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# BOSE-EINSTEIN CORRELATIONS DUE TO RESONANCE PRODUCTION AND SPACE-TIME EVOLUTION OF HADRONIC COLLISIONS

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We study in a simple model the effects of resonance production and decay on Bose-Einstein correlations of identical pions. The intermediate stage of collision is simulated by the formation process of resonances given by a single function of the proper time of a resonance. Direct pions are described as decay products of a resonance with vanishing life-time. In contradistinction to our recent work we include realistic $p_T$-distributions of resonances. The comparison with data of the EHS/NA22 Collaboration shows that the mean time of resonance formation is rather short, 0.1–0.4 fm/c. This result is almost independent of the form of the function describing resonance formation. The data of the EHS/NA22 Collaboration require also a decrease of the chaoticity parameter $\lambda$ with increasing average transfer momentum of identical pions.
It is pointed out that there are similarities between the model based on resonance formation and decay and the one based on hydrodynamical evolution of hadronic matter. We conjecture that hydrodynamical models in hadronic collisions are describing in a different language the process of resonance formation and decay.

## 1 Introduction

Bose-Einstein correlations (BEC) of identical particles, for reviews see Refs. [1-3], bring information on the space-time distribution of the region from which particles have been produced. Relationship between BEC and space-time evolution of the system has been recently intensively studied in connection with the analysis of data on heavy ion collisions [4-8]. The experience gained in analysing data on heavy ion collisions has

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shown that the effects due to resonance production and decay play an important role and have to be understood before one can see what is the underlying dynamics of heavy ion collisions.

Effects of resonance production on BEC in hadronic interactions are even more important than in heavy ion collisions. These effects have been studied over past two decades in a number of papers [9–19]. The formation time of resonances brings an important information about the first, rather unknown stage of $pp, p\bar{p}$, and $e^+e^-$ collisions. Decays of resonances to final state hadrons are described in a standard way, resonance widths being given by the data. It is not quite clear whether a pion which is created in a decay of a resonance is able to interact immediately with the full cross- section with other hadrons. Such a "formation time" is important [20] for estimating the energy density of interacting matter produced in proton-nucleus (pA) or nucleus-nucleus (AB) collisions. On the other hand this formation time consists presumably in "dressing up" of the valence partons by soft ones and it is not relevant for BEC.

There exist numerous models of the evolution of hadronic collisions [21–24] and the determination of the time scale from analysis of BEC could make the parameters in these models more accurate.

We have recently studied the effects due to resonance decays in hadronic collisions in a very simplified model [19]. The purpose of the present paper is to make that model more realistic by including transverse momenta of resonances, what will permit us to study also the transverse momentum dependence of correlation functions.

In Ref. [19] we have shown by using one simple parametrization of the time dependence of resonance formation, that the mean time for the formation of resonances is rather short, 0.2–0.4 fm/c. In the present paper we shall use a few different parametrizations and we shall show that the conclusion of the short average time of the formation of resonances is not due to a particular form of parametrization.

The present paper is organized as follows. In Sects. 2 and 3 we shall describe the formulation of our model. In Sect.4 we compare the model with data of EHS/NA22 Collaboration. Sect. 5 contains comments and conclusions. Appendix A deals with the connection of the present model to its simplified version used in Ref. [19] and in the Appendix B we discuss analogies between the model based on resonance formation and decay and hydrodynamical models. The discussion leads to the conjecture that hydrodynamical models describe in another language the dynamics of the hadronic collision based in fact only on resonance formation and decay.

## 2 A model of Bose-Einstein correlations in hadronic collisions based on resonance formation and decay.

In Ref. [19] we have studied identical particle correlations in hadronic collisions in a model where resonance formation and decay is responsible for correlations of identical pions. The model has contained a few very simplifying assumptions, the strongest of them consisting in putting transverse momenta of resonances equal to zero. In the present, improved version of the model, we shall include a realistic description of the transverse momenta of resonances.



When discussing collisions of really heavy-ions, like Pb+Pb the assumption of an intermediate system of thermalized matter [7,25] may be quite realistic. For hadronic collisions such an assumption is very debatable, although it is known since the work of Hagedorn [26], see also Ref. [27] that transverse momentum spectra of final state hadrons in pp collisions at high energy are close to thermal ones. The increase of $dN/p_T dp_T$ for pions at low values of $p_T$ can be most likely explained by resonance decays [25,28].

In the present paper we shall not assume that hadronic system formed in hadronic collisions is thermalized. We shall take instead as much information on resonance production as we are able to from the experimental data and we shall parametrize the formation of resonances by a simple function of their proper time.

Experiments show that a part of pions is produced via decays of experimentally identified resonances. The rest of pions is referred to as "direct" ones. A part of them is probably due to decays of resonances which are too broad to be identified. In describing direct pions we shall assume that they are due to decays of a short living resonance.

The probability density (Wigner distribution) for formation of a resonance R with four vector P in point with four-cordinate X will be denoted as $S_R^f(X,P)$. The probability density $S_{R\to\pi}(x;p)$ for a pion to be produced with four-momentum p in point with four-coordinate x is given as [7,19,25,28,29]

$$S_{R\to\pi}(x;p) = \frac{M_R}{4\pi\sqrt{E^{*2}-m_\pi^2}} \int \frac{d^3P}{E}\delta(P.p - M_R E^*) \int d^4X \int d\tau \Gamma e^{-\Gamma\tau} \delta^{(4)}(x-(X+\frac{P}{M_R}\tau))S_R^f(X,P). \quad (1)$$

The factor $M_R/4\pi\sqrt{E^{*2}-m_\pi^2}$ comes from the probability distribution of a decay of a resonance with mass $M_R$ to pion with the energy $E^*$ in the resonance rest frame. The probability distribution is given as

$$P(p_0,\vec{p}) = \frac{1}{4\pi\sqrt{E^{*2}-m_\pi^2}}\delta(p_0 - E^*) = \frac{M_R}{4\pi\sqrt{E^{*2}-m_\pi^2}}\delta(P.p - M_R E^*). \quad (2)$$

The probability is normalized by

$$\int P(p_0,\vec{p})\frac{d^3p}{p_0} = 1.$$

The corresponding correlation function is expressed as the Fourier transform of the Wigner distribution [4,5]

$$C(q,K) = 1 + \lambda \frac{|\Sigma_R \int d^4x S_{R\to\pi}(x,K)e^{iq\cdot x}|^2}{|\Sigma_R \int d^4x S_{R\to\pi}(x,K)|^2}, \quad (3)$$

where $p_1, p_2$ are momenta of identical pions, $q = p_1 - p_2$ and $K = \frac{1}{2}(p_1 + p_2)$ and $\lambda$ is the chaoticity parameter. Define now

$$\tilde{S}_{R\to\pi}(q;K) \equiv \int d^4x e^{iq\cdot x} S_{R\to\pi}(x;K). \quad (4)$$

Inserting $S_{R\to\pi}$ from Eq.(1) we have

$$\tilde{S}_{R\to\pi}(q;K) = \frac{M_R}{4\pi\sqrt{E^{*2}-m_\pi^2}}\int d^4x e^{iq\cdot x}\int \frac{d^3P}{E}\delta(P.K - M_R E^*)$$
$$\int d^4X \int d\tau \Gamma e^{-\Gamma\tau}\delta^{(4)}(x-(X+\frac{P}{E}\tau))S_R^f(X,P). \quad (5)$$

We shall now introduce the variable $\xi = x - X$ and integrate over $\xi$ keeping X fixed. Using

$$\int d^4\xi \int_0^\infty d\tau\Gamma e^{-\Gamma\tau}\delta^{(4)}(\xi - \frac{P}{M_R}\tau)e^{iq\cdot\xi} = \int_0^\infty d\tau\Gamma e^{-\Gamma\tau}e^{i\frac{q\cdot P}{M}\tau} = \frac{\Gamma}{\Gamma - i\frac{q\cdot P}{M_R}} \quad (6)$$

we find

$$\tilde{S}_{R\to\pi}(q;K) = \frac{M_R}{4\pi\sqrt{E^{*2}-m_\pi^2}}\int\frac{d^3P}{E}\delta(P.K - M_R E^*)\frac{\Gamma}{\Gamma - i\frac{q\cdot P}{M_R}}$$
$$\int d^4X e^{iq\cdot X}S_R^f(X;P) \quad (7)$$

We shall now rewrite preceding expressions in the form similar to that used in our earlier work [19]. The four-vectors $K$ and $P$ are expressed in terms of rapidities and transverse momenta as

$$K \equiv K(m_T\cosh(y); K_T\cos\phi, K_T\sin\phi, m_T\sinh(y))$$

$$P \equiv P(M_{RT}\cosh(Y); P_T\cos(\Phi), P_T\sin(\Phi), M_{RT}\sinh(Y)).$$

The scalar product $K\cdot P$ then becomes

$$K\cdot P = m_T M_{RT}\cosh(Y-y) - K_T P_T\cos(\Phi-\phi). \quad (8)$$

This enables us to write

$$\delta(K\cdot P - M_R E^*) = \delta\left(m_T M_{RT}\cosh(Y-y) - K_T P_T\cos(\Phi-\phi) - M_R E^*\right)$$
$$= \frac{1}{m_T M_{RT}}\delta\left(\cosh(Y-y) - \frac{K_T P_T\cos(\Phi-\phi) + M_R E^*}{m_T M_{RT}}\right). \quad (9)$$

The integration over momentum is replaced by integration over rapidity

$$\frac{d^3P}{E} = dY P_T dP_T d\Phi = dY M_{RT} dM_{RT} d\Phi.$$

The $\delta$-function in Eq.(8) has the form $\delta(f(Y))$ and $f(Y)=0$ has two solutions

$$Y_{1,2} = y \pm y_R$$
$$y_R \equiv y_R(K_T, P_T, \Phi-\phi) = \operatorname{arccosh}\left(\frac{M_R E^* + K_T P_T\cos(\Phi-\phi)}{M_{RT}m_T}\right) \quad (10)$$



In this way the $\delta$-function can be written as

$$\delta(K\cdot P - M_R E^*) = \frac{1}{m_T M_{RT}|\sinh(y_R)|}(\delta(Y-y+y_R) + \delta(Y-y-y_R))$$
$$\times\Theta(K_T P_T\cos(\Phi-\phi) + M_R E^* - m_T M_{RT}) \quad (11)$$

where the last term is due to the fact that $\cosh(y_R)\geq 1$.

Putting everyting together we have

$$\tilde{S}_{R\to\pi}(q;K) = \frac{M_R}{4\pi\sqrt{E^{*2}-m_\pi^2}}\int dY M_{RT}dM_{RT}d\Phi$$
$$\Theta(M_R E^* + K_T P_T\cos(\Phi-\phi) - m_T M_{RT})$$
$$\frac{1}{m_T M_{RT}|\sinh(y_R)|}[\delta(Y-y-y_R) + \delta(Y-y+y_R)]$$
$$\frac{\Gamma}{\Gamma - i\frac{q\cdot P}{M_R}}\tilde{S}_R^f(q;P), \quad (12)$$

where

$$\tilde{S}_R^f(q;P) = \int d^4X e^{iq\cdot X}S_R^f(X;P) \quad (13)$$

and $y_R$ is given by Eq.(9).

In the present paper, following Ref. [19], we shall assume that "direct" pions are decay products of a resonance with a very large width.

Inserting Eq.(12) for resonance contributions into Eq.(2) we obtain the correlation function $C(q,K)$.

In the Appendix A we shall show how formulas used in our earlier work are obtained in the present formalism.

## 3 Model with non-vanishing transverse momenta of resonances

We shall start here with Eqs. (11) and (12) and derive expressions for $\tilde{S}_{R\to\pi}(q;K)$ corresponding to the situation with non- vanishing transverse momenta of resonances.

Making use of the $\delta$-function in Eq.(11) we obtain

$$\tilde{S}_{R\to\pi}(q;K) = \frac{M_R}{4\pi\sqrt{E^{*2}-m_\pi^2}}\sum_{i=1,2}\int M_{RT}dM_{RT}d\Phi$$
$$\Theta(M_R E^* + K_T P_T\cos(\Phi-\phi) - m_T M_{RT})\frac{1}{m_T M_{RT}|\sinh(y_R)|}$$
$$\frac{1}{1 - i\frac{q\cdot P_i}{\Gamma M_R}}\tilde{S}_R^f(q;P_i), \quad (14)$$

where

$$P_i \equiv P_i(M_{RT}\cosh(Y_i); P_T\cos\Phi, P_T\sin\Phi, M_{RT}\sinh(Y_i)). \quad (15)$$

The model of resonance production is contained in the function $\tilde{S}^f_R(X,P)$ in Eq.(12). We assume that all resonances are produced in the point $X=0$ and that all of them are formed in an interval $(\tau, \tau + d\tau)$ in their proper time $\tau$ with the same probability density $p(\tau)$.

In order to see the dependence of results on the functional form of $p(\tau)$ we shall use here three parametrizations of $p(\tau)$:

- exponential as in Ref. [19]
- $\delta$ - function, and
- Gaussian.

### 3.1 Exponential parametrization

In this case we have

$$p(\tau) = \frac{1}{\tau_f}\exp\left(-\frac{\tau}{\tau_f}\right). \qquad (16)$$

In its proper time $\tau$ the resonance arrives at the point

$$\vec{X} = \frac{\vec{P}}{M_R}\tau \qquad X_0 = \frac{E}{M_R}\tau. \qquad (17)$$

In this way we obtain

$$S^f_R(X;P) = \int_0^\infty \mathrm{d}\tau \frac{1}{\tau_f} e^{-\tau/\tau_f}\delta^3(\vec{X} - \frac{\vec{P}}{M_R}\tau)\delta(X_0 - \frac{E}{M_R}\tau)N_R(P) \qquad (18)$$

and via Eq. (12)

$$\tilde{S}^f_R(q;P) = \int_0^\infty \mathrm{d}\tau \frac{1}{\tau_f}\exp(-\frac{\tau}{\tau_f})\exp(i\frac{q\cdot P}{M_R}\tau)N_R(P) = \frac{1}{1 - i\frac{q\cdot P}{M_R}\tau_f}N_R(P), \qquad (19)$$

where $N_R(P)$ is the distribution of resonances in the four-momentum P. Inserting Eq.(18) into Eq.(13) we find

$$\tilde{S}_{R\to\pi}(q;K) = \frac{M_R}{4\pi\sqrt{E^{*2} - m_\pi^2}} \sum_{i=1,2} \int M_{RT}\mathrm{d}M_{RT}\mathrm{d}\Phi \; \Theta(M_R E^* + K_T P_T \cos(\Phi - \phi) - m_T M_{RT}) \frac{1}{m_T M_{RT}|\sinh(y_R)|} \frac{1}{1 - i\frac{q\cdot P_i}{\Gamma_R M_R}} \frac{1}{1 - i\frac{q\cdot P_i}{M_R}\tau_f} N_R(P_i), \qquad (20)$$

where $y_R$ is given by Eq. (10) and four-momenta $P_1, P_2$ are given by Eq. (15) with rapidities $Y_1, Y_2$ determined by Eq. (10).



### 3.2 Parametrization with the $\delta$-function

In this case we have

$$p(\tau) = \delta(\tau - \tau_f)$$

and

$$\tilde{S}^f_R(q;P) = \int_0^\infty \mathrm{d}\tau p(\tau)\exp(i\frac{q\cdot P}{M_R}\tau) = \exp(i\frac{q\cdot P}{M_R}\tau_f).$$

Instead of Eq.(19) we get

$$\tilde{S}_{R\to\pi}(q;K) = \frac{M_R}{4\pi\sqrt{E^{*2} - m_\pi^2}} \sum_{i=1,2} \int M_{RT}\mathrm{d}M_{RT}\mathrm{d}\Phi \; \Theta(M_R E^* + K_T P_T \cos(\Phi - \phi) - m_T M_{RT}) \frac{1}{m_T M_{RT}|\sinh(y_R)|} \frac{1}{1 - i\frac{q\cdot P_i}{\Gamma_R M_R}} \exp(i\frac{q\cdot P_i}{M_R}\tau_f) N_R(P_i) \qquad (21)$$

### 3.3 Gaussian parametrization

In this case

$$p(\tau) = \frac{1}{\sqrt{2\pi\sigma^2}}\exp[-(\tau - \tau_f)^2/2\sigma^2].$$

The Fourier transform can be taken explicitly only in the case when $\tau_f/\sigma$ is much larger than 1, and we can replace the lower bound in

$$\int_0^\infty \mathrm{d}\tau p(\tau)\exp\left(i\frac{(q\cdot P)}{M_R}\tau\right)$$

by $-\infty$. In this approximation we obtain

$$\tilde{S}_{R\to\pi}(q;K) = \frac{M_R}{4\pi\sqrt{E^{*2} - m_\pi^2}} \sum_{i=1,2} \int M_{RT}\mathrm{d}M_{RT}\mathrm{d}\Phi \; \Theta(M_R E^* + K_T P_T \cos(\Phi - \phi) - m_T M_{RT}) \frac{1}{m_T M_{RT}|\sinh(y_R)|} \frac{1}{1 - i\frac{q\cdot P_i}{\Gamma_R M_R}} \exp\left(i\frac{q\cdot P_i}{M_R}\tau_f\right)\exp\left(-\frac{1}{2}\sigma^2\left(\frac{q\cdot P}{M_R}\right)^2\right) N_R(P_i) \qquad (22)$$

We shall parametrize $N_R(P)$ in terms of rapidity and transverse momentum

$$N_R(P) = B_R \frac{\mathrm{d}\sigma_R}{\mathrm{d}Y P_T \mathrm{d}P_T \mathrm{d}\Phi} = B_R \frac{1}{2\pi} 2b.\exp(-bP_T^2)\left(\frac{\mathrm{d}\sigma_R}{\mathrm{d}Y}\right)_{Y=Y_{Ri}}, \qquad (23)$$

Table 1: Parameters of resonance production cross-sections in $\pi^+ p$ and branching ratios of resonances to $\pi^-$.

| *Resonance* | $B$ | $E^*[GeV]$ | $b[GeV^{-2}c^2]$ | $\tau_d[GeV^{-1}]$ | $\sigma_R^0[mb]$ |
|---|---|---|---|---|---|
| $\rho_0$ | 1 | 0.385 | 2.7 [30] | 6.66 | 1.74 [30] |
| $\rho^-$ | 1 | 0.385 | 2.7 g[30] | 6.66 | 1.3 g[30] |
| $\omega$ | 0.9 | 0.263 | 2.7 [30] | 118.6 | 2.6 [30] |
| $f_2$ | 0.57 | 0.635 | 2.0 g[31] | 5.41 | 0.3 g[31] |
| $K^{*0}$ | 0.66 | 0.319 | 3.5 [30] | 20.0 | 0.7 [30] |
| $\eta$ | 0.286 | 0.180 | 2.0 g | $832.10^3$ | 2.0 g[33] |
| $\Delta^0$ | 0.33 | 0.265 | 7.0 g[32] | 8.3 | 0.7 g[31] |
| $\Delta^-$ | 1.0 | 0.265 | 7.0 g[32] | 8.3 | 0.7 g[31] |
| $\sigma$ | 1 | 0.385 | 2.7 g | 0 | $\frac{r}{1-r}\sum(B\sigma_r)$ |

where $B_R$ is the branching ratio for the decay of resonance R to the particle whose correlation we are studying ($\pi^-$).

In this way we arrive for the case of the exponential parametrization of $p(\tau)$ at the final expression

$$\begin{aligned}\tilde{S}_{R\to\pi}(q;K) &= \frac{M_R}{4\pi\sqrt{E^{*2}-m_\pi^2}}\sum_{i=1,2}\int M_{RT}\mathrm{d}M_{RT}\mathrm{d}\Phi \\ &\Theta(M_R E^* + K_T P_T\cos(\Phi-\phi) - m_T M_{RT}) \\ &\frac{1}{m_T M_{RT}|\sinh(y_R)|}\frac{1}{1-i\frac{q\cdot P_i}{\Gamma_R M_R}} \\ &\frac{1}{1-i\frac{q\cdot P_i}{M_R}\tau_f}B_R\frac{1}{2\pi}2b_R.e^{-b_R P_T^2}\left(\frac{\mathrm{d}\sigma_R}{\mathrm{d}Y}\right)_{Y=Y_{Ri}} \qquad (24)\end{aligned}$$

When inserted into Eq.(2) this gives the correlation function $C(q;K)$.

Note that resonance decay and resonance formation enter Eq.(19) in the case of the exponential parametrization of $p(\tau)$ in the same way. For resonance decay this is dictated by the decay law and for resonance formation it follows from the specific assumption in Eq.(15) which describes resonance formation by an exponential law. Resonance decay widths $\Gamma_R$ are taken from the data and $\tau_f$, which can be written as $1/\Gamma_f$, is a free parameter of our model which is to be determined from the data on correlations of identical pions.

## 4 Comparison with data on C($q_L$) at a fixed value of $K_T$

We shall now compare resulting correlations for the three parametrizations of $p(\tau)$. In order to make the comparison realistic we have taken parameters of resonance production and decay from data. Parameters are summarized in Table 1.



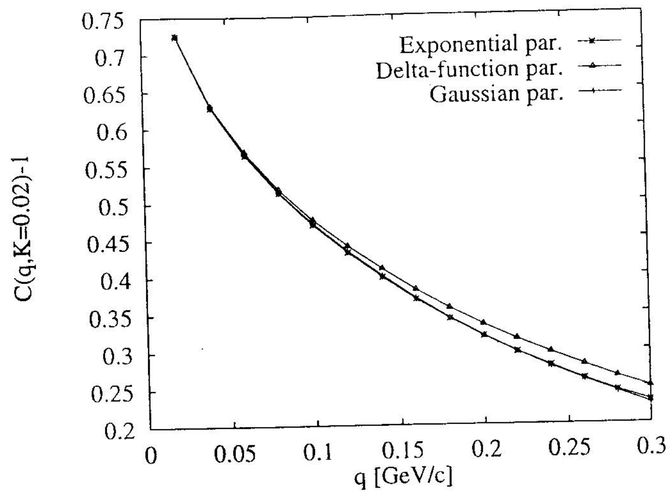


Fig. 1. Comparison of correlation functions calculated with exponential, gaussian with the width $\sigma$ = 0.3 fm/$c$ and $\delta$-function parametrizations of resonance formation at $K_T$ = 20 MeV/$c$, $\tau_f$ = 0.3 fm/$c$ and $r$ = 0.8.

Correlation functions for the three parametrizations are presented in Fig.1. In all three cases we have $\tau_f$ = 0.3 fm/$c$ and the ratio $r$ of direct to all pions is given as $r = 0.8$. The differences between parametrizations are rather small.

Our calculations of $\pi^-\pi^-$ correlations are based on Eq.(19). Since the three parametrizations used lead to rather similar results we shall use in what follows only the exponential one. The longitudinal momentum of the pair of pions $K_L$ is put equal to zero, what means also that $y = 0$ and the transverse momentum $K_L$ is taken as a parameter in $C(q;K_T)$. The contribution of a resonance R is then given as

$$\begin{aligned}\tilde{S}_{R\to\pi}(q;K) &= \frac{M_R}{4\pi\sqrt{E^{*2}-m_\pi^2}}\sum_{i=1,2}\int M_{RT}\mathrm{d}M_{RT}\mathrm{d}\Phi \\ &\Theta(M_R E^* + K_T P_T\cos(\Phi-\phi) - m_T M_{RT})\frac{1}{m_T M_{RT}|\sinh(y_R)|} \\ &\frac{1}{1-i\frac{q\cdot P_i}{\Gamma_R M_R}}\frac{1}{1-i\frac{q\cdot P_i}{M_R}\tau_f}B_R\left(\frac{\mathrm{d}\sigma_R}{\mathrm{d}Y P_T \mathrm{d}P_T \mathrm{d}\Phi}\right)_{Y=Y_{Ri}}, \qquad (25)\end{aligned}$$

where the four-vector $P_1$ is given by $(y_R, P_T, \Phi)$ and $P_2$ by $(-y_R, P_T, \Phi)$, $\tau_d = 1/\Gamma_R$ is the resonance decay time and $y_R$ in $Y_{R1} = y_R$ and $Y_{R2} = -y_R$ is calculated via Eq.(9). Since $M_{RT} = \sqrt{M_R^2 + P_T^2}$ we have also $M_{RT}\mathrm{d}M_{RT} = P_T\mathrm{d}P_T$.

For the four-vector $q = (0, \vec{0}, q_L)$ (in what follows $q_L$ will be denoted simply as q)

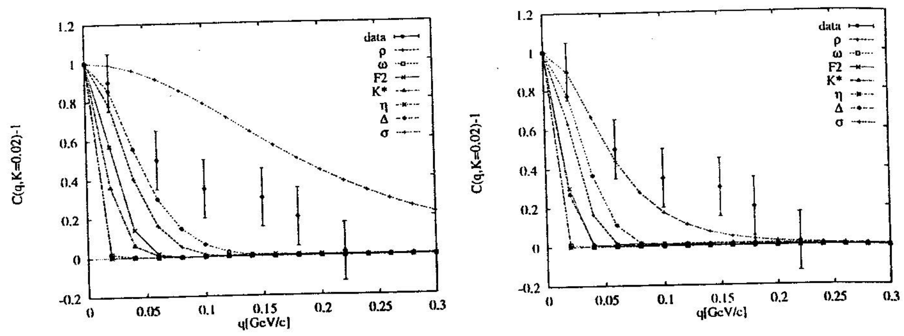


Fig. 2. Contribution of individual resonances at $K_T = 20$ MeV/c, formation time of resonances $\tau_f = 0.3$ fm/c (left) and $\tau_f = 1.0$ fm/c (right).

we have

$$q \cdot P_1 = -qM_{RT}\sinh(y_R), \qquad q \cdot P_2 = +qM_{RT}\sinh(y_R)$$

The symbol $B_R$ in Eq.(24) denotes the branching ratio of $R \to \pi^- X$.

Parameters of resonance production cross-sections are to a large extent given by excellent data of EHS/NA22 Collaboration [30,31,32] but in a few cases we shall have to guess the relevant information from other data at similar energies.

Cross-section for resonance production will be parametrized as

$$\frac{d\sigma_R}{dY P_T dP_T d\Phi} = \frac{1}{2\pi} 2b.\exp(-bP_T^2)\left(\frac{d\sigma_R}{dY}\right)_{Y=Y_{Ri}} . \qquad (26)$$

Around $Y = 0$ cross-sections for resonance production vary slowly so we shall rather use the value of cross-section at $Y = 0$

$$\left(\frac{d\sigma_R}{dY}\right)_{Y=Y_{Ri}} \approx \left(\frac{d\sigma_R}{dY}\right)_{Y=0} \equiv \sigma_R^0. \qquad (27)$$

The parameter r in Tab.1 gives the ratio of directly produced to all negative pions. According to the detailed study of resonance production by LEBC-EHS [33] Collaboration the ratio r is about 0.5.

When analysing the information on the fraction of $\pi^-$ coming from resonance decays one takes into account resonance which can be identified among final state hadrons, like $\rho, \omega, \eta, K^*, f_2, \Delta$ and pions which cannot be attributed to decays of these resonances are referred to as "direct" ones. By itself this does not give the dynamical origin of "direct" pions. In a similar way as in Ref. [19] we shall assume that direct pions are "direct" pions. In a similar way as in Ref. [19] we shall assume that direct pions are originated by a broad resonance of very large width, that means $\tau_d = 0$, and the mass of the $\rho$.

Values of parameters of resonance cross-sections used in our calculations are given in Tab.1. The letter "g" indicates that the corresponding value has been guessed on the basis of data.



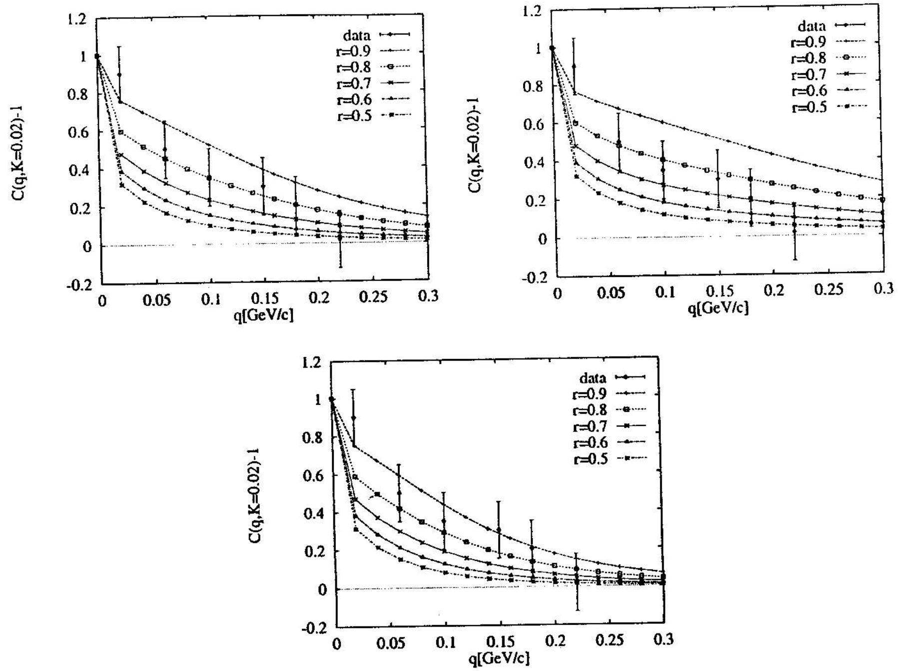


Fig. 3. Dependence of the correlation function on the ratio $r$ of direct to all pions, $K_T =$ 20MeV/c, a) $\tau_f = 0.2fm/c$ , b) $\tau_f = 0.3fm/c$, c) $\tau_f = 0.4fm/c$.

We shall now present results of our calculation of $C(q; K_T)$ and compare the results with data of EHS/NA22 Collaboration [34].

In Fig.2 we present contributions of individual resonances to the correlation function $C(q, K)$ for $K = K_T$=20 MeV. As expected and as found also in Ref. [19] resonances with large life-times lead to a very narrow $C(q, 0)$. On the other hand direct pions, originated in our model by decays of a resonance with a vanishing width give a rather broad $C(q, 0)$. For the case of $\tau_f$=0.3fm/c shown in Fig.2a mixing of resonances with direct pions can give a reasonable description of the data. We shall study this issue in more detail below. On the other hand, in the case of $\tau_f$=1 fm/c, Fig.2b, the contribution of direct pions is already more narrow than the data what shows that in this case there is no mixture of resonances and direct pions which could agree with the data of EHS/NA22 Collaboration [34].

In Fig.3 we compare the data with our results obtained with a few values of the parameters $r$ and $\tau_f$. The comparison indicates that increasing $\tau_f$ can be compensated to some extent by increasing $r$. This is also natural, since inreasing $\tau_f$ makes the region from which pions are emitted broader, whereas increasing $r$ makes it more narrow.

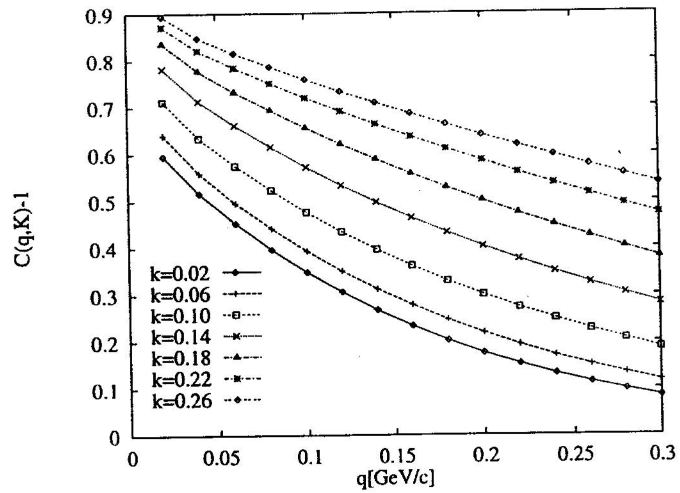


Fig. 4. Plot of correlation functions for different $K_T$, formation time of resonances $\tau_f = 0.3 fm/c$, $r = 0.8$.

Values of $r$ corresponding to larger values of $\tau_f$ are close to $r \approx 0.85 - 0.9$ what may be too high, taking into account the estimated $r \approx 0.5$ in Ref. [33]. Note that even for $\tau_f$= 0.2 fm/c the value of $r$ is almost 0.8 what may be still too large.

As a consequence the present results indicate that the formation time of resonances $\tau_f$ is surprisingly short of about 0.1–0.2 fm/c and that the ratio of direct to all pions is probably a bit larger than the experimental estimate [33] $r \approx 0.5$.

In Fig.4 we plot the correlation function $C(q, K_T)$ as function of $q$ for a set of fixed values of $K_T$. With increasing $K_T$ the shape of $C(q, K_T)$ as a function of $q$ is more flat. This is natural, since for increasing $K_T$ the contribution of long living resonances $\eta$ and $\omega$ decreases because of their three body decays leading to softer transverse momentum spectra of pions from their decays.

On the other hand the correlation functions in Fig.4 can be made consistent with the data only assuming the presence of the chaoticity parameter $\lambda = \lambda(K_T)$ with a rather strong dependence on the average transverse momentum $K_T$.

In Fig.5 we present a comparison of our results for a few values of $K_T$ with the data of Ref. [32]. In each of Figs.5 we present the data [32], the calculated $C(q, K_T)$ which is much higher than the data in particular for larger values of $K_T$ and finally the correlation function multiplied by a suitably chosen value of the chaoticity parameter: $\lambda(K_T)C(q, K_T)$. Values of $\lambda(K_T)$ required by the data are presented in Table 2 and shown also in Fig.6 together with a Gaussian fit of $\lambda(K_T)$

$$\lambda(K_T) = \exp\left(-\frac{K_T^2}{2\sigma_K^2}\right),$$



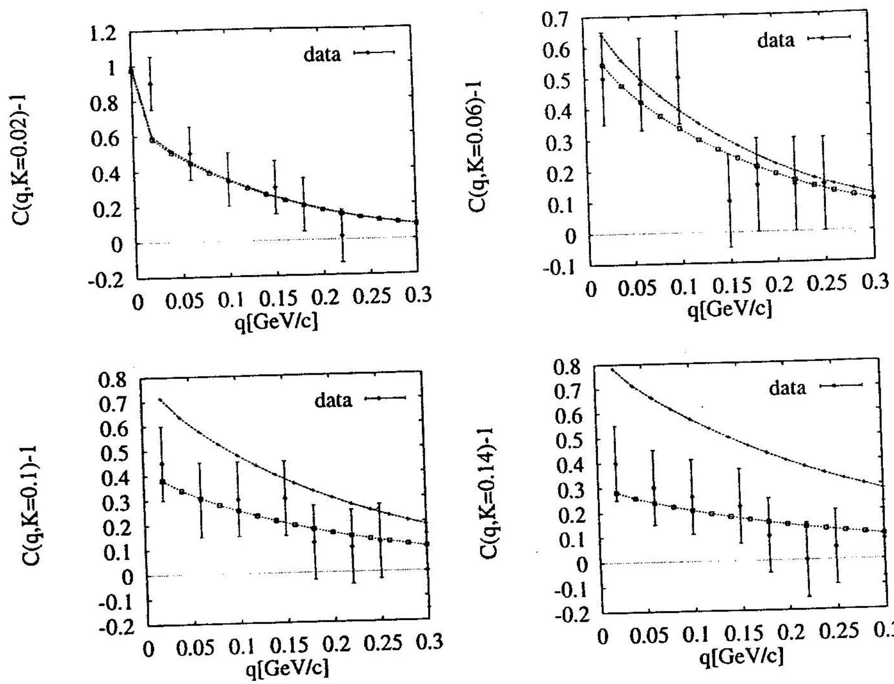


Fig. 5. Calculated correlation functions $C(q, K_T)$ compared with the data for different values of $K_T$. The upper curve corresponds to $\lambda = 1$, for the lower curve the parameter $\lambda(K_T)$ determined by the data.

Table 2: Dependence of the choaticity parameter $\lambda$ on $K_T$).

| $K_T$(MeV/c) | 20 | 60 | 100 | 140 | 180 | 220 | 260 |
|---|---|---|---|---|---|---|---|
| $\lambda$ | 0.976 | 0.838 | 0.533 | 0.360 | 0.258 | 0.208 | 0.118 |
| $\Delta\lambda$ | 0.05 | 0.15 | 0.05 | 0.07 | 0.10 | 0.07 | 0.07 |

where $\sigma_K^2 = 0.104$ GeV/c.

Results presented in Figs.4,5 and 6 and in Table 2 show that the present model can be consistent with the data only at the price of assuming the presence of chaoticity parameter $\lambda(K_T)$ strongly dependent on $K_T$. We shall come back to this point in in the next section.

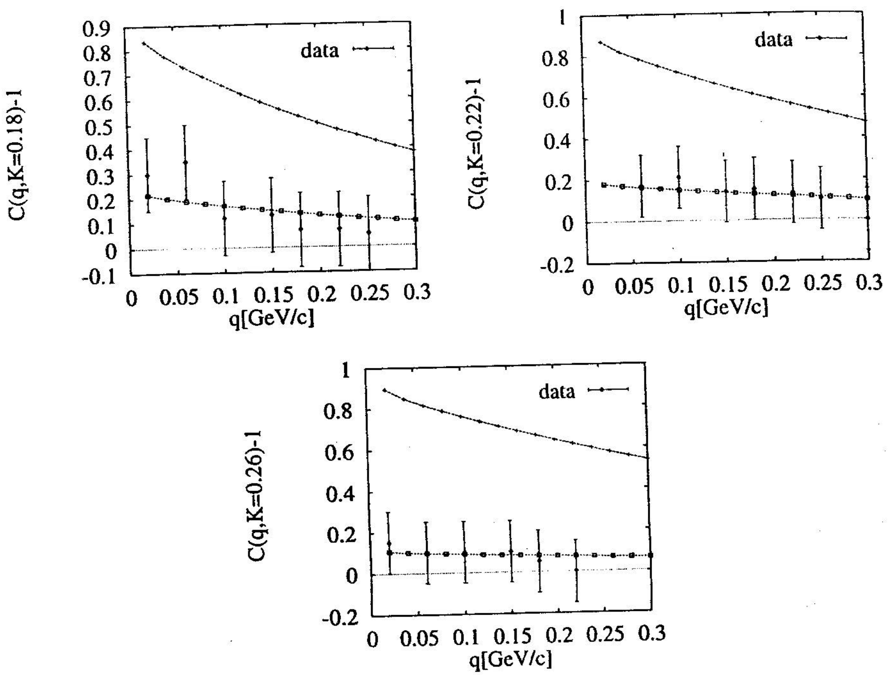


Fig. 5. (continued)

## 5 Comments and conclusions

It should be stressed that the present model is only a rough approximation. In a more ambitious model one should start with the amplitudes for resonance production and amplitudes for resonance decays to the observed pions. Such a study will hopefully be performed in the near future, but it will be technically much more complicated than the present simplified model. One can also hope that a model based on amplitudes and not on Wigner density distributions will naturally lead to the strong dependence of $\lambda$ on $K_T$ required by the data.

In the present paper and in Ref. [19] we have shown that the experimental data [34] of the EHS/NA22 Collaboration on $\pi^-\pi^-$ correlations can be reasonably described when one takes explicitly into account contributions of known resonances decaying into $\pi^-$.In the model we have also obtained without any additional assumptions the change of slope of the correlation function $C(q_L;K_T)$ with increasing $K_T$.

The effects of resonance production on correlations of identical pions have been studied in detail also by Lednický and Progulova [14].

In their model they do not consider the formation time of resonances as used in Ref.



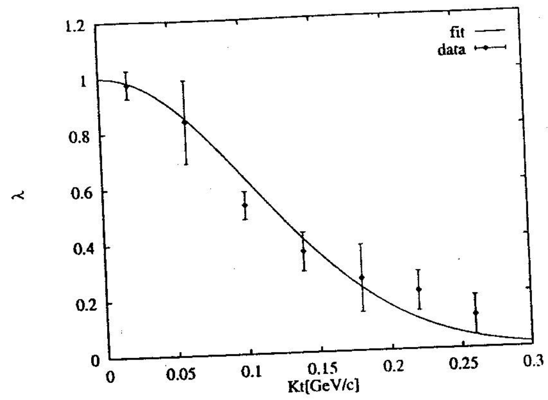


Fig. 6. Dependence of $\lambda(K_T)$ described by a Gaussian.

[19] but introduce instead as a parameter the effective radius $r_0$ of the region in which resonances are produced. Their analysis has lead to the value

$$r_0 = 0.55 \pm 0.08(stat.) \pm 0.10(syst.) \text{ fm}.$$

This can be compared with results of Ref. [19], where all resonances are produced in the same point but due to the formation time they decay in the distance roughly given by $z_f \approx \sinh(y_R)\tau_f$, where $y_R$ is the rapidity of a resonance which gives as a decay product a pion with low momentum in the cms of collision. For the $\rho$ meson we have [19] $y_R \approx$1.67 and taking $z_f \approx 0.55$ we obtain $\tau_f \approx 0.2$fm/c which corresponds to results obtained in Ref. [19]. More studies along these lines are certainly desirable.

We have not studied the single particle spectra, as done in detail in Ref. [30], since it is known that these spectra can be desribed via resonance decays. We are however well aware of the fact that simultaneous description of both single particle spectra and correlations is substantial for the determination of model parameters and we shall come back to this issue in the near future.

The problem which remains to be clarified is a rather strong dependence of the chaoticity parameter $\lambda(K_T)$ on $K_T$. The problem is probably caused by the fact that we are not working with the amplitude containing resonance production and decay but with Wigner functions corresponding to probability densities. As shown by B.Andersson and M.Ringer [36] in their study of correlations based on the Lund model where amplitudes for production of multiparticle system are known, the chaoticity parameter is calculable.To work with amplitudes in a model of resonance production and decay is in principle possible, but technically rather difficult.

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## Appendix A: Relationship to the model with vanishing transverse momenta of resonances

In the present section we shall start with the general formula Eq.(11) and study approximations which lead to the simplified model of Ref. [19]. In this model transverse momenta of resonances vanish and only slow pions in the c.m.s. of hadronic collisions were studied. In this frame we thus have $\vec{K} = 0$, $K_0 = m \equiv m_\pi$, $y \equiv y_\pi = 0$. The general expression Eq.(11) considered for simplicity for a resonance R decaying to two pions then simplifies to

$$\tilde{S}_{R\to\pi}(q;K) = \frac{M_R}{4\pi\sqrt{E^{*2}-m_\pi^2}}\frac{1}{m_T M_{RT}|\sinh(y_R)|} \times\left[\frac{\Gamma}{\Gamma - iq\frac{P(y_R)}{M_R}}\tilde{S}^f_R(q;P(y_R)) + \frac{\Gamma}{\Gamma - iq\frac{P(-y_R)}{M_R}}\tilde{S}^f_R(q;P(-y_R))\right], \quad \text{(A1)}$$

where $P(y_R)$ is the four-momentum of a resonance with mass $M$ and rapidity $Y = y_R$. Dynamics of resonance formation has been described in Ref. [19] by a single parameter $\tau_f$. The probability that a resonance has been already formed in its proper time $\tau$ has been expressed in Eq.(15).

Taking into account Lorentz time dilatation the function $S^f_R(x;P)$ for a resonance with rapidity $y_R > 0$ is given as

$$S^f_R(x;P) = \frac{1}{z_f}\exp(-z/z_f)\delta(T - \frac{1}{v_R}z)\Theta(z), \quad \text{(A2)}$$

where $z_f = \sinh(y_R)\tau_f$, $v_R = \tanh(y_R)$. Suppose now that the resonance R with mass $M_R$ decays into two pions with masses $m_\pi = m$. Suppose further that the pion taking part in Bose-Einstein correlations is show in the c.m.s. of hadronic collisions. This corresponds to the case $\vec{K} = 0, y = 0$. In this situation $E^* = M_R/2$ and

$$y_R = \operatorname{arccosh}(\frac{M_R}{2m}), \qquad \cosh(y_R) = \frac{M_R}{2m}$$

$$\sinh(y_R) = \sqrt{\left(\frac{M_R}{2m}\right)^2 - 1}, \qquad y_R = \ln\left(\left(\frac{M_R}{2m}\right) + \sqrt{\left(\frac{M_R}{2m}\right)^2 - 1}\right)$$

Assuming further that the interfering pions have the same energy, that means $q_0 = k_{10} - k_{20} = 0$ we find for the two terms in Eq.(9)

$$\tilde{S}^f_R(q;P(y_R)) = \frac{1}{2z_f}\int_0^\infty e^{-iqz}e^{-z/z_f}\mathrm{d}z = \frac{1}{2(1+iqz_f)} \quad \text{(A3)}$$



$$\tilde{S}^f_R(q;P(-y_R)) = \frac{1}{2z_f}\int_{-\infty}^0 e^{-iqz}e^{z/z_f}\mathrm{d}z = \frac{1}{2(1-iqz_f)} \quad \text{(A4)}$$

Inserting that into Eq.(11) and making use of $q\cdot P(y_R) = -qM_R\sinh(y_R)$ and $q\cdot P(-y_R) = qM_R\sinh(y_R)$, we find

$$\begin{aligned}\tilde{S}_{R\to\pi}(q;K) &= \frac{M_R}{4\pi\sqrt{E^{*2}-m_\pi^2}}\frac{1}{2m_T M_{RT}|\sinh(y_R|)} \left[\frac{1}{1+iqz_d}\cdot\frac{1}{1+iqz_f} + \frac{1}{1-iqz_d}\cdot\frac{1}{1-iqz_f}\right] \\ &= \frac{M_R}{4\pi\sqrt{E^{*2}-m_\pi^2}}\frac{1}{m_T M_{RT}|\sinh(y_R)|}\cdot\frac{1-q^2 z_f z_d}{[1+(qz_f)^2][1+(qz_d)^2]},\end{aligned} \quad \text{(A5)}$$

where $z_d = \sinh(y_R)\tau_d = \sinh(y_R)/\Gamma$. This is up to the factor $M_R/(4\pi\sqrt{E^{*2}-m_\pi^2})$, which has been by error omitted in Ref. [19], the structure of resonance contribution to the correlation function used in Ref. [19]. The absence of this factor does not modify the resulting correltions very much, since the resonnces we consider have similar masses and values of $E^*$ are also rather similar.

Note that $z_d$ is the distance travelled by the resonance R moving with velocity $v_R = \mathrm{tgh}(y_R)$ during the Lorentz dilated time $t_d = \cosh(y_R)\tau_d$.

## Appendix B: Relationship between models based on resonance formation and decay and hydrodynamical models

In very interesting papers [36,37] the EHS/NA22 Collaboration have recently analyzed their data on single pion distributions and correlations of $\pi^-$ pairs. The analysis has been based on the hydrodynamical model [38–41]. In what follows we shall refer to this model as to CLZ one (Csörgö, Lörstad, Zimányi). In this model the authors use a few parameters characterizing the hydrodynamical evolution of the matter formed in $\pi$N interactions at 250 GeV/c. The time-evolution of the system is given by the probability distribution of hadronization (freeze- out) in the proper time of parts of the system. This distribution is described by a Gaussian function with mean value $\tau_f$ and dispersion $\Delta\tau$. Using the EHS/NA22 data the authors of Ref. [30] have obtained

$$\tau_f = 1.4 \pm 0.1\text{fm}/c \qquad \Delta\tau \geq 1.3 \pm 0.3\text{fm}/c.$$

The value of $\tau_f$ is rather close to the mean life- time of the $\rho$-meson, $\tau_\rho = \hbar/150$ MeV= 1.33 fm/c. As pointed out by the authors of Ref. [30] the values of $\tau_f$ and $\Delta\tau$ found indicate that the emission process occurs during almost all of hydrodynamical evolution.

It is also worth noting that for an exponential decay process described by the probability density $p(\tau) = \frac{1}{\tau_f}\exp\left(-\frac{\tau}{\tau_f}\right)$ we get $\langle(\Delta\tau)^2\rangle = \langle\tau^2\rangle - \langle\tau\rangle^2 = \tau_f^2$, what would make $\Delta\tau \approx \tau_f$ quite natural.

The CLZ refer to their model [38–41] sometimes as to the core-halo model, where the halo corresponds to long living resonances, like $\omega, \eta$, etc. and the core which is described by hydrodynamics. The halo is responsible for the decrease of the chaoticity parameter $\lambda$, since the contribution of these resonances is non- vanishing only for relative momenta smaller than the experimental resolution.

In our model we have allowed for a possibility of a formation time for resonances. The resulting value of this time is rather short, about 0.1–0.4fm/c. In our model all resonances start to be formed in one point and it is quite possible that the value of the formation time is just taking into account the dimensions of the $\pi$p collision region. The small value of the formation time is in our opinion an evidence against a presence of truly hydrodynamical processes, since the introduction of temperature requires thermalization and within very short formation times the thermalization can hardly be established. Formation times larger than or at least comparable with the mean decay time of resonances like the $\rho$-meson could be compatible with thermalization and hydrodynamical evolution in $\pi$p interactions. Although our parametrization of the time-dependence of the formation of resonances is rather primitive, it would be quite interesting to learn what formation times are required by the data on Bose-Einstein correlations in hadron-nucleus and light ion induced collisions.

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