acta physica slovaca vol. 48 No. 5, 583 – 594 October 1998

SUPERPOSITIONS OF DISPLACED FOCK STATES: PROPERTIES AND GENERATION

A.-S.F.Obada[1], G.M. Abd Al-Kader
Mathematics Department Faculty of Science Al-Azhar University, Nasr City, Cairo, Egypt.

Received 19 May 1998, in final form 4 August 1998, accepted 18 August 1998

The superposition of displaced Fock states (DFS's) is discussed and various moments are calculated. The s-ordered quasiprobability distribution function (QDF) for the superposition of DFS's is investigated. The s-parameterized charactristic function (CF) for the superposition of two DFS's are considered. The Glauber second-oreder coherence function is calculated. The squeezing properties of the superposition of DFS's are studied. Analytical and numerical results for the quadrature component distributions are presented. A generation scheme is discussed.

1 Introduction

There are two kinds of states that play a fundamental role in quantum optics of a single boson mode: the Fock (number) state and the coherent state [1]. The number state $|n\rangle$ is determined by its photon number while the phase is completely random. The amplitude of the field has a zero expectation value in this state. On the other hand for the coherent state $|\alpha\rangle$ the phase is determined and the amplitude of the field has a non-zero value. Experiments have been performed to prepare the Fock states and coherent states through motional dynamics of the centre of mass of trapped ions [2]. Displaced Fock states (DFS's) $|\alpha, n\rangle$ are also important states in quantum optics [3, 4], defined by the action of the displacement operator $D(\alpha)$ on the number state $|n \neq 0\rangle$. In one sense the DFS's is obtained from a number state by adding a nonzero value to the field amplitude [4]. By displacing in phase space, a field amplitude is added to this state, and the photon number has now a contribution from the coherent component of the field. They become phase dependent because of the phase of the displacement operator. They can be regarded as a generalized class of the Fock and coherent states. They may form a complete basis, and have interesting and unusual physical properties [4, 5]. The experimental observtion of the DFS's is still to be done.

The DFS, $|\alpha, n>$, is defined by

$$|\alpha, n\rangle = D(\alpha)|n\rangle \tag{1.1}$$

[1] E-mail address: Obada@frcu.eun.eg

0323-0465/98

with $D(\alpha)$ the displacement operator, given by [1]

$$D(\alpha) = \exp(\alpha a^+ - \alpha^* a), \qquad \alpha = |\alpha| e^{i\theta} \tag{1.2}$$

where a (a^+) is the annihilation (creation) operator of the boson field.
The scalar product $\langle \beta, m | \alpha, n \rangle$ is given by, [4]

$$\langle \beta, m|\alpha, n\rangle = \begin{cases} \langle\beta|\alpha\rangle\sqrt{\frac{n!}{m!}}(\alpha-\beta)^{m-n}L_n^{m-n}(|\alpha-\beta|^2), & m>n \\ \langle\beta|\alpha\rangle\sqrt{\frac{m!}{n!}}(\beta^*-\alpha^*)^{n-m}L_m^{n-m}(|\alpha-\beta|^2), & n>m \end{cases} \tag{1.3}$$

where the scalar product of two coherent states has the well known value $\langle\beta|\alpha\rangle = \exp[-\frac{1}{2}(|\alpha|^2 + |\beta|^2) + \alpha\beta^*]$, and $L_m^\sigma(x)$ is the Laguerre polynomial

$$L_m^\sigma(x) = \sum_{s=0}^{m} \binom{m+\sigma}{m-s} \frac{(-x)^s}{s!} \tag{1.4}$$

The generation of nonclassical states of light is at the heart of quantum optics. In particular the superpositions of quantum states [6], are also considered as an important type of nonclassical states. These states are of particular interest because they possess various nonclassical properties, such as squeezing and sub-Poissonian statistics.Non-classical properties of states generated by the excitations of even and odd coherent states of light have been studied in ref. [9]. Also generating Schödinger-cat-like states by means of conditional measurements on a beam splitter have been proposed in [10]. It may then be of interest to consider superpoisitions of the DFS's to see if there is any enhancement of some nonclassical properties. Previously, two kinds of superpositions of DFS's, i.e., $|\psi_1>=\frac{1}{\sqrt{2}}(|\alpha,n>+|\alpha,k>)$ and $|\psi_2>=N(|\alpha,n>+|-\alpha,n>)$ with N the normalization constant, have been studied [7]. The generation of special superpositions of the DFS's via the driven Jaynes-Cummings model have been discussed [8].

In this paper we study another type of superposition of the DFS's.

This paper is organized as follows. In section 2 we discuss the construction and properties of superposition of DFS's and illustrate the photon number probability distributions. In section 3 we discuss the statistical properties of superposition of a pair of DFS's. We discuss the s-parameterized quasi-probability distribution function. We study some applications for the s-ordered characteristic function: namely; moments and squeezing. Also we discuss the quadrature component distributions for these states. Finally in section 4 we present a generatation scheme for these states.

2 Superposition of DFS's

We consider a superposition of these states in the form

$$|\Psi_k\rangle = A_k \sum_{j=0}^{k-1} C_j |\alpha_j, n_j\rangle \tag{2.1a}$$

where C_j are constants, with A_k as the normalization constant given by

$$|A_k|^{-2} = \sum_{j,l=0}^{k-1} C_j C_l^* \langle \alpha_l, n_l | \alpha_j, n_j \rangle \tag{2.1b}$$

It is interesting to see the connection between this superposition and a similar state defined by Agrawal and Tara [4d]. There they define the state

$$|\Lambda\rangle = \frac{1}{N} a^{+n} |\beta\rangle, \qquad \text{with} \qquad N = \sqrt{\langle\beta|a^n a^{+n}|\beta\rangle} \tag{2.2}$$

after little algebra, we get

$$|\Lambda\rangle = \frac{1}{N}\sum_{j=0}^{n} \binom{n}{j} \beta^{*n-j}\sqrt{j!}|\beta, j\rangle \tag{2.3}$$

which is a special case of eqn.(2.1) when $\alpha_j = \beta$ We first obtain the photon statistics for the states of equation (2.1). To begin we set

$$|\Psi_k\rangle = \sum_{m=0}^{\infty} \mathbf{a}_m(k)|m\rangle \tag{2.4}$$

where

$$\mathbf{a}_m(k) = |A_k| \sum_{j=0}^{k-1} C_j \langle m|\alpha_j, n_j\rangle \tag{2.5}$$

The expectation value in this expression can be calculated from (1.3) when we let $\beta = 0$.
The photon number distribution $P(l)$ is

$$\begin{aligned} P(l) &= |\langle l|\Psi_k\rangle|^2 \\ &= |\mathbf{a}_l(k)|^2 \end{aligned} \tag{2.6}$$

Numerical calculations, for $k = 2$, show that these distributions oscillate and as l becomes large the distributions are damped. As n_j increase the distributions are still oscillatory. We note that fast oscillations appear in $P(l)$ with decreasing of α_0.

We now examine the quadrature squeezing properties of $|\Psi_k\rangle$. The quadrature components can be expressed in terms of creation and annihilation operators as follows:

$$X_1 = \frac{1}{2}(a + a^+), \qquad X_2 = \frac{1}{2i}(a - a^+) \tag{2.7}$$

To calculate the moments of the quadratures in our state, one has to find average values of products of the operators a and a^+ in these states, on the form (dropping the surperscript k)

$$< a^{+p} a^q > = \sum_{s,r=0}^{\infty} \mathbf{a}_r \mathbf{a}^*_s \sqrt{\frac{r!}{(r-q)!}}\sqrt{\frac{s!}{(s-p)!}}\delta_{s-p,r-q} \tag{2.8}$$

As is well known squeezing is said to exist whenever $(\Delta X_j)^2 < \frac{1}{4}$ for (j=1 or 2), with the variance $\langle(\Delta X_j)^2\rangle = \langle X_j^2\rangle - \langle X_j\rangle^2$.

The squeezing is best parameterized by the q parameter defined as

$$q_i = \frac{\langle(\Delta X_i)^2\rangle - 0.25}{0.25}, \qquad i = 1,2 \tag{2.9}$$

such that squeezing exits for $-1 < q_i < 0$.

Next we consider a phase space quasiprobability distribution functions (QDF's) for our states. We study the s-parameterized QDF which is given in ref. [5]. For the density operator $\rho = |\Psi_k><\Psi_k|$ the s-parameterized QDF has the form:

$$F(\beta, s) = \frac{2}{\pi}\sum_{j=0}^{\infty}(-1)^j \frac{(1+s)^j}{(1-s)^{j+1}}|<\beta, j|\Psi_k>|^2 \tag{2.10}$$

The general representation function $F(\beta, s)$ that may be identified with the weight functions $Q(\beta)$ Husimi function ,$W(\beta)$ Wigner function and $P(\beta)$ the Glauber-Sudershan function when the order parameter assumes the values $s = -1, 0, +1$ respectively.

3 Superposition of a pair of DFS's

In the previous section we have derived a general expression for the s-parameterized QDF of superpositions of DFS's (2.1). To shed some light on eq.(2.1), we take the case of the superposition of two DFS's $\{k = 2\}$, namely

$$|\Psi_2\rangle = A_2\{C_0|\alpha_0, n_0\rangle + C_1|\alpha_1, n_1\rangle\} \tag{3.1}$$

we choose $C_0 = 1$, $C_1 = \exp(i\psi)$ with ψ as a parameter and $alpha_1 = -\alpha_0$. We take different number of photons in the two DFS's of eq.(3.1). We choose $n_0 > n_1$ in our analysis, which is the main difference from the work in ref. [7, 9]. Note that the resulting states (odd or even states) depend on n_0 and n_1, the normalization constant A_2 is obtained from (2.1b).

The s-ordered characteristic function (CF) $C(\lambda, s)$ is defined by [14], $C(\lambda, s) = Tr[\rho D(\lambda)]\exp(\frac{s}{2}|\lambda|^2)$ with $D(\lambda)$ given by eqn.(1.2). The density operator corresponding to the state (3.1) is $\rho = |\Psi_2><\Psi_2|$. By using the operator identities, we can write the s-ordered CF in the form,

$$\begin{aligned} C(\lambda, s) &= |A_2|^2 \exp[\frac{s}{2}|\lambda|^2]\Big\{\big[\exp\{-\frac{1}{2}|\lambda|^2 + \alpha_0^*\lambda - \alpha_0\lambda^*\}L_{n_0}[|\lambda|^2]\big] \\ &+ \exp[-\frac{1}{2}|\lambda|^2 - \alpha_0^*\lambda + \alpha_0\lambda^*]L_{n_1}[|\lambda|^2] + \exp(i\psi)\sqrt{\frac{n_1}{n_0}}\exp\{-\frac{1}{2}|\lambda - 2\alpha_0|^2\} \\ &\times (\lambda - 2\alpha_0)^{n_0-n_1}L_{n_1}^{n_0-n_1}[|\lambda - 2\alpha_0|^2] + \exp(-i\psi)\sqrt{\frac{n_1}{n_0}}\exp\{-\frac{1}{2}|\lambda + 2\alpha_0|^2\} \\ &\times [-(\lambda + 2\alpha_0)^*]^{n_0-n_1}L_{n_1}^{n_0-n_1}[|\lambda + 2\alpha_0|^2]\Big\} \end{aligned} \tag{3.2}$$

Thus the s-ordered CF is obtained; and from it we can calculate any expectation value for the field operators.

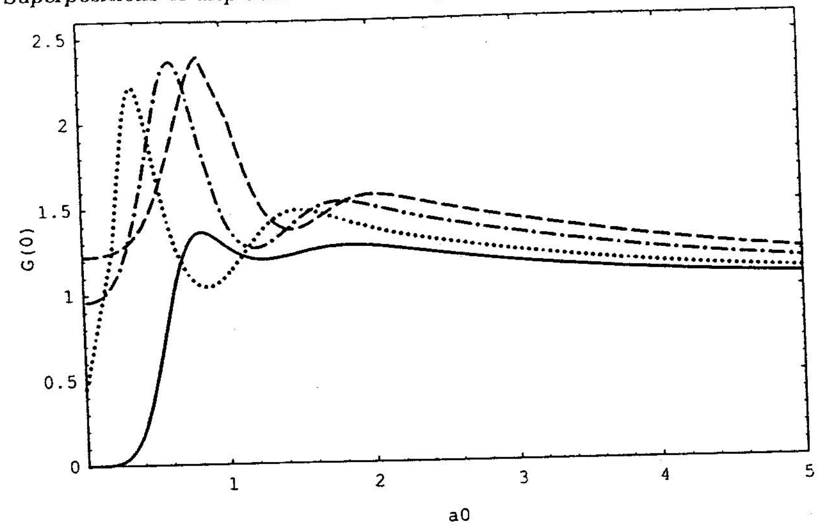

Fig. 1. Coherence function $g^{(2)}$ against the displacement parameter α_0, , for $\psi = 0$. The number of photons has the values: $n_1 = 1$, $n_0 = 1$ (solid curve), $n_1 = 1$, $n_0 = 2$ (dotted curve) $n_1 = 1$, $n_0 = 4$ (chained curve) and $n_1 = 1$, $n_0 = 6$ (dashed curve).

3.1 Photon statistics

Here we focus our attention on the autocorrelation function $g^{(2)}$ defined by Glauber as follows:

$$g^{(2)} = \frac{\langle a^{+2}a^2\rangle}{\langle a^+a\rangle^2}.$$

We plot the autocorrelation function $g^{(2)}$ in Fig.1 against the displacement parameter α_0. We assume the parameters as follows: $n_1 = 1$ with $n_0 = 1, 2, 4, 6$, for $\psi = 0$. We note that sub-Poissonian light exists for $n_0 = n_1 = 1$ (solid curve) and $n_0 = 2$ with $n_1 = 1$. The super-thermal light exist only with $\alpha_0 < 1$. Also when the displacement parameter α_0 is increased the Poissonian behaviour is persistent. Numerical calculations show that super-Poissonian light exists for even $n_0 - n_1$. The super-thermal light exists only with $n_0 = n_1 = 0$.

3.2 Squeezing

Discussion of normal squeezing is given here through investigation of the parameter q_2.

In Fig.2 we plot q_2 against the displacement parameter α_0. We assume the parameters as follows: $n_1 = 0$ $n_0 = 0, 2, 4, 6$. We find that the maximum squeezing in the case of $n_0 = n_1 = 0$, i.e, coherent states. Increasing of n_j implies the demolishing of squeezing. Also the squeezing of q_1 does not exist. Thus as might be expected there is no second-order squeeing for these states for $n_j \geq 1$.

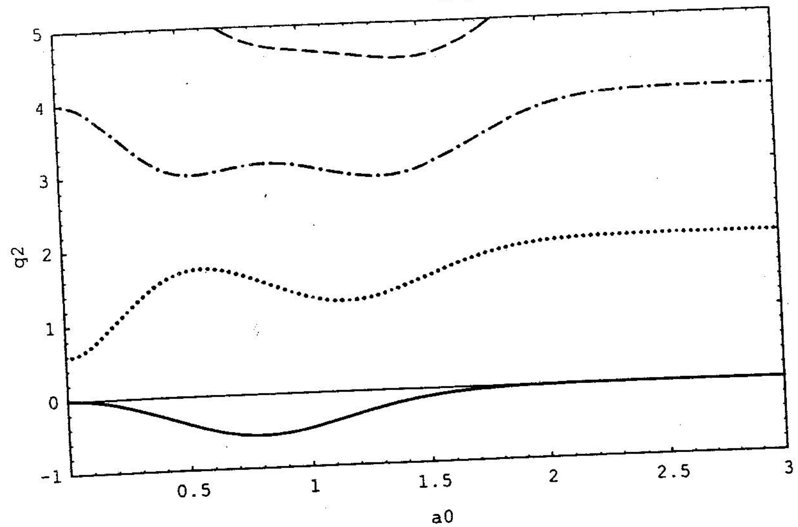

Fig. 2. Squeezing parameter q_2 against the displacement parameter α_0, , for $\psi = 0$. The number of photons has the values: $n_1 = 0$, $n_0 = 0$ (solid curve), $n_1 = 0$, $n_0 = 2$ (dotted curve) $n_1 = 0$, $n_0 = 4$ (chained curve) and $n_1 = 0$, $n_0 = 6$ (dashed curve).

3.3 s-ordered QDF

From eqn.(2.10) we choose to discuss the Wigner function ($s = 0$). Also the s-ordered QDF is defined as a Fourier transformation of the s-ordered CF [14].

The Wigner function for $n_0 = n_1 = 2$, $\alpha_0 = 3$ for $\psi = 0$ are shown in Fig.3a. From the plots, two separated peaks surrounded by almost circular contours and an oscillatory regime between them can be seen. The separation of the two peaks is seen to increase with increasing α_0, but the oscillatory regime increases with increasing n_j. It is observed that the plot is symmetric about $X = Re(\beta) = 0$, $Y = Im(\beta) = 0$.

In Fig.3b we plot the Wigner function with $n_0 = 2$, $n_1 = 1$, $\alpha_0 = 1$, for $\psi = 0$. Here we notice the asymmetry where we have a peak and a crater. However, it is symmetric about $X = Re(\beta) = 0$.

3.4 Quadrature distributions

In order to calculate the quadrature component distribution for the superposition state (i.e., the phase-parameterized field strength distribution) we write

$$P(x, \Phi) = |\langle x, \Phi|\Psi_2\rangle|^2 \tag{3.3}$$

which can be measured in balanced homodyne detection [10]. We first expand the eigenstate $|x, \Phi\rangle$ of quadrature component

$$x(\Phi) = \frac{1}{\sqrt{2}}(e^{-i\Phi}a + e^{i\Phi}a^+) \tag{3.4}$$

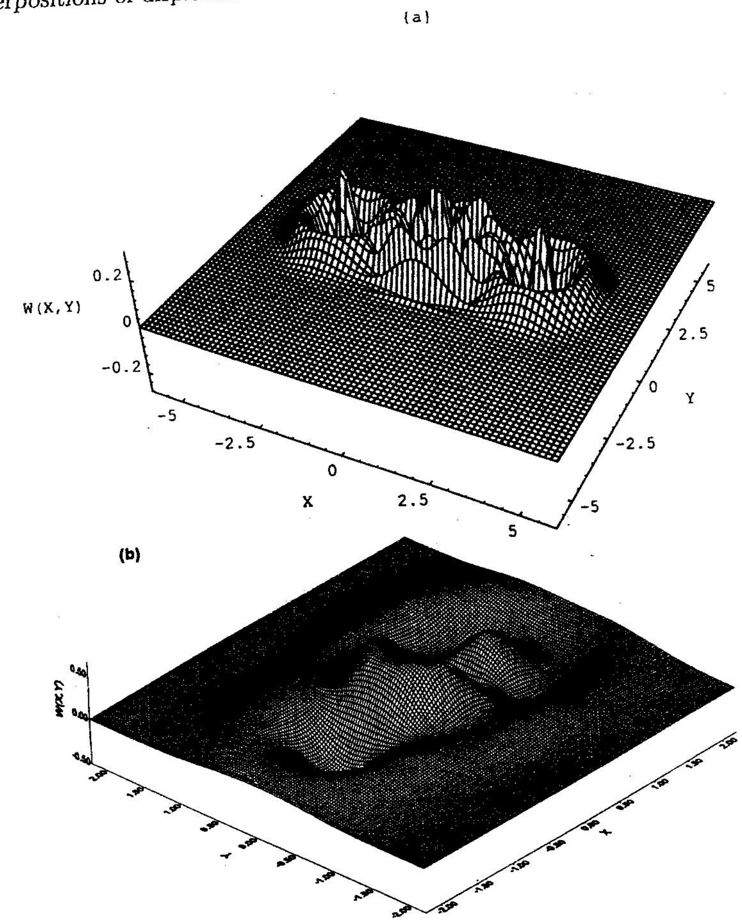

Fig.3. Wigner function (i.e., $s = -1$) the two DFS's superposition state, for $\psi = 0$, with; (a) $n_0 = n_1 = 2$, $\alpha_0 = 3$; (b) $n_0 = 2$, $n_1 = 1$, $\alpha_0 = 1$. Here $X = Re(\beta)$ and $Y = Im(\beta)$.

in the photon number basis as [10]

$$|x, \Phi\rangle = \frac{1}{\pi^{\frac{1}{4}}} \exp(-\frac{1}{2}x^2) \sum_{j=0}^{\infty} \frac{e^{i\Phi j}}{\sqrt{2^j j!}} H_j(x)|j\rangle \tag{3.5}$$

By using Eqns.(2.1) and (3.5) we have the quadrature component distribution (3.3) in the form

$$P(x, \Phi) = \frac{1}{\pi^{\frac{1}{2}}} \exp(-x^2) \sum_{j,l=0}^{\infty} \frac{\cos[\Phi(l-j)]}{\sqrt{2^{(l+j)} j! l!}} [\langle j|\Psi_2\rangle][\langle\Psi_2|l\rangle] H_j(x) H_l(x) \tag{3.6}$$

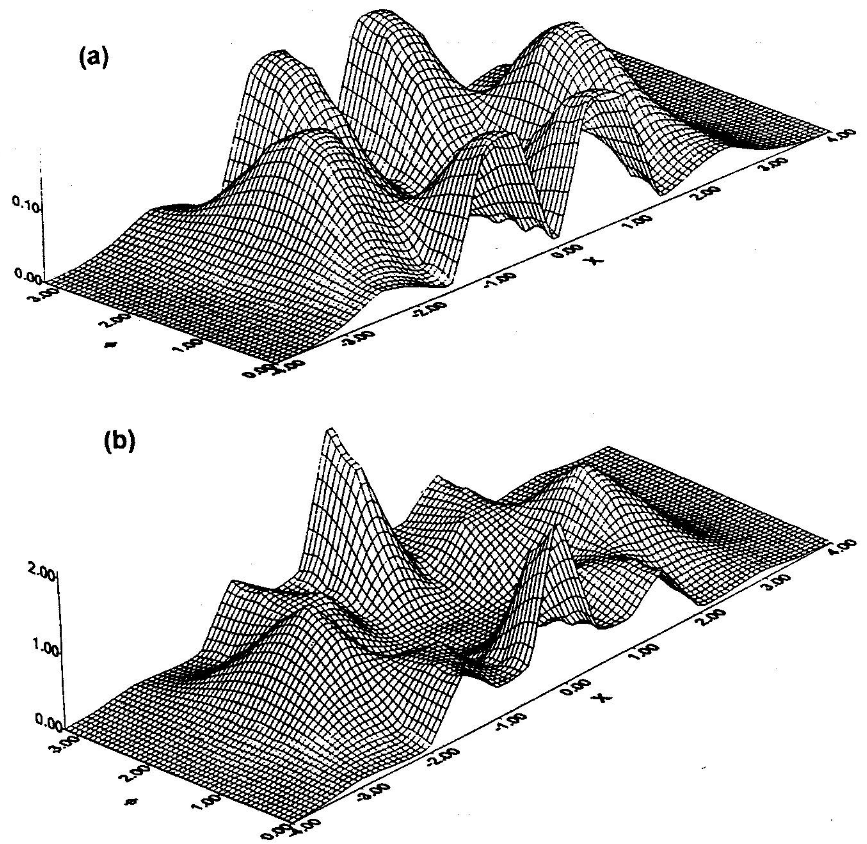

Fig. 4. Quadrature distribution $P(x, \Phi))$ of the pair of DFS's superpoition state $|\Psi\rangle$ with $\alpha_0 = 0.5$, $\psi = 0$. The number of photons are assumed as: (a) $n_0 = n_1 = 1$, (b) $n_0 = n_1 = 2$.

In Fig.4 we plot the phase-parameterized field strength distribution (quadrature component) distribution $P(x, \Phi)$ with (a) $n_0 = n_1 = 1$, $\alpha_0 = 0.5$, (b) $n_0 = n_1 = 2$, $\alpha_0 = 0.5$, for $\psi = 0$. In general the figures for $P(x, \Phi)$ are symmetric around $x = 0$ and $\Phi = \frac{\pi}{2}$.

In Fig.4a the middle two peaks diminish by the increase in Φ while the outer peaks build up for the range $[0, \frac{\pi}{2}]$. In Fig.4b. we see that the outer peaks diverge as Φ increases in $[0, \frac{\pi}{2}]$. For Φ near $\frac{\pi}{2}$ the quadrature component distribution $P(x, \Phi)$ exhibits two separated peaks or more, whereas for Φ close to 0 or π an interference pattern is observed. Numerical calculations show that for different values of ψ the features shown in these figures change.

4 Generation scheme

After the discussion of the properties of the superposition of the DFS's, we wish to consider the production of such superposition. Let a two-level ion of mass M move in a harmonic potential of frequency ω_x in the x-direction. Let $a(a^+)$ stand for the annihilation (creation) operator of the vibrational quanta in the x-direction. Then the position operator is given by $x = \Delta x_0(a + a^+)$ with $\Delta x_0 = (2\omega_x M)^{-\frac{1}{2}}$ the width of the harmonic ground state. In this scheme 2 beams of lasers applied along the x-axis are required to manipulate the motion of the atom: they are detuned by $\pm\omega_x$. In the rotating wave approximation the hamiltonian for this system is given by

$$\mathbf{H} = \omega_x a^+ a + \frac{\omega_0}{2}\sigma_z - (\mu E^-(x,t)\sigma_- + h.c) \tag{4.1}$$

The frist two terms describe the external and internal free motion of the ion and the last term stands for the atom-field interaction. The dipole matrix element μ and the transition frequency ω_0 of the 2-level ion, and the operators $\sigma_z = |e\rangle\langle e| - |g\rangle\langle g|$, $\sigma_+ = |e\rangle\langle g|$, $\sigma_- = |g\rangle\langle e|$ where $|e\rangle$ and $|g\rangle$ are the atomic excited and ground state repectively. The negative frequency part of the driving electric field is given by

$$E^-(x,t) = E_1 e^{i[(\omega_0-\omega_x)t - k_1 x + \phi_1]} + E_2 e^{i[(\omega_0+\omega_x)t - k_2 x + \phi_2]} \tag{4.2}$$

where E_i and ϕ_i indicate amplitudes and phases of the driving beams. When the trapping frequency is much larger than the other characteristic frequencies, and providing that the field is resonant with one of the vibrational side-bands, then the ion-field interaction can be described by a non-linear Jaynes Cummings model (JCM) [15]. Accordingly, in the interaction picture the Hamiltonian (4.1) takes the form

$$\begin{aligned}\mathbf{H_I} \;=\; & -\sum_{j=0}^{\infty}\{\Omega_1 e^{i\phi_1} e^{-\eta_1^2/2}\frac{(i\eta_1)^{2j+1}}{j!(j+1)!}(a^+)^{j+1}a^j \\ & + \;\Omega_2 e^{i\phi_2} e^{-\eta_2^2/2}\frac{(i\eta_2)^{2j+1}}{j!(j+1)!}(a^+)^j a^{j+1}\}\sigma_- + h.c.\end{aligned} \tag{4.3}$$

with $\Omega_j = \mu E_j$ the Rabi frequencies and $\eta_l^2 = \frac{k_l^2}{2M}(\frac{1}{\omega_x})$ the Lamb-Dicke parameters, and they describe the ratio between the single photon recoil energy and the energy-level spacing in the harmonic oscillator strength. In the Lamb-Dicke limit where the vibrational amplitude of the ion is much smaller than the laser wavelength it is sufficient to keep the frist few terms in (4.3) and one works with an effective Hamiltonian $\acute{\mathbf{H}}_\mathbf{I}$ of the form

$$\acute{\mathbf{H}}_\mathbf{I} = -(2g_1 a^+ + 2g_2 a)\sigma_- + h.c. \tag{4.4a}$$

where

$$g_j = i\Omega_j e^{i\phi_j}\eta_j^2 e^{-\eta_j^2/2}, \qquad j = 1,2 \tag{4.4b}$$

the exponentials may be put equal to 1 because of the smallness of the η_j^2's.

When we look at (4.4) it is noticed that the first terms (+ its h.c.) is the usual JCM Hamiltonian. It describes the first red-side band resonance. While the second term (+ its h.c.) is the first blue-side band resonance. It is the counter-rotating term which is not present in the cavity Q.E.D. The motional and electronic dynamics may be decoupled in the Hamiltonian (4.4) by adding another interaction [16]; and we finish up with

$$\bar{\mathbf{H}}_I = -\{(2(g_1 + g_2^*)a^+ + 2(g_1^* + g_2)a\}(\sigma_+ + \sigma_-) \tag{4.5}$$

under this Hamiltonian any atom prepared in the state $\frac{1}{\sqrt{2}}(|e\rangle + |g\rangle)$ which can be generated from the ground state by applying a $\frac{\pi}{2}$ carrier pulse will stay in this state and will be left unchanged [16]. Thus the dynamics is reduced to that of the motional degrees of freedom only. Under this Hamiltonian the motional dynamics evolves towards the DFS's $|\alpha, m\rangle$ when it is prepared initially in the Fock state $|m\rangle$. The state $|m\rangle$ can be prepared with very high efficiency according to recent experiments [2]. The preparation of superposition of these states can be done according to the scheme described below.

We start from

$$|\Psi(0)\rangle = \sum_{n=0}^{m} c_n |n, g\rangle \tag{4.6}$$

This state can be generated by successive applications of an external classical driving field and a quantized field (similar to (4.4)) as described in detail in ref. [17]. Applying classical field (carrier) for a duration time τ_1 whose evolution operator takes the form

$$\begin{aligned} U_1(\tau_1) &= \cos\Omega_1\tau_1|e\rangle\langle e| - ie^{i\theta_1}\sin\Omega_1\tau_1|e\rangle\langle g| \\ &- ie^{-i\theta_1}\sin\Omega_1\tau_1|g\rangle\langle e| + \cos\Omega_1\tau_1|g\rangle\langle g| \end{aligned} \tag{4.7}$$

where Ω_1 is the Rabi frequency in this case, θ_1 is a phase, on the state (4.6) and taking $\Omega_1\tau_1 = \frac{\pi}{2}$, $\theta_1 = \frac{\pi}{2}$, then we get

$$|\Psi(\tau_1)\rangle = U_1(\tau_1)|\Psi(0)\rangle = \sum_{n=0}^{m} \frac{c_n}{\sqrt{2}}|n\rangle \otimes (|e\rangle + |g\rangle) \tag{4.8}$$

the internal state ($|e\rangle+|g\rangle$) will remain constant under the Hermiltonian (4.5). Applying this Hermiltonian for a time duration τ_2. The state $|\Psi(\tau_1)\rangle$ evolves to

$$|\Psi(\tau_2)\rangle = |\Psi(\tau_2 + \tau_1)\rangle = U_1(\tau_2)|\Psi(\tau_1)\rangle = \sum_{n=0}^{m} \frac{c_n}{\sqrt{2}}|\alpha, n\rangle \otimes (|e\rangle + |g\rangle) \tag{4.9}$$

where $\alpha = 2i(g_1 + g_2^*)\tau_2$. The state in (4.9) is a superposition of displaced Fock states but with the same displacement α. This state is equivalent to that discussed in ref. [4(d)]

We choose the polarization in the quantized field so that it affects the excited state only as described in ref.[2] and apply the field for a duration τ_3 which generates the state

$$|\Psi(\tau_3)\rangle = \acute{U}_2(\tau_3)|\Psi(\tau_2)\rangle = \sum_{n=0}^{m} \frac{c_n}{\sqrt{2}}[|\beta, n\rangle|e\rangle + |\alpha, n\rangle|g\rangle] \tag{4.10}$$

where $\alpha = \alpha + 2i(\acute{g}_1 + \acute{g}_2^*)\tau_3$.

After that we apply a carrier pulse for a duration τ_4 with the evolution operator (4.7). It produces the following state

$$\begin{aligned} |\Psi(\tau_4)\rangle &= \sum_{n=0}^{m} \frac{c_n}{\sqrt{2}}[|\beta, n\rangle(\cos\Omega_1\tau_4|e\rangle - ie^{-i\theta_2}\sin\Omega_1\tau_4|g\rangle) + |\alpha, n\rangle(-ie^{i\theta_2} \\ &\times \sin\Omega_1\tau_4|e\rangle + \cos\Omega_1\tau_4|g\rangle)] \\ &= \sum_{n=0}^{m} \acute{c}_n[(\cos\Omega_1\tau_4|\beta, n\rangle - ie^{i\theta_2}\sin\Omega_1\tau_4|\alpha, n\rangle)|e\rangle) \\ &+ (\cos\Omega_1\tau_4|\alpha, n\rangle - ie^{-i\theta_2}\sin\Omega_1\tau_4|\beta, n\rangle)|g\rangle] \\ &= \sum\{(C_{1n}|\beta, n\rangle + C_{2n}|\alpha, n\rangle)|e\rangle + (D_{1n}|\beta, n\rangle + D_{2n}|\alpha, n\rangle)|g\rangle \end{aligned} \tag{4.11}$$

Detecting the atom in either of its electronic states gives the desired superposition $\sum_{n=0}^{m}[C_n|\beta, n\rangle + K_n|\alpha, n\rangle]$.

5 Conclusions

We have discussed the properties and a generation scheme of DFS superposition states. In particular, we have given the expression for the photon number distribution and QDF's.

The three dimensional plots of the Wigner function for some parameters have been illustrated for the state of superposition of two DFS's showing nonclassical and interference effects. Several moments have been calculated. The second-order correlation function $g^{(2)}$ have been investigated numerically. The squeezing properties for these states have been discussed. We have analyzed the quadrature component distributions for the pair of DFS's superposition state and have presented analytical and numerical results. We have found that the basic features of a pair of DFS's superposition state, such as the appearance of two separated peaks and an interference pattern. A generation scheme for these states has been presented, depending on motional dynamics of center of mass of trapped ions.

Acknowledgments The authors wish to express their appreciation to the referee for their valuable comments that helped to improve the paper.

References

[1] R.J. Glauber: *Phys. Rev.* **130** (1963) 2529 ; R.J. Glauber: *Phys. Rev.* **131** (1963) 2766 ; J. Perina: *Quantum statistics of linear and non-linear optical phenomena* Reidel-Dordrecht, Holland 1984; D.F. Walls, and G.J. Milburn: *Quantum Optics* Springer-Verlag, Berlin 1994; W.H. Louisell: *Quantum statistical properties of radiation* Wiley, New York, 1973

[2] (a) D. Leibfried, D.M. Meekhof, B.E. King, C. Monroe, W.M. Itano, D.J. Wineland: *Phys. Rev. Lett.* **77** (1996) 4281

(b) D.M. Meekhof, C. Monroe, B.E. King, W.M. Itano, D.J. Wineland: *Phys. Rev. Lett.* **76** (1996) 1796

(c) C. Monroe, D.M. Meekhof, B.E. King, D.J. Wineland: *Science* **272** (1996) 1131

(d) W.M. Itano, C. Monroe, D.M. Meekhof, D. Leibfried, B.E. King, D.J.Wineland D.J.: *SPIE Proc.* **2995** (1997) 43

[3] M. Boiteux, A. Levelut: *J. Phys. A: Math.Gen.* **6** (1973) 589 ; S.M. Roy, V. Singh: *Phys. Rev. D* **25** (1982) 3413 ; M.V. Satyanarayana: *Phys. Rev. D* **32** (1985) 400

[4] (a) A. Wunsche: *Quantum Opt.* **3** (1991) 359

(b) F.A.M. De Oliveira,M.S. Kim, P.L. Knight, V. Bužek: *Phys. Rev. A* **41** (1990) 2645

(c) G.M. Abd Al-Kader: *J. Egypt. Math. Soc.* **2** (1994) 89

(d) G.S. Agarwal, K. Tara: *Phys. Rev. A* **43** (1991) 492

[5] N. Moya-Cessa, P.L. Knight: *Phys. Rev. A* **48** (1993) 2479

[6] V.V. Dodonov, I.A. Malkin, V.I. Man'ko: *Physica A* **72** (597) ; B. Yurke, D. Stoler: *Phys. Lett. A* **57** (1986) 13 ; J. Janszky, A.V. Vinogradov: *Phys. Rev. Lett.* **64** (1990) 2771 ; V. Bužek, P.L. Knight: *Optics Commun.* **81** (1991) 331 ; V. Bužek, A. Vidiella-Barranco, P.L. Knight: *Phys. Rev. A* **45** (1992) 6570 ; C.C. Gerry: *Optics Commun.* **91** (1992) 247 ; *J. Mod. Optics* **40** (1993) 1053; E.E. Hach III, C.C. Gerry: *Phys. Rev. A* **49** (1994) 490 ; *J.Mod. Optics* **40** (1993) 2351; D. Mogilevtsev, S.K. Ya: *Optics Commun.* **132** (1996) 452 ; M. Ban: *Phys. Lett. A* **193** (1994) 121 ; *Phys. Rev. A* **51** (1995) 1604; A.-S. F. Obada, Z.M. Omar: *I.C.T.P. Publ.* (*Trieste*) (1995) IC/95/302; *Phys. Lett. A* **227** (1997) 349; M.S. Abdalla, M.H. Mahran, A.-S. F. Obada: *J. Mod. Optics* **41** (1994) 1889 ; A.-S.F. Obada, M.H. Mahran, F.A.A. El-Orany, M.S. Abdalla: *Internal. J. Theor. Phys.* **35** (1996) 1393 A. Joshi, A.-S.F. Obada: *J. Phys. A* **30** (1997) 81

[7] H. Moya-Cessa: *J. Mod. Optics* **42** (1995) 1741

[8] B. Zheng, C.C. Guo: *Quantum Semiclass. Opt.* **8** (1996) 951

[9] V.V. Dodonov, Y.A. Korennoy, V.I. Man'ko, Y.A. Moukhin: *Quantum Semiclass. Opt.* **8** (1996) 413

[10] M. Dakna, T. Anhut, T. Opatrný, L. Knöll, O.-G. Welsch: *Phys. Rev. A* **55** (1997) 3184

[11] Special issue on *Quantum phase and phase dependent measurements*, of *Physica Scripta* **T48** (1993) pp 1-142; R. Lynch: *Phys. Rep.* **256** (1995) 367

[12] S.M. Barnett, D.T. Pegg: *Phys. Rev. A* **39** (1989) 1665 ; *J. Mod. Optics* **36** (1989) 7; S.M. Barnett, D.T. Pegg: *Phys. Rev. A* **1990** (41) 3427

[13] A.V. Chizhov, B.K. Murzakhmetov: *Phys. Lett. A* **176** (1993) 33 ; A.V. Chizhov, Ts. Gantsog, B.K. Murzakhmetov: *Quantum Opt.* **5** (1993) 85

[14] see the review M. Hillery, R.F. O'Connell, M.O. Scully, E.P. Wigner: *Phys. Rep.* **106** (121) a nd itemerences therein; K.E. Cahill, R.J. Glauber: *Phys. Rev.* **177** (1969) 1857, 1882 h.-W. Lee: *Phys. Rep.* **259** (1995) 147

[15] W. Vogel, R.L. de Matos Filho: *Phys. Rev. A* **52** (1995) 4214 ; R.L. de Matos Filho, W. Vogel: *Phys. Rev. Lett.* **76** (1996) 608

[16] S.-C. Gou, J. Steinbach, P.L. Knight: *Phys. Rev. A* **55** (1997) 3719 ; J. Steinbach, J. Twamley, P.L. Knight: *Phys. Rev. A* **56** (1997) 4815

[17] C.K. Law, J.H. Eberly: *Phys. Rev. Lett.* **76** (1996) 1055 ; S.A. Gardiner, J.I. Cirac, P. Zoller: *Phys. Rev. A* **5** (1997) 1683 B. Kneer, C.K. Law: *Phys. Rev. A* **57** (1998) 2096