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CAVITY-INDUCED ATOM-ATOM CORRELATION FOR TWO UNIDENTICAL ATOMS[1]

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The mutual coherence of pairs of two-level atoms is studied for the case of unidentical atoms in a lossy cavity. It is established that the cavity can induce a correlation or anticorrelation of the atomic dipoles depending on the nature of the atom-atom and atom-cavity detunings. The cavity-induced atom-atom correlation is clearly manifested in the spectra of the cavity field and of the fluorescence field.

1. Introduction

In a recent experiment [1] it was demonstrated that interference effects analogous to Young's two-slit experiment can be observed in the light scattered by a pair of trapped atoms which are coherently excited by a weak laser field. The fundamental nature of this experiment and rapid technical advances in the cooling of trapped ions [2] has prompted a number of theoretical studies on the mutual coherence of atomic pairs [3, 4, 5, 6]. In particular, it has been established that the fringe visibility may be significantly enhanced by coupling the atoms to a single standing-wave resonator mode [4]. This is of particular significance in the regime of strong driving where incoherent scattering processes begin to dominate and the fringe visibility approaches zero, indicating that the singly-excited symmetric and antisymmetric atomic Dicke states are equally populated. The effect of the cavity, however, is to introduce a two-stepped path involving coherent atom-cavity interaction followed by cavity decay which serves to preferentially populate the symmetric atomic states. In this way, the mutual coherence is actually increased by incoherent decay processes [5]. This enhancement of mutual coherence through cavity decay is most clearly seen when the atoms are incoherently excited so that the atom-atom correlations are induced solely by the atom-cavity interactions [6]. In this case, it is found that the interference pattern produced by the fluorescing atoms contains an intensity minimum at line center despite the setup being entirely symmetric.

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It has long been established that mutual atomic coherence may be generated via dipole-dipole interaction. When a pair of two-level atoms are separated by a distance comparable to or less than their transition wavelength a dipole-dipole interaction, mediated by the continuum of modes [7], may induce simultaneous atomic transitions [8]. A similar behaviour can also be observed for a pair of two-level atoms in a perfect cavity [9]. In this latter case, however, the atoms are correlated through their interaction with the quantized cavity field rather than the free-space continuum of modes. In this paper we shall study the mutual coherence of two *unidentical* atoms coupled by their interaction with a single resonator mode. Such a situation might arise in a Paul or linear ion trap where the localized ions are different isotopes of the same element; that is, the ions have approximately the same mass, charge and dipole moments but have markedly different transition energies [10]. Consequently, it is possible to envisage a situation of two trapped atoms which are cooled to their ground motional state and coupled to a single cavity mode with equal vacuum Rabi frequency but with different atom-cavity detunings. It is this system which is the focus of the present paper. We shall establish that the cavity can induce either a correlation or an anticorrelation of the atomic dipoles depending on the nature of the individual atom-cavity detunings. This is a direct consequence of the coherent coupling of the singly-excited symmetric and antisymmetric atomic Dicke states, which arises when the individual atom-cavity detunings have different values. A direct confirmation of the atom-atom correlation is found in the fluorescence and cavity field spectra which display additional peaks not found in the case of a single atom or of two identical atoms coupled to the resonator mode.

2. Model

We consider a pair of two-level atoms interacting with a single-mode cavity field with annihilation and creation operators a and $a^\dagger$. The excited and ground states are, respectively, denoted by $|e_{A,B}\rangle$ and $|g_{A,B}\rangle$ for the atoms A and B. Let us denote by $|g_A, g_B, n\rangle$ a state of the combined atom-cavity field system, where both of the atoms are in their ground states and n photons are present in the cavity. For the case of a single atomic excitation the atom-field system can either be in the Dicke symmetric state $|s, n\rangle$ or antisymmetric state $|a, n\rangle$.

In this paper we assume that the vacuum Rabi frequencies are equal for each atom. For two unidentical atoms an equal atom-cavity coupling can be obtained for an atom-atom separation of an integer number of mode wavelengths if the two atoms are different isotopes of the same element. Under this assumption, the Hamiltonian of the system in the frame rotating with *cavity frequency* ω_c is

$$H = \sqrt{2}\hbar\kappa(a^\dagger\sigma_s^- + a\sigma_s^+) - \hbar\Delta_A|g_A\rangle\langle g_A| - \hbar\Delta_B|g_B\rangle\langle g_B|, \tag{1}$$

where κ is the atom-field coupling constant and Δ_A and Δ_B are detunings between the cavity field and atomic transitions for atoms A and B. The transition operator σ_s^+ is defined as

$$\sigma_s^+ = \frac{1}{\sqrt{2}}(|e_A\rangle\langle g_A| + |e_B\rangle\langle g_B|) \tag{2}$$

and σ_s^- is its hermitian conjugate. It is easily seen from the Hamiltonian (1) that when the two identical atoms are resonantly coupled with the single-mode cavity field, i.e. $\Delta_A = \Delta_B = 0$, the atomic evolution is restricted to the ground-ground, symmetric and excited-excited states.

For later analysis we rearrange the Hamiltonian (1) as

$$H = \sqrt{2}\hbar\kappa(a^\dagger\sigma_s^- + a\sigma_s^+) - \hbar\Delta\sigma_s^o - \hbar\Delta_c\sigma_a^o \ , \tag{3}$$

where $\Delta = \frac{1}{2}(\Delta_A + \Delta_B)$ and $\Delta_c = \frac{1}{2}(\Delta_A - \Delta_B)$ and we have introduced this new operators

$$\sigma_s^o = |g_A\rangle\langle g_A| + |g_B\rangle\langle g_B| \ \ , \ \ \sigma_a^o = |g_A\rangle\langle g_A| - |g_B\rangle\langle g_B|. \tag{4}$$

In the case of identical atoms we see that $\Delta_c = 0$ and the operator σ_a will play no role in the dynamical evolution of the atom-cavity system. However, for unidentical atoms with $\Delta_c \neq 0$ we shall later see that the operator σ_a plays a crucial role and leads to qualitatively different behaviour from that found in the case of identical atoms. The reason for this is that it generates a coherent coupling of the singly-excited symmetric and antisymmetric atomic Dicke states, as evident in the relation

$$\sigma_a^o|s,n\rangle = -|a,n\rangle \ \text{ and } \ \sigma_a^o|a,n\rangle = -|s,n\rangle, \tag{5}$$

with $|n\rangle$ denoting the number of photons in the cavity mode. This, of course, is not a cavity-induced effect but merely represents the different evolution frequencies of the individual atoms modulating their relative phase. Such behaviour is in stark contrast to that of the operator σ_s which merely maps the Dicke states $|s\rangle$ and $|a\rangle$ onto themselves. From this argument it seems reasonable to expect that the effect of the coherent coupling of the Dicke states, brought about by the presence of unidentical atoms, will be maximized when the atoms are oppositely detuned from the resonant cavity frequency so that $\Delta = 0$. This is the case which we shall examine in detail for the remainder of this paper. In particular, we shall investigate the behaviour of two unidentical atoms which are coupled to a lossy resonator mode and incoherently excited by broadband radiation. We assume the rates, p, of incoherent excitation to be identical for each atom. We also assume equal rates, γ, of spontaneous atomic decay by restricting our analysis to the case where the atoms are different isotopes of the same element. Finally to clarify the influence of the atom-cavity coupling we consider well-separated atoms so that dipole-dipole interactions may be neglected. The evolution of the density operator ρ for the atom-cavity system is then governed by the master equation

$$\frac{\partial\rho}{\partial t} = -\frac{i}{\hbar}[H,\rho] + L_{field}\rho + L_{atom}\rho \ , \tag{6}$$

where the Hamiltonian is defined in Eq.(3), the field Liouvillian is [7]

$$L_{field}\rho = -\frac{\gamma_f}{2}(a^\dagger a\rho - 2a\rho a^\dagger + \rho a^\dagger a), \tag{7}$$

with the cavity photon decay rate γ_f, and the atomic Liouvillian has the form

$$L_{atom}\rho = \qquad -\frac{\gamma}{2}\sum_{i=A,B}(|e_i\rangle\langle e_i|\rho - 2|g_i\rangle\langle e_i|\rho|e_i\rangle\langle g_i| + \rho|e_i\rangle\langle e_i|)$$

$$-\frac{p}{2}\sum_{i=A,B}(|g_i\rangle\langle g_i|\rho - 2|e_i\rangle\langle g_i|\rho|g_i\rangle\langle e_i| + \rho|g_i\rangle\langle g_i|). \tag{8}$$

3. Spectra

We consider the far-field spectra of the cavity and fluorescence fields in the steady state to provide an unambiguous measure of the atom-atom interaction as mediated by the cavity. To begin our discussion, we focus on the cavity field spectrum. The normalized cavity field spectrum $S_c(\omega)$ is simply the Fourier transform of the field correlation function in the steady state

$$S_c(\omega) = \mathcal{N}\int \langle a^\dagger(0)a(\tau)\rangle_{ss} e^{i\omega\tau} d\tau \ , \tag{9}$$

where $\mathcal{N}$ is the normalization factor. In what follows, we shall concentrate on the regime of weak excitation of the atom-cavity field system to make the study of atomic coherences more transparent [6].

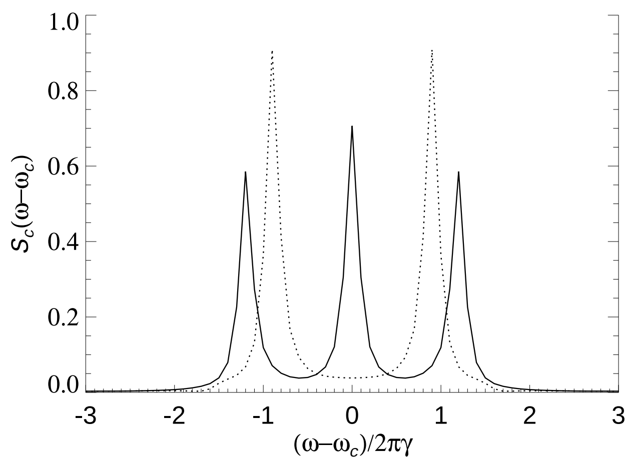

Fig. 1. Spectra for the cavity fields. The atomic decay rate $\gamma = \gamma_f$. The atom-field coupling $\kappa = 4\gamma_f$. The incoherent pump rate $p = 0.02\gamma_f$. When the two resonant atoms are in the cavity (Dotted line). When two unidentical atoms are off-resonant, $\Delta_c = 5\gamma_f$ and $\Delta = 0$ (Solid line). ω_c is the frequency of the cavity field.

In Fig. 1 we plot the cavity field spectrum for the parameters $p = 0.02\gamma_f$, $\gamma = \gamma_f$, $\kappa = 4\gamma_f$, and $\Delta = 0$ for the two distinct cases of unidentical atoms with $\Delta_c = 5\gamma_f$ and identical atoms $\Delta_c = 0$. Numerical calculations show that in both cases the cavity-mean photon number and atomic excitation is approximately 0.01. Clearly, for such weak excitations the behaviour of the atom-cavity field system may be described using only the states of zero and one excitation: $|g_A, g_B, 0\rangle, |s, 0\rangle, |g_A, g_B, 1\rangle$, and $|a, 0\rangle$. The

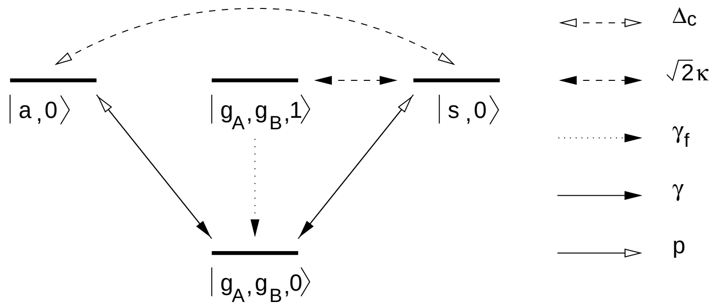

Fig. 2. Schematic representation of the dynamics for atom-cavity field states the incoherent pump is very weak.

states and couplings of the atom-cavity field system in the regime of weak excitation are shown schematically in Fig. 2.

Inspection of Fig. 1 reveals that for identical atoms the cavity field spectrum is a two-peaked structure and the individual peaks are separated by $2\sqrt{2}\kappa$. Thus, the cavity field spectrum for two identical atoms is qualitatively similar to that produced by a single atom coupled to a resonator mode and differs only quantitatively by a factor $\sqrt{2}$ in the magnitude of the vacuum Rabi splitting. The case of two unidentical atoms, however, produces a cavity field spectrum which is markedly different *qualitatively* from the single-atom case. In particular, Fig. 1 reveals that a third peak occurs when two unidentical atoms are coupled to the resonator mode. Additionally, the normal-mode vacuum splitting from two unidentical atoms is more pronounced than that which occurs for two identical atoms.

To analyze the new spectral features arising from the unidentical atoms we consider first the simple case of two different atoms coupled to a lossless cavity and neglect the effects of the weak pump. The Hamiltonian of this system is given by Eq.(3). For $\Delta = 0$, the case under consideration, it is straightforward to show the eigenvalues of the Hamiltonian $\hbar\omega_o$ and $\hbar\omega_\pm$ are

$$\omega_o = 0 \quad , \quad \omega_\pm = \sqrt{\Delta_c^2 + 2\kappa^2}. \tag{10}$$

The central peak in the spectrum is naturally due to the eigenvalue $\hbar\omega_0$. The sidebands, on the other hand, are attributed to the effect of vacuum Rabi splitting in a manner similar to single-atom and identical two-atom systems. For unidentical atoms we find that the magnitude of the splitting is enhanced by the presence of a non-zero value for Δ_c as evident in Eq.(10). This enhanced frequency shift is analogous to that found with non-resonant coherent excitation of a single atom [11].

The physical mechanism which results in the extra peak of Fig. 1 can be ascertained by inspection of the approximate states and couplings depicted in Fig. 2. We begin our analysis by considering the simple case $\Delta_c = 0$ where the singly-excited antisymmetric Dicke state $|a,0\rangle$ is uncoupled from every other state due to destructive quantum interference [6]. A subsequent dressed-state analysis of the remaining two coupled states $|g_A, g_B, 1\rangle$ and $|s,0\rangle$ shows that a cavity decay can occur at the distinct energies $\hbar(\omega_c \pm \kappa)$, leading to the familiar two-peaked spectrum. For two unidentical atoms, however, a different picture emerges. In this case, we find that all three singly-excited states are coherently coupled. Thus, it is to be expected that there exist three eigenvalue solutions rather than the usual two. After a straightforward algebra, we find that the three eigenstates corresponding to these eigenvalues are

$$
\begin{aligned}
|\omega_o\rangle &= \frac{1}{\sqrt{\Delta_c^2+2\kappa^2}}[\sqrt{2}\kappa|g_A,g_B,1\rangle + \Delta_c|a,0\rangle]\ , \\
|\omega_\pm\rangle &= \frac{\pm 1}{\sqrt{2(\Delta_c^2+2\kappa^2)}}[\sqrt{2}\kappa|g_A,g_B,1\rangle \pm \sqrt{\Delta_c^2+2\kappa^2}|s,0\rangle + \Delta_c|a,0\rangle]. \qquad (11)
\end{aligned}
$$

It is apparent that each of these eigenstates may result in a cavity photon decay, thereby producing a three-peaked cavity field spectrum. This, of course, is in stark contrast to the case of identical atoms where the eigenstate $|a,0\rangle$ does not participate in cavity decoherence events.

Let us now consider the spectrum of the fluorescence field, which is defined as

$$
S_F(\omega) = \mathcal{N}\int \langle(|e_A\rangle\langle g_A|_0 + \mathrm{e}^{i\theta}|e_B\rangle\langle g_B|_0)(|g_A\rangle\langle e_A|_\tau + \mathrm{e}^{-i\theta}|g_B\rangle\langle e_B|_\tau)\rangle_{ss}\mathrm{e}^{i\omega\tau}d\tau, \quad (12)
$$

where the subscript $t=0,\tau$ denotes the time dependence of the transition operators and the phase factor θ describes the relative distance from the observing point to the atoms [12]. If the observing point is equidistant from the two atoms then $\theta=0$, and $|e_A\rangle\langle g_A|_t + \mathrm{e}^{i\theta}|e_B\rangle\langle g_B|_t$ is the symmetric-state transition operator $\sigma_s^+(t)$. On the other hand, when the distances from each atom to the observing point differ by $c\pi/\omega_c$ (c: speed of light) then $\theta=\pi$, and $|e_A\rangle\langle g_A|_t + \mathrm{e}^{i\theta}|e_B\rangle\langle g_B|_t$ is the antisymmetric-state transition operator $\sigma_a^+(t)$. Depending on the observing point, the spectrum is thus related to different components of the atomic dynamics. Specifically, we are able to separate spontaneous emission events $|a,0\rangle \to |g_A,g_B,0\rangle$ from $|s,0\rangle \to |g_A,g_B,0\rangle$ by choosing an appropriate position for the measurement.

In Figs. 3a and 3b the spectra for the fluorescence fields have been plotted for the same parameters as in Fig. 1 for positions corresponding to $\theta=0$ and π. When $\theta=0$, only spontaneous emission processes from the symmetric Dicke states will contribute to the spectrum due to a destructive interference of the possible decay paths from the antisymmetric states. Consequently, we may deduce from Eq. (11) that a two-peaked spectrum will result as only the eigenstates $|\omega_\pm\rangle$ contain symmetric-state components. Similarly, a two-peaked spectrum will result when $\Delta_c=0$; that is, the case of identical atoms. The two peaks arising from unidentical atoms will, nevertheless, be shifted from those arising from identical atoms in a similar manner to that found in the cavity field spectrum. In Fig. 3b, we plot the fluorescence spectra arising at $\theta=\pi$. At such positions we are exclusively measuring decay processes arising from the antisymmetric states. As the three singly-excited eigenstates arising from unidentical atoms each contain an antisymmetric component we might expect the spectrum at $\theta=\pi$ to comprise three peaks occurring at the frequencies ω_o and $\omega_\pm$. For identical atoms, however, only a single peak at the frequency ω_o will occur as shown in Fig. 3b. Thus, further qualitative

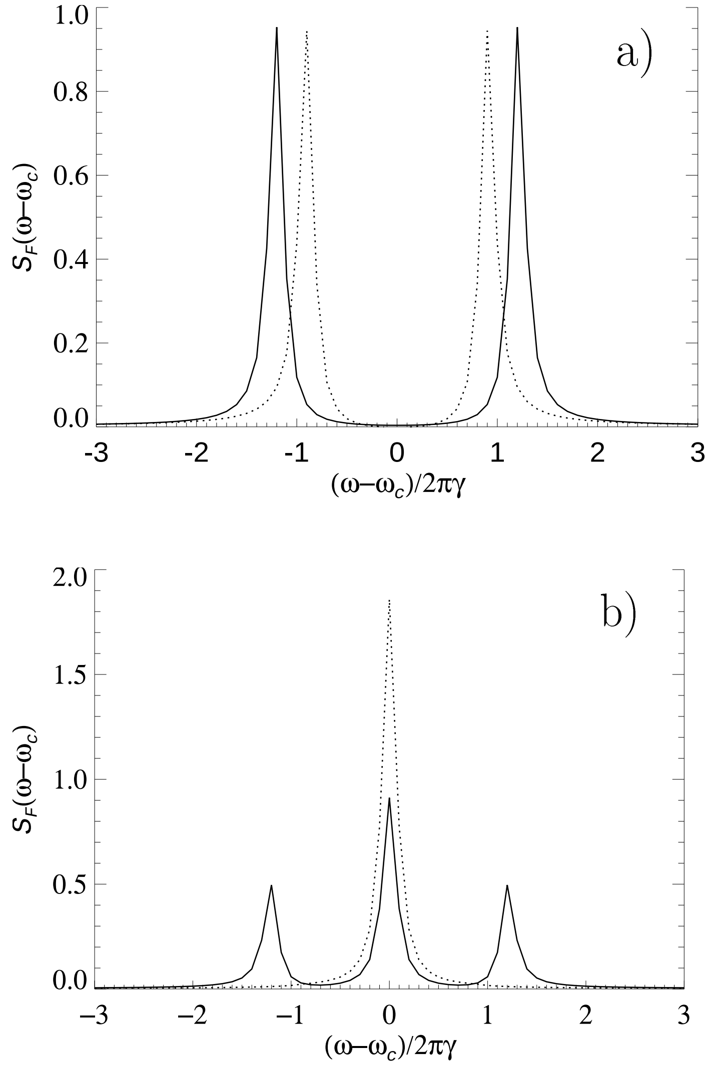

Fig. 3. Spectra for the fluorescence fields. The atomic decay rate $\gamma = \gamma_f$. The atom-field coupling $\kappa = 4\gamma_f$. The incoherent pump rate $p = 0.02\gamma_f$. Phase factors $\theta = 0$ (a) and $\theta = \pi$ (b). The solid lines are for two identical atoms with $\Delta_c = \Delta = 0$. The dotted lines are for two unidentical atoms with $\Delta = 0$ and $\Delta_c = 5$.

signatures of the cavity-induced atomic correlations are evident in the measured spectral profiles when the two atoms are unidentical.

4. Remarks

Previously, a number of proposals have been made for the observation of cavity-induced correlations in the fluorescence field. In particular, quantum statistical properties such as the fringe visibility [4,6], and position-dependent correlation functions [11] have been shown to exhibit qualitative signatures of atom-atom correlations. However, such proposals are limited in their applicability due to the difficulties of detecting a weak field which is distributed over the entire 4π solid angle. A more suitable candidate for the experimental verification of mutual atomic coherence would be a system whose quantum statistics could be unambiguously detected in the cavity field.

In this paper we have studied continued the search for experimentally measurable signatures of cavity-induced atomic coherences. Specifically, we have calculated the spectral profiles of the cavity output and fluorescence fields for the case of two unidentical atoms which are coupled to a single-mode resonator field and incoherently excited using broadband radiation. We have established that the cavity-induced mutual atomic coherence is manifest in both the cavity *and* fluorescence spectra. This result is of particular significance as for the first time a qualitative signature of atom-atom correlations is predicted in the cavity field spectrum, thereby easing the experimental realization of cavity-induced atomic correlations.

For the sake of a clear analysis we have taken very weak excitation of the atom-cavity field system. However when the excitation is too weak, the fluorescence and cavity fields are also too weak to evade from detection. We have checked that the discussions in this paper do not change much even the pump is as strong as $p = 0.2\gamma_f$, in which case the mean-photon number of the cavity is 0.1.

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