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QUANTUM ERASER AND THE DECOHERENCE TIME OF A LOCAL MEASUREMENT PROCESS[1]

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We propose an implementation of the quantum eraser, based on a recent experimental scheme by Eichmann *et al.* [1] involving two four–level atoms. In our version a continuous broad band excitation (BBE) field drives the two trapped atoms and information about which atom scattered the light is stored in the internal degrees of freedom of the atoms. Entanglement of the two atoms after the detection of the photon is intimately connected to the availability of this "which path" information. We also show that the quantum eraser can be used to measure the decoherence time of a local measurement process.

1. Introduction

The theory of light scattering from two atoms has first been dealt with by Heitler [2] in the context of coherent scattering. A recent paper by Eichmann *et al.* [1] reports on the first observation of interference effects in the light scattered from two trapped ions. In the experiment two $^{189}\mathrm{Hg}^+$ ions were tapped along the axis of a linear trap [1, 3, 4]. The $^{189}\mathrm{Hg}^+$ ion has an interesting internal level structure with lower state $6s^2s_{1/2}$ and an excited state $6p^2s_{1/2}$, both degenerate with respect to the magnetic quantum number $m_j = \pm 1/2$. The internal structure has the consequence that the resonance fluorescence contains $\pi-$ and $\sigma-$ polarized light ($|\Delta m_j| = 0$ and $|\Delta m_j| = 1$), when the incident light is linearly polarized. Because of this level structure, the interference properties of the scattered light can be observed by polarization sensitive detection [1, 5]. The interference pattern comes only from the coherent part of the scattered field, while the incoherent part gives no contribution.

The explanation in Ref.[1] (see also [5]) indicates that the experiment offers the possibility to obtain which way information by exploring the internal structure of the

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atom. This description is, however, only valid as long as the atom is not saturated. The assumption is that only one photon was scattered at a time. This allows, in principle (by comparing the initial internal states with the final states), to decide if interference is possible or not. However, one must be careful with such a description. Because of the continuous monochromatic excitation laser, the assumption of independently scattered photons, i.e. one at a time, is not valid. For a monochromatic excitation we have a coherent oscillation between the ground state and the excited state [6, 7, 8]. Due to the interaction with the reservoir, after a few Rabi cycles, the atom reaches a steady state. In this paper we want to answer the question what happens to the interference picture when the atoms have evolved in a steady state. When is it necessary to talk about saturated atoms or how long is the description of scattering of independent photons valid [9, 10]? Is it possible in the long time regime, in particular, to get "which path" information about the photon? The clarification of these problems is important for a possible realization of the quantum eraser [11-15], which stores erasable information about which path the particle had taken. We hope to answer these questions and give possible schemes how the experimental arrangement of Eichmann *et al.* can be used to implement a quantum eraser.

Section 2 is devoted to the dynamical behavior of a 4–level atom where the two–fold degenerate excited and ground states are driven by a near resonant linearly polarized laser. Section 3 discusses the interference pattern of two driven 4–level atoms, especially in the saturated or steady state regime, and clarifies whether it is still possible to obtain which way information when there is no interference. Based on these considerations, we show how a quantum eraser can be implemented when the two atoms are driven continuously, as in the experiment of Ref.[1]. As it turns out, it is possible even in the long time regime to adopt their interpretation if one uses a broad band excitation (instead of a cw one) of the two atoms. In this case, independent excitation and scattering events (one photon at a time) take place. In section IV we study the effect of a broad band excitation field on the two atoms. Section 5 is devoted to the discussion of a quantum eraser model which can be used in the experiment. Finally, in Section 6 we show how one can detect quantum coherence of mesoscopic or macroscopic systems with a quantum eraser or, in general, in interference experiments and we propose the quantum eraser as a tool to measure the decoherence time of a local measurement process.

2. Dynamical behavior of a driven 4–level atom.

The dynamical equations or master equation for a driven 4–level atom were first derived by Polder and Schurmann [7] in the context of resonance fluorescence and by Walls *et al.* [8] in the context of interference in their analysis of [1]. In this section we use the results of Polder and Schurmann [7] to derive explicitly the time dependence of expectation values of the electric field components in different polarization directions. These results are important when we consider interference effects in the resonance fluorescence of the two atoms, especially in the steady state regime.

We consider an atom at rest, at the position $\mathbf{r} = 0$, which is coupled to the electro-

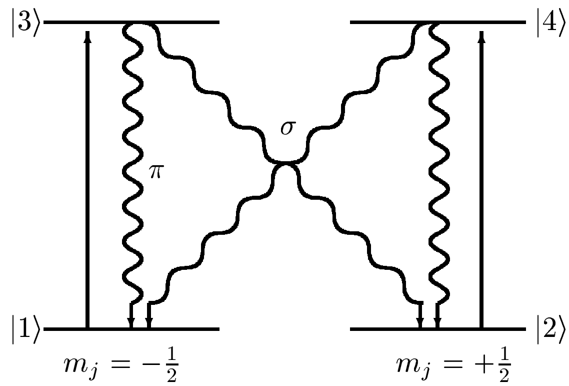

Fig. 1. Internal structure of the four–level atom with the relevant polarization sensitive transitions.

magnetic field by the interaction Hamiltonian,

$$H = -\vec{\mu} \cdot \vec{\mathbf{E}}(\mathbf{r} = 0, \mathrm{t}). \tag{2.1}$$

Here $\vec{\mu}$ is the dipole operator and the electric field is the sum of the classical x–polarized field and the quantized field of the reservoir in free space. The classical field is given by

$$\vec{\mathbf{E}}_{\mathbf{c}}(\mathbf{r}, \mathrm{t}) = \hat{\mathbf{e}}_{\mathbf{x}} \mathrm{E_o}(\mathbf{r}) \cos(\omega \mathrm{t}). \tag{2.2}$$

We denote the two ground states with $|1\rangle$ ($m_j = -1/2$) and $|2\rangle$ ($m_j = +1/2$), and the excited states with $|3\rangle$ ($m_j = -1/2$) and $|4\rangle$ ($m_j = +1/2$). The frequency splitting between the ground and the excited states is ω_0. We consider the reservoir to be at zero temperature coupled to the atoms in the Markoff approximation. The interaction with the classical field is given by,

$$H_c = -\vec{\mu} \cdot \vec{\mathbf{E}}_{\mathrm{c}} = -\frac{1}{2} \mathrm{E_o} \hat{\mathrm{e}}_{\mathrm{x}} \cdot (\vec{\mu}^{(-)} \mathrm{e}^{\mathrm{i}\omega \mathrm{t}} + \vec{\mu}^{(+)} \mathrm{e}^{-\mathrm{i}\omega \mathrm{t}}), \tag{2.3}$$

where the dipole operator, $\vec{\mu}^{(-)}$, is related to the lowering operator operator as

$$\vec{\mu}^{(-)} = \mu \big\{ \hat{\mathbf{x}}(|1\rangle\langle 4| + |2\rangle\langle 3|) + i\hat{\mathbf{y}}(|1\rangle\langle 4| - |2\rangle\langle 3|) + \hat{\mathbf{z}}(|1\rangle\langle 3| - |2\rangle\langle 4|) \big\}, \ \vec{\mu}^{(+)\dagger} = \vec{\mu}^{(-)} \ . \tag{2.4}$$

The x–polarized ($\sigma^{(-)}\mu_x$) and the y–polarized ($\sigma^{(-)}\mu_y$) scattered fields are σ–polarized ($|\Delta m_j| = 1$), while the z–polarized scattered field ($\sigma^{(-)}\mu_z$) is π–polarized ($|\Delta m_j| = 0$) – see Fig. 1. Only the x–polarized scattered field is coherent to the incident light, while the y– or z–component scattered fields are not coherent. The two incoherent parts behave in the same way (have, e. g., the same spectra) therefore it is sufficient to consider only one incoherent component [7]. We restrict our consideration to the coherent E_x–component and the incoherent E_y-component. The matrix elements of the atomic density operator satisfy the equations,

$$
\begin{aligned}
\frac{d\hat{\rho}_{14}}{dt} &= (i\Delta - 3\gamma)\hat{\rho}_{14} + iv(\rho_{44} - \rho_{11}), \\
\frac{d\hat{\rho}_{23}}{dt} &= (i\Delta - 3\gamma)\hat{\rho}_{23} + iv(\rho_{33} - \rho_{22}), \\
\frac{d\rho_{11}}{dt} &= 4\gamma\rho_{44} + 2\gamma\rho_{33} + iv(\hat{\rho}_{41} - \hat{\rho}_{14}), \\
\frac{d\rho_{22}}{dt} &= 4\gamma\rho_{33} + 2\gamma\rho_{44} + iv(\hat{\rho}_{32} - \hat{\rho}_{23}), \\
\frac{d\rho_{33}}{dt} &= -6\gamma\rho_{33} + iv(\hat{\rho}_{23} - \hat{\rho}_{32}), \\
\frac{d\rho_{44}}{dt} &= -6\gamma\rho_{44} + iv(\hat{\rho}_{14} - \hat{\rho}_{41}). \qquad (2.5)
\end{aligned}
$$

Here $\rho_{kl} = \rho_{lk}^*$, $\hat{\rho}_{14} = \rho_{14}e^{-i\omega t}$ and $\hat{\rho}_{23} = \rho_{23}e^{-i\omega t}$. The equations for $\rho_{24}, \rho_{13}, \rho_{12}$ and ρ_{34} necessary for the calculation of $\langle E_z(t)\rangle$ are not needed because $\langle E_y(t)\rangle = \langle E_z(t)\rangle$. In Eqs. (2.5) $\Delta = \omega_0 - \omega$ is the detuning and $v = \vec{\mu}\cdot\vec{\mathbf{E}}_{\mathbf{o}}(0)/2\hbar$ is the interaction parameter. The decay rate $2\gamma = 4/3\mu^2\omega_0^3c^{-3}\hbar^{-1}$ is one third of the spontaneous decay rate of the upper to lower level. We obtain the following expressions for the relevant elements of the density operator in the weak field limit, $v^2 \ll \Delta^2 + 9\gamma^2$,

$$
\begin{aligned}
\rho_{14}(t) + \rho_{23}(t) &= e^{i\omega t}\frac{v(\Delta - 3i\gamma)}{(9\gamma^2 + \Delta^2 + 2v^2)}[1 - e^{-(3\gamma - i\Delta)t}], \\
\rho_{14}(t) - \rho_{23}(t) &= e^{i\omega t}\frac{v(\Delta - 3i\gamma)}{(9\gamma^2 + \Delta^2 + 2v^2)}e^{-4v^2\gamma t/(9\gamma^2+\Delta^2)}[\rho_{22}(0) - \rho_{11}(0)], \\
(\rho_{44})_{ss} &= \frac{1}{2}\frac{v^2}{(9\gamma^2 + \Delta^2 + 2v^2)}. \qquad (2.6)
\end{aligned}
$$

Here ss stands for steady state. For symmetric initial conditions $\rho_{14}(t) - \rho_{23}(t)$ vanishes for all times. It should also be noted that in this expression the longtime limit should be taken before the weak field ($v = 0$) limit. The expression for the radiated field is related to the dipole operator and is given by,

$$
\begin{aligned}
\langle E_x^{(+)}\rangle &= \Theta(t_r)\Psi(\vec{\mathbf{r}})_{\mathrm{x}}\langle\sigma_{\mathrm{x}}^{(+)}(\mathrm{t_r})\rangle, \\
&= \Theta(t_r)\Psi(\vec{\mathbf{r}})_{\mathrm{x}}\langle\rho_{14}(\mathrm{t_r}) + \rho_{23}(\mathrm{t_r})\rangle, \\
&= \Theta(t_r)\Psi(\vec{\mathbf{r}})_{\mathrm{x}}\frac{v(\Delta - 3\mathrm{i}\gamma)}{(9\gamma^2 + \Delta^2 + 2v^2)}\mathrm{e}^{\mathrm{i}\omega\mathrm{t_r}}, \\
\langle E_y^{(+)}\rangle &= \Theta(t_r)\Psi(\vec{\mathbf{r}})_{\mathrm{y}}\langle\sigma_{\mathrm{y}}^{(+)}(\mathrm{t_r})\rangle, \\
&= \Theta(t_r)\Psi(\vec{\mathbf{r}})_{\mathrm{y}}\langle\rho_{14}(\mathrm{t_r}) - \rho_{23}(\mathrm{t_r})\rangle, \\
&= \Theta(t_r)\Psi(\vec{\mathbf{r}})_{\mathrm{y}}\frac{v(\Delta - 3\mathrm{i}\gamma)}{(9\gamma^2 + \Delta^2 + 2v^2)}\mathrm{e}^{(\mathrm{i}\omega - 4v^2\gamma/(9\gamma^2+\Delta^2))\mathrm{t_r}}. \qquad (2.7)
\end{aligned}
$$

Here $t_r = t - r/c$ is the retarded time. For the intensity we give only the steady state

($t \gg \gamma^{-1}$) results,

$$\begin{aligned}\langle E_x(t)E_x(t)\rangle &= 2|\vec{\mathbf{\Psi}}(\vec{\mathbf{r}})|^2\langle\rho_{44}\rangle_{\rm ss} = |\vec{\mathbf{\Psi}}(\vec{\mathbf{r}})|^2\frac{\mathrm{v}^2}{(9\gamma^2+\Delta^2+2v^2)},\\ \langle E_y(t)E_y(t)\rangle &= \langle E_x(t)E_x(t)\rangle,\end{aligned} \tag{2.8}$$

with

$$\vec{\mathbf{\Psi}}(\vec{\mathbf{r}}) = \frac{\omega_{\rm o}^2}{4\pi \mathrm{r}^3\epsilon_{\rm o}\mathrm{c}^2}\left((\vec{\mu}\times\vec{\mathbf{r}})\times\vec{\mathbf{r}}\right).$$

Eqs. (2.7) show that the expectation value of the incoherent part of the electric field vanishes in the steady state. We can only have fluctuations from this part in steady state, and we note that the steady state is reached within a few Rabi cycles or Raman transitions in the incoherent part of the spectrum.

3. Interference of light scattered from two independent 4–level atoms

Here we consider the question of the interference of light scattered from two independent 4–level atoms. We will consider the experimental situation of Ref.[1] and assume the atoms are at rest at positions $\vec{\mathbf{r}}_{\mathbf{A}}$ and $\vec{\mathbf{r}}_{\mathbf{B}}$. We will also neglect thermal fluctuations of the center of mass of the atoms. The two atoms are driven by an x–polarized monochromatic weak laser, so we can use the results of the previous section. The interaction Hamiltonian is

$$H = -(\vec{\mu}_{\mathbf{A}}\cdot\vec{\mathbf{E}}(\vec{\mathbf{r}}_{\mathbf{A}},\mathbf{t}) + \vec{\mu}_{\mathbf{B}}\cdot\vec{\mathbf{E}}(\vec{\mathbf{r}}_{\mathbf{B}},\mathbf{t})). \tag{3.1}$$

For a monochromatic transition we have a coherent oscillation between the ground and the excited states [7, 8]. This implies, if we can neglect direct interaction between the two atoms, that each atom is driven independently. We can therefore use the equations of Section 2 for each atom which leads to an entanglement between the two atoms after the absorption of the photon. The initial density operator for the two independent atoms is given by

$$\rho(t) = \rho_A(t)\otimes\rho_B(t). \tag{3.2}$$

The field scattered from the two atoms has now two contributions to the resonance fluorescence and the intensity at the detector at position $\vec{\mathbf{r}}$ is given by

$$\langle I(\vec{\mathbf{r}},\mathrm{t})\rangle = \langle \mathrm{I_A}(\vec{\mathbf{r}},\mathrm{t})\rangle + \langle \mathrm{I_B}(\vec{\mathbf{r}},\mathrm{t})\rangle + \left\{\langle\vec{\mathbf{E}}_{\rm A}^{(-)}(\vec{\mathbf{r}}-\vec{\mathbf{r}}_{\rm A},\mathrm{t})(\vec{\mathbf{E}}_{\rm B}^{(+)}(\vec{\mathbf{r}}-\vec{\mathbf{r}}_{\rm B},\mathrm{t})\rangle + \mathrm{c.c.}\right\}. \tag{3.3}$$

The scattered fields from atoms **A** and **B** are given in terms of the lowering operators of the two atoms as

$$\vec{\mathbf{E}}_{\rm A,B}^{(+)}(\vec{\mathbf{r}}-\vec{\mathbf{r}}_{\rm A,B},\mathrm{t}) = \Theta(\mathrm{t}-\mathrm{r_{A,B}}/\mathrm{c})\vec{\mathbf{\Psi}}(\vec{\mathbf{r}})\sigma_{\rm A,B}^{(-)}\left(\mathrm{t}-\frac{|\vec{\mathbf{r}}-\vec{\mathbf{r}}_{\rm A,B}|}{\mathrm{c}}\right). \tag{3.4}$$

The last two terms of Eq.(3.3) are responsible for the interference. We want to see the effect of detecting a specific polarization direction of the scattered field, say the x–polarization. The x–polarized part of the scattered field is responsible for the coherent part of the spectrum [7], and therefore interference is expected in this polarization. From Eq. (3.3) we can express these terms as

$$\begin{aligned} &\langle \vec{\mathbf{E}}^{(-)}_{\mathrm{x,A}}(\vec{\mathbf{r}}-\vec{\mathbf{r}}_{\mathrm{A}},\mathrm{t})\vec{\mathbf{E}}^{(+)}_{\mathrm{x,B}}(\vec{\mathbf{r}}-\vec{\mathbf{r}}_{\mathrm{B}},\mathrm{t}) + \mathrm{h.c.}\rangle = \\ &\Theta(t-r_A/\mathrm{c})\Theta(\mathrm{t}-\mathrm{r_b}/\mathrm{c})|\vec{\mathbf{\Psi}}(\vec{\mathbf{r}})|^2[\langle\sigma^{(+)}_{\mathrm{x,A}}(\mathrm{t}')\sigma^{(-)}_{\mathrm{x,B}}(\mathrm{t}'+\tau)\rangle + \langle\sigma^{(-)}_{\mathrm{x,A}}(\mathrm{t}')\sigma^{(+)}_{\mathrm{x,B}}(\mathrm{t}'+\tau)\rangle]. \end{aligned} \quad (3.5)$$

Here we have introduced the retarded time $t' = t - |\vec{\mathbf{r}} - \vec{\mathbf{r}}_{\mathrm{A}}|/\mathrm{c}$ and the delay time $\tau = (|\vec{\mathbf{r}} - \vec{\mathbf{r}}_{\mathrm{A}}| - |\vec{\mathbf{r}} - \vec{\mathbf{r}}_{\mathrm{B}}|)/\mathrm{c}$. Using the results of Section 2 we find

$$\begin{aligned} &\Theta(t-r_A/\mathrm{c})\Theta(\mathrm{t}-\mathrm{r_B}/\mathrm{c})|\vec{\mathbf{\Psi}}(\vec{\mathbf{r}})|^2[\langle\sigma^{(+)}_{\mathrm{x,A}}(\mathrm{t}')\sigma^{(-)}_{\mathrm{x,B}}(\mathrm{t}'+\tau)\rangle + \langle\sigma^{(-)}_{\mathrm{x,A}}(\mathrm{t}')\sigma^{(+)}_{\mathrm{x,B}}(\mathrm{t}'+\tau)\rangle] \\ = \ &\Theta(t-r_A/c)\Theta(t-r_B/c)|\vec{\mathbf{\Psi}}(\vec{\mathbf{r}})|^2\frac{\mathrm{v}^2(9\gamma^2+\Delta^2)}{(9\gamma^2+\Delta^2+2v^2)^2}\cos(\omega\tau), \end{aligned} \quad (3.6)$$

for the interference term. As expected, the coherent part of the spectrum gives rise to interference.

Next, we turn our attention to the incoherent part of the scattered field and consider, e.g., the y–polarized component. The z–polarized field has the same properties, so it suffices to deal with only one of them. Treating the y–component in the same fashion as the x–component we get,

$$\begin{aligned} &\langle E^{(-)}_{y,A}(\vec{\mathbf{r}}-\vec{\mathbf{r}}_{\mathrm{A}},\mathrm{t})\mathrm{E}^{(+)}_{\mathrm{y,B}}(\vec{\mathbf{r}}-\vec{\mathbf{r}}_{\mathrm{B}},\mathrm{t})\rangle + \mathrm{h.c.} = \\ &\Theta(t-r_A/c)\Theta(t-r_B/c)|\Psi\vec{\mathbf{r}})|^2\tfrac{\mathrm{v}(9\gamma^2+\Delta^2)}{(9\gamma^2+\Delta^2+2v^2)}\mathrm{e}^{-8\mathrm{v}^2\gamma\mathrm{t}'/(9\gamma^2+\Delta^2)}\cos(\omega\mathrm{t}) \to 0. \end{aligned} \quad (3.7)$$

We note that the observation of an interference pattern usually requires several scattered photons. On the other hand, the excitation time of an incoherent photon is related to $t_c = \frac{(9\gamma^2+\Delta^2)}{4v^2\gamma}$ so we must assume $t \gg t_c$ to have any incoherent excitation. The consequence is that Eq. (3.7) goes to zero and the incoherent part does not contribute to the interference. At this point it is worth comparing our results to the interpretation of Eichmann *et al.* [1]. Both considerations lead to the same conclusion, viz., the existence of interference in the coherent part of the spectrum and no interference in the incoherent part. There is however an important difference between the two approaches. According to our results, the presence or lack of interference is a consequence of the steady state behavior of the two atoms. In the interaction of a monochromatic laser with an atom coupled to a reservoir steady state is reached in a few Rabi cycles. In the steady state regime, however, there is no which way information any longer, and one thus can not invoke which way arguments to explain the presence or lack of interference [1, 5, 8].

4. Interference due to the lack of "which way" information

We now look for a possible modification of the experiment of Eichmann *et al.* [1] to implement a *"which way"* experiment. First we can not consider a continuous monochromatic driving field, the infinite long coherence time in such a field leads to coherent Rabi flopping of the atoms with spontaneous decay, leading to a steady state of the two atoms after some time. Therefore, we need laser pulses weak enough to excite only one atom per pulse and separated well enough to complete spontaneous emission before the next pulse arrives or, alternatively, we can use a continuous broad band incoherent excitation. The interaction of a 4–level atom with a broad band excitation is discussed in detail in Ref.[9], we therefore give only the main results. The coherence time of the broad band field is given by $\tau_c = 1/\Delta$ where Δ is the bandwidth (not to be confused with the detuning in the previous sections). We assume that $T_p \ll \gamma^{-1}$ so we can neglect stimulated emission by the broad band field. Furthermore, since $\tau_c \ll T_p$, we can regard the broad band field as a reservoir which leads to absorption of one photon at a time, followed by spontaneous decay. For such a system the interpretation in [1] is applicable, and we can talk about interference effects due to the indistinguishability of the possible paths. In other words, for such a system a which way argument is applicable which is required for the implementation of a quantum eraser [12, 13, 14, 15]. We note that a broad band excitation has the advantage over laser pulses, in that it allows a continuous monitoring of the atoms. With a broad band excitation the atoms do not saturate. The interaction Hamiltonian with the broad band driving field E_D is,

$$H_{int}(t) = -(\vec{\mu} \cdot \vec{\mathbf{E}}_{\rm D}({\rm t})) = -(\vec{\mu}^{(-)} \cdot \vec{\mathbf{E}}_{\mathbf{D}}^{(+)} + \vec{\mu}^{(+)} \cdot \mathbf{E}_{\mathbf{D}}^{(-)}). \tag{4.1}$$

For appropriate parameters E_D satisfies the relation (see Cohen-Tannoudji [9])

$$\langle E_D^{(-)}(t) E_D^{(+)}(t') \rangle = \frac{\gamma_D}{2} \delta(t - t'), \tag{4.2}$$

which is just the Markoff's approximation. We take $\mathbf{E}_D$ to be z–polarized and obtain the master equation,

$$\frac{d}{dt}\rho(t) = -\gamma_D \sum_{k,k'=A,B} \{\sigma_{zk}^{(-)}\sigma_{zk'}^{(+)}\rho(t) + \rho(t)\sigma_{zk}^{(-)}\sigma_{zk'}^{(+)} - 2\sigma_{zk}^{(+)}\rho(t)\sigma_{zk'}^{(-)}\}, \tag{4.3}$$

and for the interaction with the vacuum we have,

$$\frac{d}{dt}\rho(t) = -\gamma \sum_{k=A,B} \{\sigma_{zk}^{(-)}\sigma_{zk}^{(+)}\rho(t) + \rho(t)\sigma_{zk}^{(-)}\sigma_{zk}^{(+)} - 2\sigma_{zk}^{(+)}\rho(t)\sigma_{zk}^{(-)}\}. \tag{4.4}$$

The interaction with $\mathbf{E}_D$ leads to mixed terms which connect the two atoms, while for the vacuum we have no mixed terms. The mixed terms indicate that there is correlation between the two atoms, while for the vacuum each atom is independently coupled to the reservoir and there is no correlation.

In the case of a continuous broad band excitation, independent scattering events occur for which we can apply which way arguments and a quantum eraser can be implemented. Unlike in the previous section, we now consider a z–polarized broad band driving field. This makes the incoherent part of the spectrum σ–polarized and the coherent part π–polarized. With the broad band excitation, we may consider a single absorption process at a time and neglect stimulated emission since we assumed $T_p \ll \gamma^{-1}$. Therefore we treat a single absorption process followed by a spontaneous emission.

We consider the initial condition for the density operator,

$$\rho_{AB}(t_o) = |1\rangle_{AA}\langle 1| \otimes |1\rangle_{BB}\langle 1|. \tag{4.5}$$

After the absorption of one photon at time t' of the broad band excitation field, the density operator of the two atoms, **A** and **B**, is entangled due to the nonlocal behavior of a single absorbed photon. The coupling to the vacuum leads to decay of the excited states. So, immediately after the absorption of a photon at time t', the density operator evolves according to a master equation which describes the interaction of an excited atom with the vacuum and we obtain,

$$\begin{aligned}
\frac{d\rho_{44}(t)}{dt} &= -6\gamma\rho_{44}(t),\\
\frac{d\rho_{41}(t)}{dt} &= -3\gamma\rho_{41}(t),\\
\frac{d\rho_{11}(t)}{dt} &= 2\gamma\rho_{33}(t) + 4\gamma\rho_{44}(t),\\
\frac{d\rho_{22}(t)}{dt} &= 2\gamma\rho_{44}(t) + 4\gamma\rho_{33}(t),\\
\frac{d\rho_{21}(t)}{dt} &= -3\gamma\rho_{43}(t),\\
\frac{d\rho_{43}(t)}{dt} &= -6\gamma\rho_{43}(t).
\end{aligned} \tag{4.6}$$

With these equations we get the following time evolution of the density matrix,

$$\begin{aligned}
\rho_{44}(t) &= \rho_{44}(0)e^{-6\gamma t},\\
\rho_{43}(t) &= \rho_{43}(0)e^{-6\gamma t},\\
\rho_{21}(t) &= \frac{1}{2}\rho_{43}(0)(e^{-6\gamma t} - 1),\\
\rho_{11}(t) &= \frac{1}{3}\rho_{33}(0)(1 - e^{-6\gamma t}) + \frac{2}{3}\rho_{44}(0)(1 - e^{-6\gamma t}),\\
\rho_{22}(t) &= \frac{1}{3}\rho_{44}(0)(1 - e^{-6\gamma t}) + \frac{2}{3}\rho_{33}(0)(1 - e^{-6\gamma t}),\\
\rho_{41}(t) &= \rho_{41}(0)e^{i\omega t - 3\gamma t}.
\end{aligned} \tag{4.7}$$

The other matrix elements are related so that ρ_{33} satisfies the same equation as ρ_{44} and ρ_{32}, ρ_{42}, and ρ_{31} satisfy the same equation as ρ_{41}. From the previous section, the interference is given by

$$|\vec{\mathbf{\Psi}}(\vec{\mathbf{r}})|^2 \langle \sigma_{\mathrm{A}}^{(+)}(\mathrm{t}')\sigma_{\mathrm{B}}^{(-)}(\mathrm{t}' + \tau) + \sigma_{\mathrm{A}}^{(-)}(\mathrm{t}')\sigma_{\mathrm{B}}^{(+)}(\mathrm{t}' + \tau)\rangle. \tag{4.8}$$

The coherent part is related to the z–polarization or $\pi - (|\Delta m_j| = 0)$ transition, while the incoherent part is related to x– or y–polarization or $\sigma - (|\Delta m_j| = 1)$ transition. With these equations, we obtain for the interference term in the coherent scattering the expression,

$$\begin{aligned} & |\vec{\mathbf{\Psi}}(\vec{\mathbf{r}})|^2 \Big\langle \sigma^{(+)}_{\mathrm{A,z}}(\mathrm{t}')\sigma^{(-)}_{\mathrm{B,z}}(\mathrm{t}'+\tau) + \sigma^{(-)}_{\mathrm{A,z}}(\mathrm{t}')\sigma^{(+)}_{\mathrm{B,z}}(\mathrm{t}'+\tau)\rangle \\ = \; & |\vec{\mathbf{\Psi}}(\vec{\mathbf{r}})|^2 \mathrm{Tr}_{\mathrm{A,B}} \Big\{ \rho^{\mathrm{A}}_{13}(\mathrm{t}')|1\rangle_{\mathrm{AA}}\langle 3|3\rangle_{\mathrm{AA}}\langle 1| + \rho^{\mathrm{B}}_{31}(\mathrm{t}'+\tau)|3\rangle_{\mathrm{BB}}\langle 1|1\rangle_{\mathrm{BB}}\langle 3| \Big\} + \mathrm{c.c.} \\ = \; & |\vec{\mathbf{\Psi}}(\vec{\mathbf{r}})|^2 \cos(\omega\tau)\mathrm{e}^{-6\gamma(\mathrm{t}'+\frac{\tau}{2})}. \end{aligned} \tag{4.9}$$

The coherent part of the scattered field, thus, has a coherence time of $(6\gamma)^{-1}$. As we see, for π-polarized scattering, there are two indistinguishable ways which lead to the same final state and therefore interference takes place.

We consider now the case of σ-polarized scattering. We get for the incoherent part of the x–polarized scattered light,

$$\begin{aligned} & |\vec{\mathbf{\Psi}}(\vec{\mathbf{r}})|^2 \mathrm{Tr}_{\mathrm{AB}} \left\{ \langle \rho^{\mathrm{A}}_{13}(\mathrm{t}')|1\rangle_{\mathrm{AA}}\langle 3|3\rangle_{\mathrm{AA}}\langle 2|\rho^{\mathrm{B}}_{31}(\mathrm{t}'+\tau)|2\rangle_{\mathrm{BB}}\langle 3|2\rangle_{\mathrm{BB}}\langle 3| + \mathrm{c.c.} \right\} \\ & = |\vec{\mathbf{\Psi}}(\vec{\mathbf{r}})|^2 \cos(\omega\tau)\mathrm{e}^{-6\gamma(\mathrm{t}'+\frac{\tau}{2})} \mathrm{Tr}_{\mathrm{AB}}(|1\rangle_{\mathrm{AA}}\langle 2|1\rangle_{\mathrm{BB}}\langle 2| + |1\rangle_{\mathrm{BB}}\langle 2|1\rangle_{\mathrm{AA}}\langle 2|) = 0. \end{aligned} \tag{4.10}$$

Thus the σ–polarized scattered light can not lead to interference. The reason is that the scattering process brings one of the two atoms to a final state which is orthogonal to the initial state. From another point of view, the scattering of a σ-polarized photon leads to two distinguishable paths for the photon, giving no interference. As emphasized above, it is important to recognize that impossibility to observe interference and entanglement of distinguishable states are connected by our final density operator. The entanglement of the atomic density operator is necessary for the observation of no interference, because it expresses the fact that, at least in principle, it is possible to obtain a which way information. The mere availability of this information, i.e. the possibility that it is knowable, makes the interference impossible.

5. Quantum eraser

After these preliminaries we now show that a modification of the experiment in [1] allows one to implement a quantum eraser with a delayed choice set up, in the sense proposed originally [14]. The first requirement is a non–unitary time evolution to erase the information through an irreversible process, and the second is a measurement of the second order (or intensity) correlation function. The first condition is required because a unitary time evolution is reversible and in any reversible process information is not "lost" and can be recovered by an inverse transformation which is again unitary. The second condition is required because of the orthogonality of the photon states $|\gamma_A\rangle$ and $|\gamma_B\rangle$ of the photons scattered from atom **A** and **B**. A detection of the photons $|\gamma_A\rangle$ and $|\gamma_B\rangle$ reduces the infinite number of possible ways they can take to one specific way, they have actually taken.

Irreversibility is brought in by a non–unitary transformation such as a decay process, which is detected, so there is a non–unitary state reduction. Because of the internal

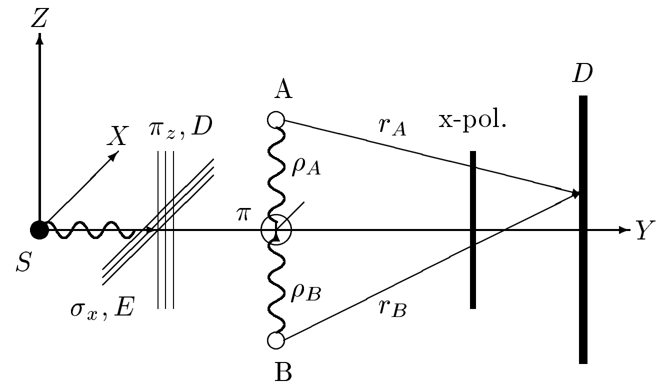

Fig. 2. Arrangement for the quantum eraser. S is the light source which can have different polarization directions, σ_x is a σ_x-polarizer for the scattering photon and π is a π- polarization dependent detector for the erasing photon.

structure of the 4–level atoms, we do not need additional levels to erase the information, the 4–level structure is enough for the realization of a quantum eraser. We use the same experimental setup that was originally suggested by Scully and Drühl, with a slight variation, where a π–polarization sensitive detector is placed equidistantly between atoms **A** and **B** (Fig. 2). The detection scheme allows us to distinguish between a π–polarized erasing photon and a σ–polarized interference photon. The erasing photon produces a final density matrix which is entangled only between indistinguishable states of the two atoms, and the measurement of the erasing photon is necessary to ensure the specific final state of the two–atom density operator.

The two atom system is driven with broad band light, as in the previous section, and $T_D(6\gamma) \approx 10$, where T_D is the travel time of the interference photon to the detector. This sets a limitation on the experimental apparatus, since it defines the position of the detector of the interference pattern. This arrangement is necessary because we want to ensure that there is sufficient time for the atomic density operator to evolve into the specific entangled density operator between distinguishable states after the scattering of the interfering photon. In addition, we apply an intense σ-polarized pulse before the detection of the interference photon and we want to make sure that there is enough time to observe the erasing photon before the interfering photon.

We start to drive the atoms at t_o and after a time $T_P + (5\gamma)^{-1}$ we apply the strong short pulse which excites only one of the atoms. After this there is sufficient time around $(5\gamma)^{-1}$ to detect the erasing photon before detecting the interfering photon. This ensures that we have the required final state whith entanglement only between indistinguishable states of the two atoms.

To express the above qualitative description mathematically we consider the second order correlation function at times t_o and $t_o + T_P + (5\gamma)^{-1}$, that is just before applying the second laser pulse which is assumed to be σ-polarized. The second order correlation function is

$$\begin{aligned} G^{(2)}(\vec{\mathbf{r}},\mathrm{t}';\vec{\rho},\tau) &= \mathrm{Tr}_{A,B}\big\{\hat{\rho}(t_o)[(\hat{E}_A^{(-)\sigma}(\vec{\mathbf{r}},\mathrm{t}')+\hat{\mathrm{E}}_{\mathrm{B}}^{(-)\sigma}(\vec{\mathbf{r}},\mathrm{t}))\ (\hat{\mathrm{E}}_{\mathrm{A}}^{(-)\pi}(\vec{\rho},\tau)+\hat{\mathrm{E}}_{\mathrm{B}}^{(-)\pi}(\vec{\rho},\tau))] \\ & [(\hat{E}_A^{(+)\pi}(\vec{\rho},\tau)+\hat{\mathrm{E}}_{\mathrm{B}}^{(+)\pi}(\vec{\rho},\tau))\ (\hat{\mathrm{E}}_{\mathrm{A}}^{(+)\sigma}(\vec{\mathbf{r}},'\mathrm{t})+\hat{\mathrm{E}}_{\mathrm{B}}^{(+)\sigma}(\vec{\mathbf{r}},\mathrm{t}'))]\big\}, \end{aligned} \tag{5.1}$$

where π denotes the π–polarized photons which are used to erase the information and σ denotes σ–polarized scattered photons which are observed at the detector.

The interfering part of the second order correlation function is given by

$$\begin{aligned} & G_{int}^{(2)}(\vec{\mathbf{r}},\mathrm{t};\vec{\rho}\tau) = \\ & \mathrm{Tr}\Big\{\hat{\rho}(t_o)\{E_A^{(-)\sigma}(\vec{\mathbf{r}},\mathrm{t})\{\mathrm{E}_{\mathrm{A}}^{(-)\pi}(\vec{\rho},\tau)\mathrm{E}_{\mathrm{B}}^{(+)\pi}(\vec{\rho},\tau)+\mathrm{E}_{\mathrm{B}}^{(-)\pi}(\vec{\rho},\tau)\mathrm{E}_{\mathrm{A}}^{(+)\pi}(\vec{\rho},\tau)\}\mathrm{E}_{\mathrm{B}}^{(+)\sigma}(\vec{\mathbf{r}},\mathrm{t}) \\ & +E_B^{(-)\sigma}(\vec{\mathbf{r}},\mathrm{t})\{\mathrm{E}_{\mathrm{A}}^{(-)\pi}(\vec{\rho},\tau)\mathrm{E}_{\mathrm{B}}^{(+)\pi}(\vec{\rho},\tau)+\mathrm{E}_{\mathrm{B}}^{(-)\pi}(\vec{\rho},\tau)\mathrm{E}_{\mathrm{A}}^{(+)\pi}(\vec{\rho},\tau)\}\mathrm{E}_{\mathrm{A}}^{(+)\sigma}(\vec{\mathbf{r}},\mathrm{t})\Big\}. \end{aligned} \tag{5.2}$$

Assuming the second short pulse is a one photon process, we get for time $t'' = t_o + T_P + T_{P_2} + (5\gamma)^{-1}$, immediately after the excitation,

$$G_{int}^{(2)}(\vec{\mathbf{r}},,\mathrm{t};\vec{\rho},\tau) = 4\Theta(\mathrm{t}''-(\vec{\mathbf{r}}_{\mathrm{A}}/\mathrm{c})\Theta(\mathrm{t}''-\vec{\mathbf{r}}_{\mathrm{B}}/\mathrm{c})\Theta(\mathrm{T}_0-\vec{\rho}/2\mathrm{c}))\mathrm{e}^{-6\gamma\mathrm{T}_0}\cos(\vec{\mathbf{k}}\cdot\delta\vec{\mathbf{r}}). \tag{5.3}$$

Here $T_0 = t'' - (T_P + T_{P_2} + (5\gamma)^{-1}))$ and $\delta\vec{\mathbf{r}}$ is the path difference. Immediately before the detection of the σ–polarized photon we detect the π–polarized photon which erases the information since the probability of detecting the π-polarized photon in that time is about $1 - e^{-5} \approx 1$, and the final atomic density operator is

$$\rho_A(t_f) = |1\rangle_{AA}\langle 1| \otimes |1\rangle_{BB}\langle 1| + |2\rangle_{AA}\langle 2| \otimes |2\rangle_{BB}\langle 2|. \tag{5.4}$$

Thus, the final atomic density operator contains only entanglement of the two atoms leading to interference. The which way information is erased if we detect the π-polarized erasing photons before the scattering or interfering photons. The detection of the erasure photon before the fluorescent photon is necessary because only this detection ensures the atomic system to be in the proper final state.

If we detect the π–polarized erasing photons after the scattering photons then no interference occurs because the atomic density operator is still in an entanglement of distinguishable states between the two atoms. Also, if we do not use a second correlation measurement there will be no interference. This is because of the orthogonality of the scattered π–polarized erasing photons. Their detection reduces the infinitely many possible ways to one for the scattered photons. These results were already known and discussed in [14]. Proceeding from these it is clear that if we use a σ–polarized detection scheme for the erasing photons there is some possibility that we detect the scattering or interfering photons at the erasing detector and therefore destroy the part which contains the entanglement between correlations.

6. Quantum eraser and the decoherence time of a measurement process

Finally, we show that the quantum eraser can be used to probe the decoherence time of a measurement process[19, 20, 21]. The general quantum measurement process has been dealt with in, e.g., [22, 23, 24] employing the following scheme. System S is coupled to a meter M, and the meter is coupled to the environment or reservoir. The measured system here is the Zeemann splitting of the lower two degenerate states of atom B. The initial condition for the two–atom system is

$$\rho_{AB}(0) = |1\rangle_{AA}\langle 1| \otimes |1\rangle_{BB}\langle 1|. \tag{6.1}$$

After the application of the BBE field and the subsequent decay of the excited states, we have,

$$\rho_{AB}(t) = |2\rangle_{AA}\langle 2| \otimes |1\rangle_{BB}\langle 1| + |2\rangle_{AA}\langle 1| \otimes |1\rangle_{BB}\langle 2| + 1 \leftrightarrow 2. \tag{6.2}$$

The system is coupled to the meter leading to the entanglement between S and M,

$$\begin{aligned}\rho_{S-M} &= |2\rangle_{AA}\langle 2| \otimes |1\rangle_{BB}\langle 1| \otimes |m_1\rangle\langle m_1| \\ &+ |2\rangle_{AA}\langle 1| \otimes |1\rangle_{BB}\langle 2| \otimes |m_1\rangle\langle m_2| + 1 \leftrightarrow 2. \end{aligned}\tag{6.3}$$

Here $|m_1\rangle$ and $|m_2\rangle$ are pointer states. The meter is coupled to the environment and the off–diagonal elements decay rapidly, with a decoherence time γ_{dec}^{-1} of the pointer states. As a consequence, the interference term in the second order correlation function of the quantum eraser will decay rapidly due to the coupling to the meter. The erasing pulse, applied at a time δt after starting the measurement process, will restore the interference but with a reduced visibility at t_f. The amount of reduction in fringe visibility is a quantitative measure of the decoherence time. Due to the coupling of the meter system to the environment, the entanglement between S and M decays very rapidly with the same decay rate γ_{dec}. The system-meter state which is "macroscopic" is then given by,

$$\begin{aligned}\rho_{S-M} &= |2\rangle_{AA}\langle 2| \otimes |1\rangle_{BB}\langle 1| \otimes |m_1\rangle\langle m_1| \\ &+ e^{-\gamma_{dec}t}|2\rangle_{AA}\langle 1| \otimes |1\rangle_{BB}\langle 2| \otimes |m_1\rangle\langle m_2| + 1 \leftrightarrow 2. \end{aligned}\tag{6.4}$$

The second order correlation function contains entanglement of correlations which are the same as in the meter system and consequently will decay rapidly due to the measurement process. The application of the erasing pulse interrupts the measurement and, since after the pulse the states have the same magnetic quantum number, the signal vanishes. After a δt measurement time and the erasing pulse afterwards, we get the following for interference, at time t_f,

$$\begin{aligned}G^2(\vec{\mathbf{r}}, t_f; \vec{\rho}, t') &= 4\Theta(t'-T_0)\Theta(t_f - r_A/c)\Theta(t_f - r_B/c) \\ &\times e^{-6\gamma(t_f - \frac{1}{2}(r_A/c - r_B/c))}e^{-6\gamma(t'-T_0)}e^{-\gamma_{dec}\delta t}\cos(\vec{\mathbf{k}}.(\vec{\mathbf{r}}_A - \vec{\mathbf{r}}_B)). \end{aligned}\tag{6.5}$$

Here T_o is the same as the time defined in Section 5. The implication of the above is that the visibility is reduced by a factor, related to $e^{-\gamma_{dec}\delta t}$, due to the measurement

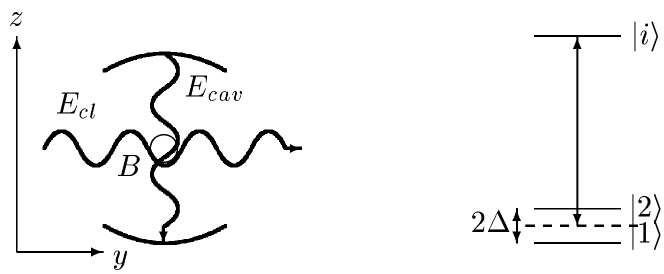

Fig. 3. Scheme for the local quantum measurement process.

process. Thus, the decoherence time is visualized by the reduction of the fringe visibility and the quantum eraser can be used as a tool to study fundamental properties such as the transition from microscopic to macroscopic systems, and the measurement process itself.

The model for the measurement scheme is as follows. A cavity is placed around atom B (with a quantized cavity field inside), and also around atom A since the presence of a cavity changes the decay rate. In addition to the cavity field, which is assumed to be in the vacuum state initially, we have a classical (coherent state E_{cl}) driving field. The cavity field, coupled to the atomic system, constitutes the meter. A magnetic field, B_0, is applied which splits the degenerate ground states. Both the classical and the cavity fields are strongly detuned from the transition frequencies of the atoms. The total Hamiltonian of atom B interacting with the cavity field and the classical field is,

$$\begin{aligned} H &= H_0 + H_{cav} + H_{cl}, \\ &= \hbar\omega_0\sigma_z + \hbar\omega a^\dagger a + \hbar\{g^*_{cav}\sigma_- a^\dagger + g_{cav}\sigma_+ a\} + +i\hbar g_{cl}E_{cl}\{\sigma_+ - \sigma_-\}. \end{aligned} \tag{6.6}$$

Here

$$\sigma_z = \frac{1}{2}(|2\rangle\langle 2| - |1\rangle\langle 1|), \tag{6.7}$$

and g_{cav} is the coupling parameter, $a^\dagger$ and a are the creation and annihilation operators for the cavity field E_{cav} respectively and $g_{cl}E_{cl}$ is the Rabi frequency of the classical driving field. The effective Hamiltonian for the system in the strong detuning limit is given by (after adiabatically eliminating the states which are not involved),

$$H_{eff} = \frac{2\hbar}{\Delta}\sigma_z\{|g_{cav}|2a^\dagger a + g^2_{cl}|E_{cl}|^2 + i(g^*_{cl}g_{cav}E^*_{cl}a - g_{cl}g^*_{cav}E_{cl}a^\dagger)\}. \tag{6.8}$$

Taking into account the strong detuning, $g_{cl}E_{cl}/\Delta \ll 1$ and $g_{cav}\langle n_{cav}\rangle/\Delta \ll 1$, we obtain

$$H_{int} = \hbar\sigma_z\frac{g^2}{\Delta}\{a^\dagger a + |E_{cl}|^2 - iE^*_{cl}a + iE_{cl}a^\dagger\}, \tag{6.9}$$

where we assumed $g_{cav} = g_{cl} = g$. The terms $\sigma_z a^\dagger a$ and $\sigma_z|E_{cl}|^2$ produce only an overall frequency shift and can be neglected. Next, the cavity field (meter) is coupled to the environment as given by

$$H_{ME} = \hbar\{a?^{\dagger} + a^{\dagger}?\}, \tag{6.10}$$

where

$$? = \sum_k g_k b_k, ?^{\dagger} = \sum_k g_k^* b_k^{\dagger}. \tag{6.11}$$

Here b and $b^{\dagger}$ are reservoir annihilation and creation operators. The meter-environment interaction determines the particular states (pointer basis) to which the meter states will reduce (approximately coherent states). We now have a complete system–meter–environment (atom, cavity field and all modes of the quantized field at zero temperature) measurement process.

The density operator for the system–meter, after tracing over the environment at zero temperature, satisfies the following equation,

$$\frac{d\rho}{dt} = \frac{g^2}{\Delta}[\sigma_z(E_{cl}a^{\dagger} - E_{cl}^* a), \rho] + \frac{\gamma_{cav}}{2}\{2a\rho a^{\dagger} - a^{\dagger}a\rho - \rho a^{\dagger}a\}. \tag{6.12}$$

We expand the density operator,

$$\rho(t) = \sum_{i,j=1}^{2} \rho_{ij}(t) \otimes |i\rangle\langle j|, \tag{6.13}$$

with the initial condition

$$\rho_{ij}(0) = \sum_{\alpha_i,\alpha_j} N_{nm}(\alpha_i, \alpha_j)\frac{|\alpha_i\rangle\langle\alpha_j|}{\langle\alpha_i|\alpha_j\rangle}. \tag{6.14}$$

The system is considered arbitrary, while the meter is in the ground state. The master equation can be solved using the normally ordered characteristic function [23, 24]. The density operator for the system atom + meter evolves into the following,

$$\begin{aligned}\rho(t) &= |2\rangle_{AA}\langle 2| \otimes |1\rangle_{BB}\langle 1| \otimes \tfrac{|\alpha_1(t)\rangle\langle\alpha_1(t)|}{\langle\alpha_1(t)|\alpha_1(t)\rangle} \\ &+ \exp\Big\{(2\alpha)^2\big(1 - \tfrac{\gamma_{cav}}{2} - e^{\frac{-\gamma_{cav}t}{2}}\big)\Big\}|2\rangle_{AA}\langle 1| \otimes |1\rangle_{BB}\langle 2| \otimes \tfrac{|\alpha_1(t)\rangle\langle\alpha_2(t)|}{\langle\alpha_1(t)|\alpha_2(t)\rangle},\end{aligned} \tag{6.15}$$

where the coherent states form the approximate pointer basis for the meter. The decoherence rate between two "macroscopic" states can be related to the distance between them [23], and in this case we get

$$\gamma_{dec} = |\alpha_1 - \alpha_2|\gamma_{cav}. \tag{6.16}$$

Finally, the visibility, at time δt after the start of the measurement and a subsequent application of the erasing pulse, is given by

$$\mathrm{V} = \exp\left(-\frac{\gamma_{dec}^2(\delta t)^2}{4}\right) . \tag{6.17}$$

To obtain this expression we have assumed $\gamma_{cav}\delta t \ll 1$ and expanded the exponent in the exponential of Eq. (6.15) to second order.

7. Conclusion

The entanglement in the two atom system, as a result of the scattering of a photon, plays a crucial role in the quantum eraser. Entanglement is connected to the nonlocal behavior of quantum systems and, in the case of interference experiments with one photon, to the nonlocal behavior of the photon itself. The entanglement term depends on whether there is interference or no interference. In the case of no interference there is an entanglement of the two atoms containing orthogonal states and therefore tracing over the atomic density operator gives vanishing result, while in the case of interference there is entanglement between populations of the two atoms and tracing over the atomic density operator gives a non–vanishing result.

It should be emphasized that, since there is entanglement in both cases, we can not use "which way" arguments for the photon. Even in the case of non–interference the detection of the photon gives no information on which path the photons has taken. If we really knew which path the photon had taken entanglement would not take place and the quantum eraser would never work. The detection of the photon at the photon detector in the case of non–interference gives us only the possibility of knowing which path the photon has taken. The quantum eraser irreversibly erases the possibility of obtaining "which way" information. In the case of interference we have no possibility of obtaining "which way" information, to begin with. The final atomic density operator contains, in the "interfering part", an entanglement between populations.

It is clear that we need the entanglement (correlations) to erase the information. If this entanglement is not stable (e. g. finite decay rates of states $|1\rangle$ and $|2\rangle$) the second pulse to erase the information must be applied before the decay of correlations. A local measurement of the state of one of the atoms, for example atom B, destroys the correlation and therefore the entanglement. In other words a local measurement on one atom and therefore an explicit knowledge of the way the photon has taken irreversibly destroys the possibility to erase the information because the entanglement leading to interference is destroyed.

To conclude, we want to point out that the quantum eraser is of fundamental interest in quantum optics because it allows us to explore two important aspects of quantum mechanics: complementarity and nonlocality. We have shown that the entanglement of nonlocal superpositions in the case of non–interference and the related entanglement of correlations in the quantum eraser can be used to measure the decoherence time of a macroscopic or mesoscopic measurement apparatus.

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