent states with an appropriate magnitude of the coherent excitation [2, 3]. Then, additionally, a ringing structure of the revivals, connected with the oscillatory photon number distribution, appears.

In turn, if the influence of the higher-order terms in (2) is significant, it may totally wash out collapses and revivals of the model. In particular, if the intensity of the coherent field is small or if the initial field is in a thermal or squeezed vacuum state, the terms connected with small n's contribute to the evolution with the greatest statistical weights. Hence, the temporal behaviour of the resonant thermal [8] or squeezed vacuum and squeezed coherent JCM with a small addition of the coherent part [9] is irregular.

Góra and Jedrzejek [10] showed that the spectrum of the Rabi frequencies of the JCM may be "linearized" for small $\bar{n}$ by detuning $\Delta = \omega_{\text{at}} - \omega$. In this way the possibility of regular dynamics of the off-resonant JCM appears for fields for which the resonant model reveals irregularity in its time evolution [11]. In particular, since the photon number distribution for a squeezed vacuum field contains only contributions from even n's, such fields accelerate twice the emergence of revivals when compared to other fields of the same intensity, to states of which both odd and even n's contribute

[11]. As in the resonant case the off-resonant Rabi frequency is a monotonic function of n.

2. The Jaynes-Cummings model with a Kerr medium

Many different extensions of the JCM have been proposed, among others the one including the effect of a Kerr-like medium on the dynamics of the model [12]-[16]. The rotating-wave approximation Hamiltonian of this system is as follows:

$$H = \omega a^\dagger a + \omega_{\mathrm{at}} S_z + g\left(a^\dagger S_- + a S_+\right) + \chi a^{\dagger 2} a^2 \,. \tag{5}$$

The symbol χ is the third-order susceptibility representing the dispersive part of the third-order nonlinearity of a Kerr medium, modeled here as an anharmonic oscillator. For $\chi = 0$ the Hamiltonian (1) is recovered.
The oscillation Rabi frequency of such a system has the form

$$\Omega_n = 2\sqrt{\left(n\chi - \frac{\Delta}{2}\right)^2 + (n+1)g^2}, \tag{6}$$

and the atomic inversion $\langle S_z(t)\rangle$ evolves according to the formula

$$\langle S_z(t)\rangle = \frac{1}{2}\left[1 - \sum_{n=0} \frac{4(n+1)g^2}{\Omega_n^2} P_n \left(1 - \cos\Omega_n t\right)\right]. \tag{7}$$

At a special choice of the detuning Δ the Rabi frequency (6) can have a minimum at $\bar{n}$. Then the situation becomes especially interesting. Namely, instead of the "usual" revivals resembling those manifested by the standard JCM, the nonlinear JCM with an initially coherent field exhibits superstructures [17]. This occurs for $\Delta = 2\bar{n}\chi + g^2/\chi$ and the Rabi frequency in such a case reads

$$\Omega_n = 2\sqrt{\left[(n-\bar{n})\chi - \frac{g^2}{2\chi}\right]^2 + (n+1)g^2}\,. \tag{8}$$

In the following, we shall consider in detail just this case only. As previously, let us expand this dispersion curve around the point $\bar{n}$

$$\Omega_n = \Omega_{\bar{n}} + \Omega_{\bar{n}}^{(2)}\,(n-\bar{n})^2 + \ldots, \quad \Omega_{\bar{n}} = 2g\sqrt{\frac{g^2}{4\chi^2} + (\bar{n}+1)}, \quad \Omega_{\bar{n}}^{(2)} = \frac{2\chi^2}{\Omega_{\bar{n}}}\,. \tag{9}$$

In fact, this expansion contains only even powers of n–$\bar{n}$. As in Eq. (2), the first term of the above expansion is responsible for rapid oscillations of the model while the remaining terms are responsible for their envelope. If we used in the series (7) the expanded Rabi frequency (9), collapses and revivals of the oscillations would be perfectly periodic. Since the first nonvanishing derivative of the frequency is the second-order one, we can now speak of the second-order revivals or the revivals of the second kind, to distinguish them distinctly from the revivals exhibited by the standard JCM or by the nonlinear JCM if the condition $\Delta = 2\bar{n}\chi + g^2/\chi$ is not satisfied. Some interesting aspects of these revivals at an initially strongly squeezed field have recently been considered by Du et al. [18].

2.1. Second-order revivals: photon mechanism

As in the case of the first-order revivals, the second-order revival occurs if at least the most heavily weighted terms of the series (7) acquire the phase difference equal to 2π (subsequent revivals occur at the phase differences being multiplicities of 2π).

Let us consider first an integer $\bar{n}$. With respect to the symmetry properties of the expanded frequency (9) the cosines in the series (7), corresponding to $n = \bar{n} - I$ and $n = \bar{n} + I$ (I - an arbitrary integer), are always in phase which is readily seen in the lower graph of Fig. 1. In fact, in this figure the exact Rabi frequencies (8) are presented;

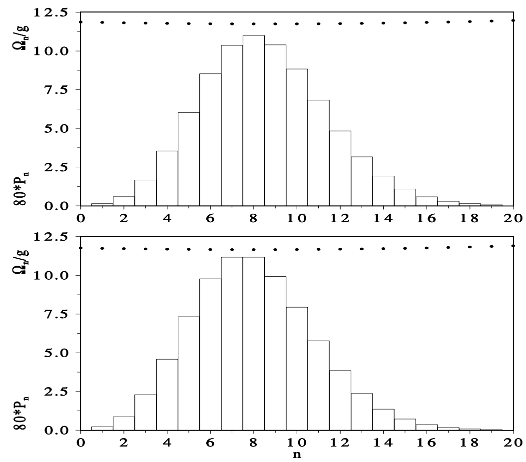

Fig. 1. Poissonian distribution P_n and the Rabi frequency (11) for $\bar{n} = 8$ (lower graph) and $\bar{n} = 8.5$ (upper graph).

for the photon-number assumed no difference between (8) and (9) would be seen in the scale of the graphs within the interval of effective summation over n. The revival period may be simply estimated as

$$(\Omega_{\bar{n}-1} - \Omega_{\bar{n}})\, T_R = (\Omega_{\bar{n}+1} - \Omega_{\bar{n}})\, T_R \approx \Omega^{(2)}_{\bar{n}}\, T_R = 2\,\pi \tag{10}$$

and reads [17]

$$T_R^{\text{integer}} = \frac{2\pi}{\Omega^{(2)}_{\bar{n}}} = \frac{\pi}{\chi^2}\Omega_{\bar{n}} = \frac{2\pi g}{\chi^2}\sqrt{\frac{g^2}{4\chi^2} + \bar{n} + 1}\,. \tag{11}$$

However, the above estimation of the revival period is not valid for a non-integer $\bar{n}$ [19]. In particular, if $\bar{n}$ is a half-integer the terms corresponding to $\bar{n} - (2k+1)/2$ and

$\bar{n}+(2k+1)/2,\ (k=0,1\ldots)$ are always in phase (upper graph of Fig. 1) and the revival period is calculated as follows:

$$\left(\Omega_{\bar{n}-\frac{3}{2}}-\Omega_{\bar{n}-\frac{1}{2}}\right)T_R=\left(\Omega_{\bar{n}+\frac{3}{2}}-\Omega_{\bar{n}+\frac{1}{2}}\right)T_R\approx 2\Omega^{(2)}_{\bar{n}}\,T_R=2\pi\,. \tag{12}$$

Hence, the general form of the revival period for a half-integer $\bar{n}$ is half of that for an integer $\bar{n}$

$$T_R^{\mathrm{half-integer}}=\frac{\pi}{\Omega^{(2)}_{\bar{n}}}=\frac{\pi g}{\chi^2}\sqrt{\frac{g^2}{4\chi^2}+\bar{n}+1}\,. \tag{13}$$

The time evolution of the atomic inversion for $\bar{n}=8$ and 8.5 is presented in the lower two graphs in Fig. 2 ($\chi=0.1g$). The time is scaled by the quantity $T_R=\frac{2\pi g}{\chi^2}\left(\frac{g^2}{4\chi^2}+\bar{n}+1\right)^{1/2}$. Therefore the first revival for the half-integer $\bar{n}$ occurs at $t/T_R=0.5$.

Let now $\bar{n}=I{+}f;\ f\in(0,1)$, but $f\neq 1/2$. There are no couples of terms oscillating in phase during the whole evolution. In comparison with the nearest neighbouring integer or half-integer $\bar{n}$, twice the number of terms have to reach a phase difference 2π and the revival period is expected to be longer. The terms corresponding to $\bar{n}-f-1$, $\bar{n}-f$ and $\bar{n}-f+1$ have the greatest weights. The first two of them become phased at $T_R^{(1)}=2\pi/\Omega^{(2)}_{\bar{n}}(1+2f)$, while the latter at $T_R^{(2)}=2\pi/\Omega^{(2)}_{\bar{n}}|1-2f|$. To have only one formula valid either for $f<1/2$ or $f>1/2$, the absolute value of the denominator for $f>1/2$ has been introduced. Both times are equal for $f=0$, i.e., if $\bar{n}$ is an integer, which case has been discussed earlier. For a fractional f (but, obviously, $f\neq 1/2$), the first revival will occur at $T_R=T_R^{(1)}\times I_1=T_R^{(2)}\times I_2$, where $I_{1,2}$ are mutually prime integers. As a consequence, we obtain the following condition $I_1/I_2=1+2f/|1-2f|$.

For $f=0.1,0.3,0.7$ and 0.9 ($\bar{n}=I+f$) the above condition leads to

$$T_R=\frac{5\pi g}{\chi^2}\sqrt{\frac{g^2}{4\chi^2}+\bar{n}+1}\,. \tag{14}$$

Its general form is multiplied by 2.5 in comparison with that for $\bar{n}=I$.

In turn, for $f=0.2,0.4,0.6$ and 0.8 one finds that

$$T_R=\frac{10\pi g}{\chi^2}\sqrt{\frac{g^2}{4\chi^2}+\bar{n}+1}\,. \tag{15}$$

In other words, this time is multiplied by five in comparison with that for $\bar{n}=I$. Since the revival periods are longer, the pictures of the superstructures are more complicated (two upper graphs in Fig. 2). In fact, as the number of digits after the decimal point grows in $\bar{n}$, the revivals periods grows as well; the superstructures get increasingly complex and become more and more blurred to some extent. The formulas (14) and (15) are also valid for $\bar{n}=f<1$ except $\bar{n}=0.1$ and 0.2. Then one has to consider quantum beats between the two meaningful terms only: $n=0$ and $n=1$, and for these $\bar{n}$'s the revival period is multiplied by 5/4 and 5/3, respectively in comparison with that for $\bar{n}=I$. In all cases the collapse times are equal to one half of the corresponding revival times.

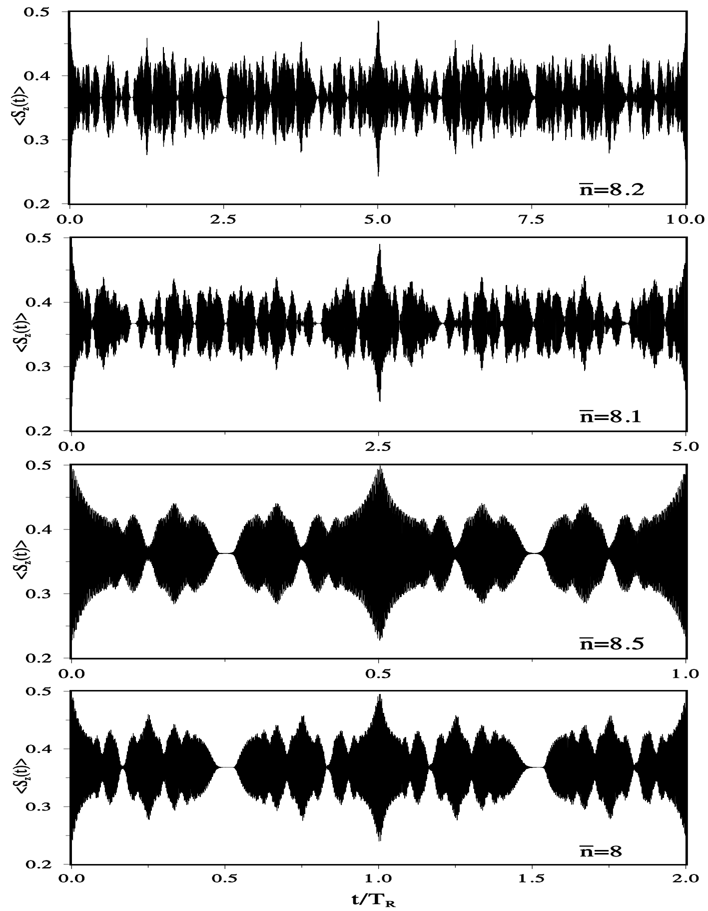

Fig. 2. Time evolution of the atomic inversion (9) for $\bar{n}$ = 8, 8.5, 8.1 and 8.2 from the bottom to the top of the figure, respectively. The time is scaled by the quantity $T_R = 2\pi g\left[g^2/(4\chi^2) + \bar{n} + 1\right]^{1/2}\chi^2$; $\chi = 0.1g$.

3. The Dicke model

Since the paper of Dicke [20] a considerable amount of attention has been devoted to the interaction of a radiation field with a small sample of A two-level atoms located within a distance much smaller than the wavelength of the radiation. Thus, all the atoms are treated as being in equivalent-mode positions. However, the wave functions of the atoms are assumed not to overlap. The collectivity of the system is then due to indirect atom-atom coupling via the field-mode. Such a system is commonly termed the Dicke model (DM). The interaction of a group of two-level atoms with a single-mode cavity field has been considered by Tavis and Cummings [21] and this particular type of the Dicke model is sometimes referred to as the Tavis-Cummings model. The latter is mathematically equivalent to the trilinear Hamiltonian describing processes of parametric conversion as well as Raman and Brillouin scattering [22] and its Hamiltonian in the rotating-wave approximation reads ($\hbar = 1$)

$$H = \omega a^\dagger a + \omega_{\rm at} S_z + g\left[a^\dagger S_- + a S_+\right] . \tag{16}$$

$S_{\pm,z} = \sum_{j=1}^{A} S_{\pm,z}^{(j)}$ are collective pseudospin raising, lowering and inversion operators, respectively. In what follows, we assume exact resonance: $\omega = \omega_{\rm at}$. In the small sample approximation the coupling coefficient g is the same for all the atoms.

The total excitation number N (the number of photons plus the number of excited atoms)

$$N = aa^\dagger + S_z + A/2 \tag{17}$$

is an integral of motion.

If the field is taken initially in a Fock state with the photon number n and the atomic system is in its ground state, then a single subspace with $N = n$ contributes to the evolution. Unlike the JCM, the general exact analytical solution for the Dicke model is not known. Although the formal solution can be written by means of the Bethe ansatz this has not led up to now to a convenient form of the solution, since the problem is reduced to an algebraic equation equivalent to the initial one. In particular, in the rotating wave approximation, the DM is exactly solvable for $N \leq 8$. Among these solutions the cases $N = 1$, 2 (A arbitrary) and $A = 1$, 2 (N arbitrary) are characterized by equidistant eigenvalues spectra and the time behaviour of such systems is strictly periodic [23]-[25]. The system with $N = A = 3$ [26]-[30] is the first in the hierarchy of those having unequidistant spectra of the eigenvalues. This anharmonicity of the eigenvalues spectrum leads to distinct collective collapses and revivals of the oscillations of the model if $3 \leq N < A$ or $3 \leq A < N$.

For $N \sim A$ collective collapses and revivals are not so well pronounced [31, 32]. Walls and Barakat [22] showed numerically that there are two limits when the eigenvalues spectrum of such systems may still be supposed to be approximately equidistant and, consequently, their evolution considered as periodic. The first case occurs if the excitation number is very much smaller than the number of atoms ($N \ll A$) and the other is the opposite of the former ($N \gg A$). It is reasonable to call them "weak" and "strong" field limits, respectively. If weaker inequalities $N < A$ and $A > N$ are satisfied we may speak of "weak" and "strong-field" domains, correspondingly. Obviously, the above determinations have nothing in common with the absolute intensity of the field; only the relation between the photon number and the number of atoms of the sample is taken into account in the above definition.

3.1. Weak-field domain

In this section we consider the time evolution of the DM for an initially Fock field. We assume that there are n photons and no excited atoms in the initial state. We start with the weak-field domain. The inversion of the atomic energy $\langle S(t)\rangle$ for the group of

n atoms is related with the inversion of the atomic energy for the whole system by the following relation [27]: $\langle S(t)\rangle = (A-n)/2 + \langle S_z(t)\rangle$. At the initial conditions assumed $\langle S_z(0)\rangle = -A/2$, i.e., $\langle S(0)\rangle = -n/2$.

We have constructed a perturbation approach to the problem in question in terms of the $SU(2)$ group representations [28, 33, 34]. In the second-order approximation for the eigenvalues and in the first-order approximation for the eigenvectors (subscript 21), with respect to the integral of motion (17) and the result for the expectation value of the photon number [30], $\langle S(t)\rangle$ is found to evolve according to the formula

$$\begin{aligned}\langle S(t)\rangle_{21}^{(w)} &= \frac{\epsilon_w n(n-1)}{16} - \sum_{p=1}^{n} C_p^n \cos\Omega_p^{(w)} t \\ &- \frac{\epsilon_w}{8}\sum_{p=1}^{n} C_p^n \left\{\left[(n-2p)^2 - 2p + 1\right]\cos\Omega_p^{(w)} t + 2(p-1)\cos\tilde{\Omega}_p^{(w)} t\right\} . \qquad (18)\end{aligned}$$

It may reach the value $n/2$; the approximate solution permits total energy transfer to the atomic subsystem. In the exact solution this inversion does not reach $n/2$ (e.g. for $n=3$ [28]).

Here the principal term corresponds to the zeroth-order approximation for the eigenvectors but contains the second-order Dicke frequencies

$$\Omega_p^{(w)} = \Omega_n^{(w)}\left\{1 + \frac{3\epsilon_w^2}{16}\left[5(p-1)(p-n) + (n-1)(n-2)\right]\right\} , \qquad (19)$$

where the carrying Rabi frequency $\Omega_n^{(w)}$ reads

$$\Omega_n^{(w)} = 2g\sqrt{A - n/2 + 1/2}\,, \qquad (20)$$

and the expansion parameter ϵ_w is $\epsilon_w = (A - n/2 + 1/2)^{-1}$.

The frequencies inside the spread are marked by the parameter p, $1 \le p \le N$; (recall that $N=n$ for an initially unexcited atomic system).

$$C_p^n = \frac{p}{2^n}\binom{n}{p} = \frac{n}{2^n}\binom{n-1}{p-1} \qquad (21)$$

is the binomial distribution multiplied by the factor $n/2$. In the zeroth-order approximation for the eigenvectors, Eq. (18) contains only the frequencies (19) related with the transitions between two neighbouring levels (p and $p-1$) of the Hamiltonian (16). From Eqs. (18) and (21) it is obvious that in the evolution of the system the most significant role is played by the terms with p close to $n/2$, i.e., by the eigenvectors with the smallest absolute values of the eigenfrequencies [35, 36].

In the first-order approximation for the eigenvectors (the terms proportional to ϵ_w) new transition frequencies $\tilde{\Omega}_p$ related with the transitions between the levels p and $p-2$ appear.

3.2. Strong-field domain

The appropriate formula for the time evolution of $\langle S_z(t)\rangle$ is directly obtainable from that for a weak field (18) after interchanging n and A:

$$\begin{aligned}\langle S_z(t)\rangle_{21}^{(s)} &= \frac{\epsilon_s A(A-1)}{16} - \sum_{p=1}^{A} C_p^A \cos\Omega_p^{(s)} t \\ &- \frac{\epsilon_s}{8}\sum_{p=1}^{A} C_p^A \left\{\left[(A-2p)^2-2p+1\right]\cos\Omega_p^{(s)} t + 2(p-1)\cos\tilde{\Omega}_p^{(s)} t\right\}\,, \quad (22)\end{aligned}$$

where

$$\Omega_p^{(s)} = \Omega_n^{(s)}\left\{1+\frac{3\epsilon_s^2}{16}[5(p-1)(p-A)+(A-1)(A-2)]\right\}, \quad \Omega_n^{(s)} = 2g\sqrt{n-A/2+1/2}, \quad (23)$$

and $\epsilon_s = (n - A/2 + 1/2)^{-1}$.

In this regime the oscillation amplitudes in Eq. (22) depend on n via the small parameter ϵ_s only. In general, however, the time behaviour of the model resembles that for an initially weak Fock field.

3.3. Second-order revivals: collective mechanism

The spreads of the Dicke frequencies (19) and (23) are responsible for the fine collective phenomena inherent in the model. The mechanism of this spread is now different from the photon number mechanism; this is due to the summation over p and has a purely cooperative origin. It is the consequence of the unequidistancy of the eigenfrequency spectrum. Due to the spread of the frequencies the oscillations dephase and rephase (collapse and revive).

If the Dicke frequency is considered as a continuous function, in the weak-field domain it has its minimum at $\bar{p} = (n+1)/2$ which simply means vanishing of its first-order derivative in this point. Since the weight function (21) (considered as a continuous function) reaches its maximum just in the same point $\bar{p}$ we expand the Dicke frequency (19) around this point,

$$\Omega_p^{(w)} = \Omega_{\bar{p}}^{(w)} + \Omega_{\bar{p}}^{(w)''}(p-\bar{p})^2, \qquad \Omega^{(w)''} = 15g^4/\Omega_{\bar{p}}^{(w)3}\,. \quad (24)$$

Since the linear term in this expansion vanishes, the appearance of revivals has to be attributed to the second-order derivative $\Omega_{\bar{p}}^{(w)''}$. In fact, all higher-order derivatives of the Dicke frequency (19) are equal to zero in the approximation considered and we deal here with purely second-order revivals.

To calculate the revival time we may neglect in Eq. (18) the contributions proportional to ϵ_w, and take into account only the terms proportional to $\cos\Omega_p^{(w)} t$. Due to the parabolic form of the frequency function (19) the pairs of the cosines with the frequencies $\Omega_p^{(w)}$ and $\Omega_{n+1-p}^{(w)}$ always oscillate in phase. Moreover, each constituent of the pair contributes to the evolution with the same weight.

The cases of odd and even n have to be treated separately. We start with odd n's. Then $\bar{p}$ is an integer number. As mentioned, the contributions with $p{=}\bar{p}{\pm}1$ are always in phase. Therefore we are interested in the phasing of those with $\Omega^{(w)}_{\bar{p}+1}$ and $\Omega^{(w)}_{\bar{p}}$. They are phased if

$$\left[\Omega^{(w)}_{\bar{p}+1} - \Omega^{(w)}_{\bar{p}}\right] T^{(w)}_{R\ \mathrm{odd}\ n} = \Omega^{(w)''}_{\bar{p}}\, T^{(w)}_{R\ \mathrm{odd}\ n} = 2\,\pi\,. \tag{25}$$

If the above condition is satisfied then, in fact, all terms engaged in the evolution acquire a common phase at this time and completeness of the revivals is expected, attributable to the vanishing of all higher-order derivatives. From (25) the revival time reads

$$T^{(w)}_{R\ \mathrm{odd}\ n} = \frac{16\pi}{15g}(A - n/2 + 1/2)^{3/2}. \tag{26}$$

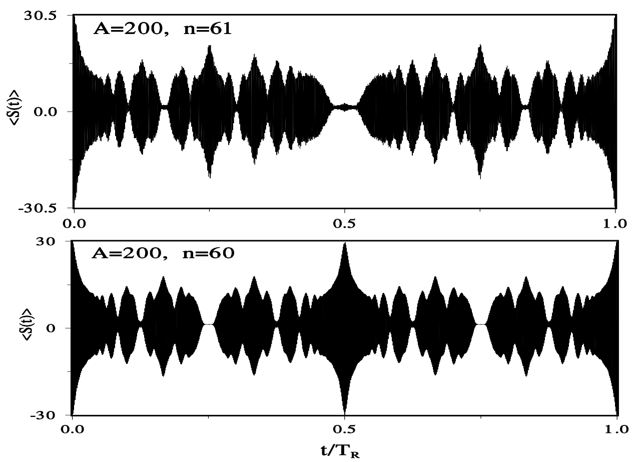

Fig. 3. Superstructures in the Dicke model in the weak-filed domain: $A=200$, $n=61$ (upper graph) and $n=60$ (lower graph). Time is scaled by the quantity $T_R = \frac{16\pi}{15g}(A - n/2 + 1/2)^{3/2}$.

For even n, the point $\bar{p}{=}(n{+}1)/2$ in which the Dicke frequency, treated as a continuous function, takes its extremum is a half-integer. The two most heavily weighted terms, $p{=}\bar{p}{\pm}1/2$, are always in phase, and in order to estimate the revival time one has to discuss the contributions related with $p{=}\bar{p}{+}1/2$ and $p{=}\bar{p}{+}3/2$. Hece we find

$$T^{(w)}_{R\ \mathrm{even}\ n} = \frac{8\pi}{15g}(A - n/2 + 1/2)^{3/2}. \tag{27}$$

It is easily verified that also in this case all terms are in phase at this time instant. An intriguing feature of the model is apparent: for a given A and for the two nearest neighbouring n the revival time is almost twice shorter for even n.

In both cases of odd and even n the collapse times are equal to one half of the corresponding revival times. Collapses and revivals recur periodically in the approximation (21) discussed. For odd n's the collapse of $\cos\Omega_p^{(w)}t$ is total and residual oscillations in the quiescent period are related with the first-order term (proportional to ϵ_w) (upper graph of Fig. 3). The latter term contains the oscillations at frequency $\tilde{\Omega}_p^{(w)}$ and evolves almost twice faster compared to the zeroth-order term. When the zeroth-order oscillations $\cos\Omega_p^{(w)}t$ collapse, the first-order oscillations $\cos\tilde{\Omega}_p^{(w)}t$ revive [30]. In turn, for even n's the collapse of the principal cosines is not total. However, this is only visible for small numbers of photons n [30].

The weak-field domain has its counterpart in the spontaneous emission of a partially inverted atomic system.

The discussion of the revival and collapse times for the strong-field domain follows the same lines as for the weak-field one. The revival time of the main term $\cos\Omega_p^{(s)}t$ is here also related with the parity, but now, of A. For a given n and for the two nearest neighbouring A the revival period is almost twice shorter for an even number of atoms than for an odd one.

The revival time of the oscillations for odd and even A are, respectively

$$T_{R\text{ odd }A}^{(s)} = \frac{16\pi}{15g}(n-A/2+1/2)^{3/2}, \quad T_{R\text{ even }A}^{(s)} = \frac{8\pi}{15g}(n-A/2+1/2)^{3/2}. \tag{28}$$

As previously, the collapses and revivals will periodically recur in the approximation discussed.

4. Conclusions

The nonlinear Jaynes-Cummings model with a Kerr medium at a special choice of the detuning starts to exhibit the revivals of the second-order. As we have shown the revival period strongly depends on whether the initial mean number of coherent photons is integer or non-integer.

In turn, the Dicke model of an assemblage of A two-level atoms, coupled in an ideal cavity to a single-mode Fock field, also exhibits second-order revivals [30]. Contrary to the photon distribution mechanism, the origin of superstructures in the Dicke model is related to the collectivity of the system. The revival period of these pure collective revivals is strongly related to the parity of n in the weak-field domain ($n<A$), and to the parity of A in the strong-field domain ($A<n$).

The mechanisms of the second-order collapses and revivals in both models are different. However, one factor connects these phenomena — discreteness of the systems.

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