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DISENTANGLEMENT-PRESERVING STATES IN MICROMASER [1]

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We consider micromaser fields which after interaction with one atom produce disentangled atom-field states. We find a special solution for which interaction with the atom has the effect of flipping the sign of the electric field. We also consider the general case and derive conditions which the field must satisfy. An example of the general solution is presented in the case that there is a trapping state at $n = 6$.

1. Introduction

While the dynamics of a micromaser with atoms injected in their upper states has been studied extensively, the situation in which the atoms enter the cavity in a coherent superposition of their upper and lower states has not. The major contribution in this area was made by Slosser and Meystre who found field states, which they called tangent and cotangent states, which are preserved when at atom traverses the cavity [1, 2]. In the absence of damping tangent and cotangent states are steady states of the micromaser field. These states can only exist if the micromaser has trapping states which separate the photon Fock space into noninteracting blocks. It is possible to create period-two steady states by putting tangent and cotangent states in adjacent blocks [2, 3].

In a somewhat different vein Julio Gea-Banacloche has considered a single two-level atom interacting with a single-mode cavity field, the Jaynes-Cummings model, and found atom-field states which remain approximately in product form for long periods of time [4]. That is, these states, which he called quasiclassical states, are initially products of atom and field states, and, even though the state changes with time, it remains, to good approximation, the product of an atomic and a field state. The atomic states in the quasiclassical states are, it should be noted, coherent superpositions of the upper

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and lower states. Because of their simple time evolution these states are useful in understanding and clarifying the dynamics of the Jaynes-Cummings model.

In this paper we combine some of the aspects of both of these perspectives. We consider a micromaser cavity with an initial field state and an atom in a coherent superposition state. We are interested in finding field states which, after interacting with one atom, yield a total atom-field state which is a product. Clearly the tangent and cotangent states are special cases of these states and have the additional property that the field state is unchanged by the passage of the atom. For other states of this type the field state will be different before and after interaction with the atom. One special case, which will be discussed in Section 2, is a state which has the sign of the expectation value of its electric field flipped by the atom. Other examples are discussed in Section 3.

2. Solution causing phase change of the cavity field

We begin by briefly reviewing some aspects of the Jaynes-Cummings model. The atom has states $|a\rangle$, with energy ω (we are using units with $\hbar = 1$), and $|b\rangle$, with energy 0. The Hamiltonian describing the atom-field system is

$$H = \omega a^{\dagger} a + \frac{1}{2}\omega(\sigma_3 + I) + g(a^{\dagger}\sigma^{-} + a\sigma^{+}). \tag{1}$$

If the atom is initially injected in the state

$$|\Psi_{\mathrm{at}}\rangle = \alpha|a\rangle + \beta|b\rangle, \tag{2}$$

and the field is initially in the state

$$|f\rangle = \sum_{n=0}^{\infty} d_n|n\rangle, \tag{3}$$

then after a time τ the state of the combined system will be

$$\begin{aligned} |f\rangle \otimes (\alpha|a\rangle + \beta|b\rangle) \quad \rightarrow \quad & \sum_{n=0}^{\infty} d_n(\alpha c_{n+1}|n\rangle - \mathrm{i}\beta s_n|n-1\rangle)|a\rangle \\ & + \sum_{n=0}^{\infty} d_n(\beta c_n|n\rangle - \mathrm{i}\alpha s_{n+1}|n+1\rangle)|b\rangle, \end{aligned} \tag{4}$$

where $s_n = \sin(g\tau\sqrt{n})$ and $c_n = \cos(g\tau\sqrt{n})$. Note that this state is in general an entangled state of the atom and the field. In order for it to be a product the coefficients d_n must satisfy some special conditions.

Rather than find the most general conditions which the d_n should satisfy, let us first look at a special case. Taking a hint from the tangent and cotangent states let us try to find field states which are rotated in phase space by their interaction with the atom (the tangent and cotangent states are rotated by an angle of zero). Note that the effect of such an interaction is to conserve the magnitude of d_n but to change its phase. Therefore, we want to find cavity field states, $|f\rangle$, such that

$$|f\rangle \otimes (\alpha|a\rangle + \beta|b\rangle) \to \mathrm{e}^{-\mathrm{i}\theta\hat{n}}|f\rangle \otimes (\alpha'|a\rangle + \beta'|b\rangle), \tag{5}$$

where $\hat{n}$ denotes the number operator $a^\dagger a$. If we employ Eq. (4) on the left hand side of Eq. (5) and then equate the coefficients of the vectors $|a\rangle$ and $|b\rangle$ separately, we obtain two reccurence relations for coefficients d_n of the cavity field

$$d_{n+1} = \mathrm{i}\frac{\alpha'\mathrm{e}^{-\mathrm{i}\theta n} - \alpha c_{n+1}}{\beta s_{n+1}} d_n \tag{6}$$

$$d_{n+1} = \mathrm{i}\frac{\alpha s_{n+1}}{\beta c_{n+1} - \beta'\mathrm{e}^{-\mathrm{i}\theta(n+1)}} d_n. \tag{7}$$

These relations must be the same for any n in order to satisfy Eq. (5) which gives in turn

$$\alpha'\beta\mathrm{e}^{-\mathrm{i}\theta n}c_{n+1} - \alpha'\beta'\mathrm{e}^{-\mathrm{i}\theta(2n+1)} + \alpha\beta'\mathrm{e}^{-\mathrm{i}\theta(n+1)}c_{n+1} - \alpha\beta = 0. \tag{8}$$

As one can see Eq. (8) is fulfilled for any n only if $\theta = 0$ or $\theta = \pi$. Let us take a closer look at the cavity fields in these two cases.

2.1. Solutions corresponding to $\theta = 0$.

Substituting $\theta = 0$ into Eq. (8) we find

$$\alpha'\beta c_{n+1} - \alpha'\beta' + \alpha\beta' c_{n+1} - \alpha\beta = 0, \tag{9}$$

which can be solved for α' and β' in two ways, each providing a different cavity field.

The first solution of Eq. (9) is $\alpha' = \alpha$ and $\beta' = -\beta$. The reccurence relations in Eqs. (6) and (7) then yield the well known tangent state

$$d_{n+1} = \mathrm{i}\frac{\alpha}{\beta}\tan\left(\frac{g\tau\sqrt{n+1}}{2}\right) d_n. \tag{10}$$

For the second solution we find that $\alpha' = -\alpha$ and $\beta' = \beta$ which after using the reccurence relations gives us the cotangent state

$$d_{n+1} = -\mathrm{i}\frac{\alpha}{\beta}\cot\left(\frac{g\tau\sqrt{n+1}}{2}\right) d_n. \tag{11}$$

The cotangent and tangent states have been studied thoroughly and their properties are well understood [1, 2]. We merely note that they can exist only when trapping states are present and the parity of the trapping states determines which of the two kinds of states is physically possible in a given subregion of Fock space. Later we provide a specific example which illustrates the case.

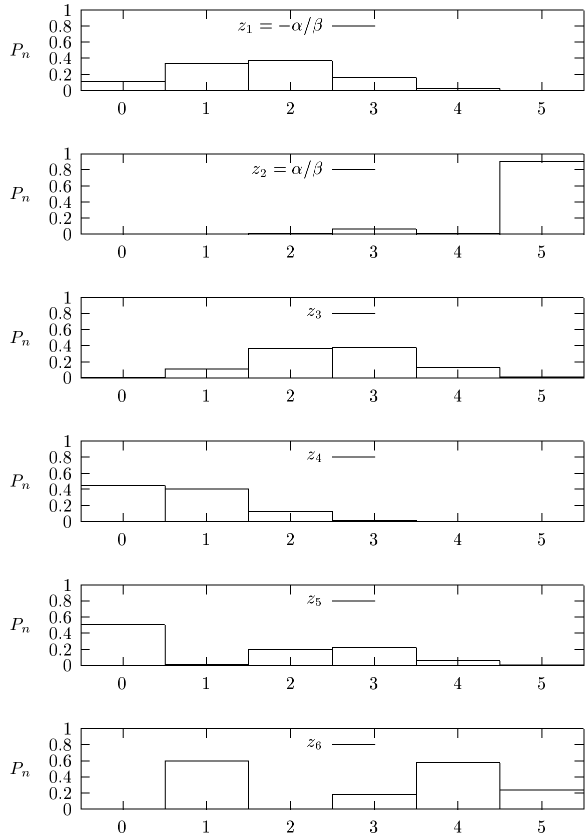

Fig. 1. Initial states. Probability $P_n = d_n^* d_n$ versus number state n is plotted for the case $\alpha = 0.8$, $\beta = 0.6$ and $g = 4.40 \times 10^4 Hz$ with a π trapping state at $|6\rangle$.

2.2. Solutions corresponding to $\theta = \pi$.

Substituting $\theta = \pi$ into Eq. (8) we find

$$\alpha' \mathrm{e}^{-\mathrm{i}\pi n} \beta c_{n+1} - \alpha'\beta' + \alpha\beta' \mathrm{e}^{-\mathrm{i}\pi n} c_{n+1} - \alpha\beta = 0 \tag{12}$$

which we can again solve in two different ways.

If $\alpha' = \alpha$ and $\beta' = \beta$ the reccurence relations give

$$d_{n+1} = \mathrm{i}\frac{\alpha}{\beta} \frac{(-1)^n - c_{n+1}}{s_{n+1}} d_n, \tag{13}$$

which takes two different forms depending on the parity of n

i) for even n we have

$$d_{n+1} = \mathrm{i}\frac{\alpha}{\beta} \tan\left(\frac{g\tau\sqrt{n+1}}{2}\right) d_n \tag{14}$$

ii) for odd n we get

$$d_{n+1} = -\mathrm{i}\frac{\alpha}{\beta}\cot\left(\frac{g\tau\sqrt{n+1}}{2}\right)d_n. \tag{15}$$

If $\alpha' = -\alpha$ and $\beta' = -\beta$ we have

$$d_{n+1} = -\mathrm{i}\frac{\alpha}{\beta}\frac{(-1)^n + c_{n+1}}{s_{n+1}}d_n \tag{16}$$

which also takes two different forms depending on the parity of n

i) for even n it reads

$$d_{n+1} = -\mathrm{i}\frac{\alpha}{\beta}\cot\left(\frac{g\tau\sqrt{n+1}}{2}\right)d_n \tag{17}$$

ii) for odd n it reads

$$d_{n+1} = \mathrm{i}\frac{\alpha}{\beta}\tan\left(\frac{g\tau\sqrt{n+1}}{2}\right)d_n. \tag{18}$$

It is clear from the analytic properties of the cotangent and tangent functions that these states are normalizeable only in the presence of trapping states. The position and the parity of the trapping states are crucial in determining the physical existence of the solutions. In particular, if we want the solution to start at $n = 0$, then we find the second solution is ruled out. A close examination reveals that if Eq. (5) is satisfied when $\alpha = -\alpha'$, $\beta = -\beta'$, and $n = 0$, then we must have that $-d_0 = c_0 d_0 = d_0$. This implies that $d_0 = 0$, which means that the entire solution vanishes.

The effect of the interaction with an atom on these field states is simply to multiply the field expansion coefficients, d_n, by $(-1)^n$. This has the effect of flipping the sign of the expectation value of any operator, such as the annihilation operator, which only has nonzero matrix elements between successive number states. In particular, this will happen to the electric field (it is proportional to $a + a^\dagger$). Note that this is true even when the field state has a large number of photons, so that interaction with a single atom could have a macroscopic effect.

Another interesting feature of these solutions shows up if we suppose that the field interacts with atoms which alternate their states between $\alpha|a\rangle + \beta|b\rangle$ and $\alpha|a\rangle - \beta|b\rangle$. Then the field $|f''\rangle$ after two interactions is given by $|f''\rangle = \sum_n (-1)^n d'_n |n\rangle$, where d'_n is the field component left after the first atom. This means that after two atoms the field has returned to its original state, and that the sign of the electric field will flip back and forth as the atoms are injected.

3. General solution

Here we shall again seek states of the micromaser which yield output states which can be written as product of field and atomic states,

$$|f\rangle \otimes (\alpha|a\rangle + \beta|b\rangle) \to |f'\rangle \otimes (\alpha'|a\rangle + \beta'|b\rangle). \tag{19}$$

However, we will now not limit ourselves to the special case considered in the previous section, but will seek to find a general solution.

The right hand side of Eq. (19) can be expressed using the Eq. (4) giving

$$\begin{aligned}\sum_{n=0}^{\infty} d'_n(\alpha'|a\rangle + \beta'|b\rangle)|n\rangle \quad = \quad & \sum_{n=0}^{\infty} \{(\alpha d_n c_{n+1} - \mathrm{i}\beta d_{n+1} s_{n+1})|a\rangle + \\ & (\beta d_n c_n - \mathrm{i}\alpha d_{n-1} s_n)|b\rangle\}\,|n\rangle. \end{aligned} \tag{20}$$

This equation will be satisfied if the field state multiplying the atomic state $|a\rangle$ is the same as that multiplying the atomic state $|b\rangle$ up to a constant factor, i. e. if

$$\alpha d_n c_{n+1} - \mathrm{i}\beta d_{n+1} s_{n+1} = z(\beta d_n c_n - \mathrm{i}\alpha d_{n_1} s_n), \tag{21}$$

where z is a complex number. If there exists a z for which the Eq. (21) is satisfied for all n then our objective is met. The values of z can, in fact, be found from the system of linear homogeneous equations which result from Eq. (21) for all different values of n. A nontrivial solution for the field components d_n exists only when the determinant of the matrix of the system is equal to zero

$$\begin{vmatrix} z\beta c_0 - \alpha c_1 & \mathrm{i}\beta s_1 & 0 & 0 & 0 & \dots \\ -\mathrm{i}z\alpha s_1 & z\beta c_1 - \alpha c_2 & \mathrm{i}\beta s_2 & 0 & 0 & \dots \\ 0 & -\mathrm{i}z\alpha s_2 & z\beta c_2 - \alpha c_3 & \mathrm{i}\beta s_3 & 0 & \dots \\ 0 & 0 & -\mathrm{i}z\alpha s_3 & z\beta c_3 - \alpha c_4 & \mathrm{i}\beta s_4 & \dots \\ & & \dots & & & \end{vmatrix} = 0. \tag{22}$$

Eq. (22) then, determines the values of the variable z. Once z is known it can be substituted into the Eq. (21) which can then be used to find the components of the initial cavity field. The most difficult part of the problem is solving Eq. (22); it is very hard to find analytical solutions for a general order of the determinant. We shall discuss a simplified case having only a limited number of non-zero field components.

4. An example

As an example we chose to have a π trapping state at $|6\rangle$ i.e. the atom-field interaction time τ satisfies $g\tau\sqrt{6} = \pi$, and we limited nonzero field components to those between states $|0\rangle$ and $|5\rangle$. We assumed in our numerical simulations that $\alpha = 0.8$, $\beta = 0.6$ and $g = 4.40 \times 10^4 Hz$. The trapping condition then implies that $\tau = 2.91 \times 10^{-5} s$. We found the general solutions of which there are six. Among them, two correspond to the case considered in Section 2.

We first solved Eq. (22) numerically, and then Eq. (21) was employed to find the field components $d_0, d_1, \dots, d_5$ for each value of the parameter z. There are six different values of z, two of them yielding cavity fields that were identified as solutions of Eq. (5). The first one corresponds to $z_1 = -\alpha/\beta$ and is the cotangent state given by Eq. (11). The second corresponds to $z_2 = \alpha/\beta$, and is identical to the state defined by Eq. (13). The probability $P_n = d^*_n d_n$ versus photon number, n, of the resulting fields has been plotted in Fig. 1.

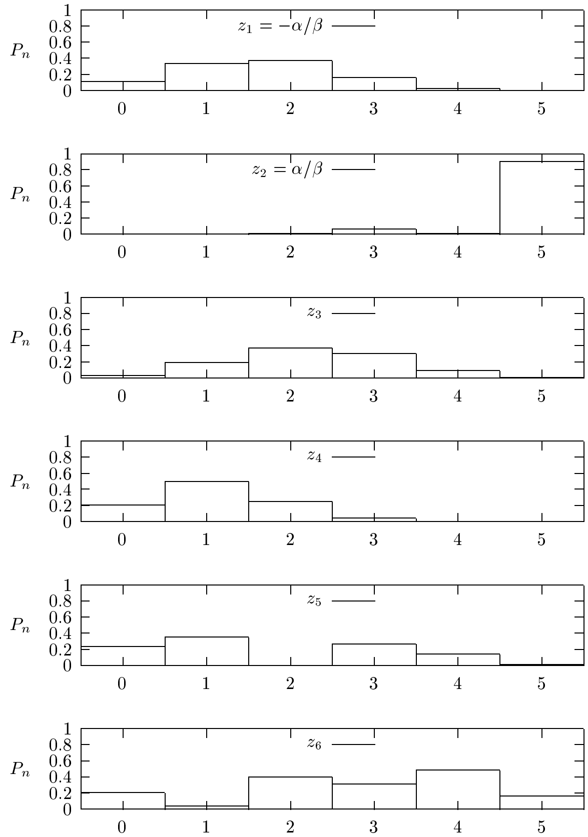

Fig. 2. Final states. Probability $P_n = d_n^* d_n$ versus number state n is plotted for the same parameters as in Fig. 1. Both states corresponding to z_1 and z_2 are left unchanged whereas that is not true for the others.

The final field states were also calculated, i.e. we let the initial states interact with one atom during a time interval τ. Probabilities P_n versus n of the cavity field states are shown in Fig. 2. As one would expect the magnitudes of the two fields corresponding to z_1 and z_2 are left unchanged since only the phase of the field changes according Eq. (5). If we compare the fields belonging to $z_3, \ldots, z_6$ to those in Fig. 1, we see that the interaction with the atom does change these fields. Clearly, the solutions of Eq. (5) are a subset of the more general set which satisfy Eq. (19).

5. Conclusions

We have studied states of the micromaser field which, upon interaction with one atom yield atom-field states which are disentangled. This means that when the atom leaves the cavity the field is in a pure rather than a mixed state. There is a large set of such states, and we have given a general method for finding them as well several explicit examples.

There are other questions which can be raised in regard to these states. Can one design a sequence of atoms which will cause the micromaser to cycle among these states, always leaving the cavity field in a pure state between atoms? Can one find states which are disentangled after the passage of two atoms? We plan to examine some of these issues in the future.

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