acta physica slovaca vol. 48 No. 5, 549 – 562 October 1998

STATIONARY SPHERICAL SHELLS AROUND KERR–NEWMAN NAKED SINGULARITIES

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Received 17 July 1998, accepted 27 August 1998

It is shown that in the field of some Kerr–Newman naked singularities a stationary spherical shell of charged dust can exist, with the specific charge being the same for all particles of the dusty shell. Gravitational attractions acting on the particles are balanced by electromagnetic repulsions in such a way that the shell is stable against radial perturbations. Particles of the shell move along orbits with constant latitude and radius. Rotation of the shell is differential. The shell is corotating relative to static observers at infinity, but it is counterrotating relative to the family of locally non-rotating observers. No such a shell can exist in the field of Kerr–Newman black holes.

1 Introduction

There is a strong evidence that black holes play the central role in astronomical phenomena connected with active galactic nuclei and some extraordinary galactic binary systems (see [1] and references therein). The most general black holes are represented by the Kerr–Newman solution of Einstein-Maxwell equations. However, the Kerr–Newman solution represents also naked singularities, i.e., spacetime singularities not hidden behind an event horizon, which could be conceivable for explaining the effects connected with quasars and active galactic nuclei along with the black-hole solutions. The conjecture of cosmic censorship [2] suggests that no naked singularities evolve from regular initial data, however, the proof and even precise formulation of the conjecture still remains one of the biggest challenges in general relativity. Therefore, it is important to consider possible astrophysical consequences of Kerr–Newman naked singularities. Of particular interest are those effects that could distinguish a naked singularity from black holes.

Motion of test particles can be considered as a first step on the way to understand physical processes governed by black holes and naked singularities, because it illustrates the geometry and electromagnetic field of the backgrounds. The behaviour of charged

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particles in charged backgrounds has been considered in a number of papers. For example, Abramowicz and Bičák [3] considered an interplay between forces acting on charged particles in the field of a Reissner-Nordström black hole; Semerák [4] compared gravitoelectric and gravitomagnetic fields of the Kerr geometry with the electric and magnetic fields of the Kerr–Newman geometry, and Semerák and Bičák [5] discussed dynamical properties of generally non-Keplerian equatorial and polar orbits around Kerr–Newman black holes. Bičák, Stuchlík and Balek discussed some aspects of the motion of test charges in both the black-hole and naked-singularity Kerr–Newman fields: general features of the radial motion, and the motion along the axis of symmetry [6], the motion in the equatorial plane [7], and the shell of incoherent charged matter falling radially from rest at infinity with zero total angular momentum onto a Kerr–Newman black hole [8].

Here, we shall show that in the field of a wide class of Kerr–Newman naked-singularity spacetimes a spherical charged shell consisting of particles with zero total angular momentum, covariant energy equal to their rest energy, and an appropriately chosen specific charge, can be stationary at a sphere with radius chosen correspondingly. The particles of the shell move along orbits with constant latitude, and their trajectories do not cross. Of course, their azimuthal coordinate varies due to the dragging. We shall also demonstrate that no stationary spherical shell of this kind can exist in the field of Kerr–Newman black holes.

In Section 2, the Kerr–Newman geometry is described in terms of the standard locally non-rotating frames, and the corresponding electromagnetic field is given. The equations of motion of charged test particles are written down, and the components of momentum and velocity of test particles in the locally non-rotating frames are given. Some relevant aspects of the latitudinal motion [9] are briefly summarized, with attention being focused on trajectories with constant latitude, which can be considered as an analogy of purely radial trajectories in non-rotating backgrounds.

In Section 3, the stationary spherical shells consisting of charged test particles will be discussed. Surprisingly enough, in a wide family of Kerr–Newman naked-singularity spacetimes such a shell can be constructed from particles of the same kind, i.e., carrying the same specific charge. However, the construction is possible only if each particle has zero total angular momentum and covariant energy equal to rest energy. At a given spacetime, the radius of the shell and the specific charge of its particles will be determined, and it will be shown that the shell is stable with respect to radial perturbations. The rotation of the shell will be given relative to both static distant observers and the locally non-rotating observers located at the radius of the spherical shell. It will be shown that the radius of the shell can be determined by the distant static observers through the redshift of photons moving along the outgoing principal null congruence of the Kerr–Newman spacetimes. Concluding remarks will be presented in Section 4.

2 The geometry and equations of motion

Using the standard Boyer-Lindquist coordinates t, r, θ, ϕ and the geometric system of units ($c = G = 1$), we give the Kerr–Newman metric in terms of tetrad of differential forms of the locally non-rotating frames (LNRF) [10]:

$$ds^2 = -\left[\omega^{(t)}\right]^2 + \left[\omega^{(r)}\right]^2 + \left[\omega^{(\theta)}\right]^2 + \left[\omega^{(\phi)}\right]^2 \qquad (1)$$

$$\omega^{(t)} = \sqrt{\Delta\Sigma/A}\, dt, \qquad (2)$$

$$\omega^{(r)} = \sqrt{\Sigma/\Delta}\, dr, \qquad (3)$$

$$\omega^{(\theta)} = \sqrt{\Sigma}\, d\theta, \qquad (4)$$

$$\omega^{(\phi)} = \sqrt{A/\Sigma}\, \sin\theta\,(d\phi - \Omega\, dt), \qquad (5)$$

where

$$\Delta = r^2 - 2Mr + a^2 + e^2, \qquad (6)$$

$$\Sigma = r^2 + a^2\cos^2\theta, \qquad (7)$$

$$A = (r^2 + a^2)^2 - a^2\Delta\sin^2\theta, \qquad (8)$$

$$\Omega = (a/A)(2Mr - e^2). \qquad (9)$$

Here M denotes the mass of the background, a — its angular momentum per unit mass, e — its electric charge. Ω is the angular velocity of locally non-rotating observers relative to distant static observers. If $M^2 \geq a^2 + e^2$, the geometry determines a black-hole spacetime, if $M^2 < a^2 + e^2$, it determines a naked-singularity spacetime. In the following we put $M = 1$; the coordinates t, r and the parameters a, e then become dimensionless.

The electromagnetic field of the Kerr–Newman background is determined by the vector potential, which is in coordinate components given in the form

$$A_\mu = -\frac{1}{\Sigma}\left(er\delta^t_\mu - aer\sin^2\theta\delta^\phi_\mu\right). \qquad (10)$$

The motion of a test particle with rest mass m, and electric charge ϵ (both being dimensionless since $M = 1$) is given by the Lorentz equation

$$m\frac{Du^\mu}{D\tau} = \epsilon F^\mu{}_\nu \qquad (11)$$

and the normalization condition

$$g_{\mu\nu}\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda} = -m^2. \qquad (12)$$

$u^\mu = dx^\mu/d\tau$ is four-velocity of the particle, and τ is its proper time, related to the affine parameter λ by the relation $\tau = m\lambda$. For photons, $m = 0$. The tensor of the electromagnetic field $F_{\mu\nu} = A_{\nu,\mu} - A_{\mu,\nu}$. The motion of uncharged particles and photons is determined by the geodesic equations.

In the integrated and separated form, the motion is determined by Carter's equations [10]:

$$\Sigma\frac{dr}{d\lambda} = \pm\sqrt{R(r)}, \quad (13)$$

$$\Sigma\frac{d\theta}{d\lambda} = \pm\sqrt{W(\theta)}, \quad (14)$$

$$\Sigma\frac{d\phi}{d\lambda} = -\frac{P_\theta}{\sin^2\theta} + \frac{aP_r}{\Delta}, \quad (15)$$

$$\Sigma\frac{dt}{d\lambda} = -aP_\theta + \frac{(r^2+a^2)P_r}{\Delta}, \quad (16)$$

where

$$R = P_r^2 - \Delta(m^2r^2 + \mathcal{K}), \quad (17)$$

$$W = (\mathcal{K} - a^2m^2\cos^2\theta) - \left(\frac{P_\theta}{\sin\theta}\right)^2, \quad (18)$$

and

$$P_r = E(r^2+a^2) - a\Phi - \epsilon e r, \quad (19)$$

$$P_\theta = aE\sin^2\theta - \Phi. \quad (20)$$

In addition to the constants of motion m, E (energy at infinity), Φ (the axial angular momentum at infinity), and $\mathcal{K}$ (related to the total angular momentum), it is convenient to introduce the constant of motion

$$Q = \mathcal{K} - (\Phi - aE)^2, \quad (21)$$

because $Q = 0$ corresponds to motion in the symmetry plane of the geometry ($\theta = \pi/2$). For null geodesics λ can be adjusted to give relation $E = \omega$, ω being the frequency of the photon at infinity.

The components of momentum relative to LNRF are

$$p^{(\alpha)} = p^\mu \omega_\mu^{(\alpha)}, \quad (22)$$

where $p^\mu = dx^\mu/d\lambda$. The components of velocity relative to the family of locally non-rotating observers are

$$v^{(r)} = \frac{p^{(r)}}{p^{(t)}} = \frac{\sqrt{A}}{\Delta}\frac{dr}{dt}, \quad (23)$$

$$v^{(\theta)} = \frac{p^{(\theta)}}{p^{(t)}} = \sqrt{\frac{A}{\Delta}}\frac{d\theta}{dt}, \quad (24)$$

$$v^{(\phi)} = \frac{p^{(\phi)}}{p^{(t)}} = \frac{A\sin\theta}{\Sigma\sqrt{\Delta}}\left(\frac{d\phi}{dt} - \Omega\right). \quad (25)$$

The latitudinal motion is independent of the charge of test particles (for a detailed analysis see [9]). It is important that the motion along constant latitude is possible out of the equatorial plane. Introducing new parameter

$$\Gamma = E^2 - m^2, \quad (26)$$

we can state that if the conditions

$$\Gamma > 0,\ Q + \Phi^2 < a^2\Gamma \quad (27)$$

are fulfilled, the motion along a constant latitude θ is determined by the relations

$$Q = -a^2\Gamma\cos^4\theta, \quad (28)$$

$$\Phi^2 = a^2\Gamma\sin^4\theta. \quad (29)$$

It can be shown (see [11]) that these conditions can be combined with the conditions of the motion on constant radial coordinate, allowing the existence of circular off-equatorial orbits in the field of Kerr–Newman naked singularities. However, in order to obtain a spherical shell, particles moving along different circles with constant latitude must have different specific charges.

3 The stationary spherical shells

There is an additional possibility of the motion along trajectories with $\theta = \text{const}$, if the conditions

$$E = m,\ Q = \Phi = 0 \quad (30)$$

are satisfied. The charge parameter

$$Z = \frac{\epsilon}{m}e \quad (31)$$

must be restricted by inspecting the equations of motion. For the constants of motion restricted by the conditions (30), Carter's equations of motion simplify to the following form:

$$\Sigma\frac{dr}{d\tau} = \pm\sqrt{\frac{R}{m^2}}, \quad (32)$$

$$\Sigma\frac{d\theta}{d\tau} = 0, \quad (33)$$

$$\Sigma\frac{d\phi}{d\tau} = -a\left(1 - \frac{r^2+a^2-Zr}{\Delta}\right), \quad (34)$$

$$\Sigma\frac{dt}{d\tau} = -a^2\sin^2\theta + \frac{(r^2+a^2)(r^2+a^2-Zr)}{\Delta}, \quad (35)$$

where

$$\frac{R}{m^2} = 2(1-Z)r(r^2+a^2) + (Z^2-e^2)r^2 - a^2e^2. \quad (36)$$

For Kerr–Newman naked singularities ($a^2 + e^2 > 1$) there is at all radii

$$\Delta = r^2 - 2r + a^2 + e^2 > 0. \tag{37}$$

A negative sign on the r.h.s. of (32) corresponds to particles that move towards the hole, a positive sign corresponds to the outward motion. Notice that it is evident from (36) that charged particles falling from infinity must have

$$Z \le 1. \tag{38}$$

(Particles with $Z > 1$ can reach infinity only with $E > m$.)

The turning points of the radial motion, determined by equation $R(r) = 0$, are given by the condition

$$Z = Z^{t}_{\pm}(r; a, e) \equiv \frac{1}{r}\left[r^2 + a^2 \pm \sqrt{\Delta\,(r^2 + a^2)}\right] \tag{39}$$

which restricts values of the charge parameter Z.

The "effective potentials" $Z^{t}_{\pm}(r; a, e)$, determining regions forbidden for the radial motion, are independent of the latitudinal coordinate θ. Especially this property of the effective potentials enables construction of a stationary spherical shell, if the charge parameter of its particles corresponds to an extreme point of the effective potentials. Now, we have to discuss properties of the functions $Z^{t}_{\pm}(r; a, e)$.

Clearly, for the naked-singularity spacetimes these functions are well defined for all radii, since $\Delta > 0$. Both the functions $Z^{t}_{\pm}(r; a, e)$ diverge at $r = 0$; there is $Z^{t}_{-} \to -\infty$, and $Z^{t}_{+} \to \infty$. Further, for $r \to \infty$, there is $Z^{t}_{-} \to 1$, while $Z^{t}_{+} \to \infty$. The zero point of $Z^{t}_{-}(r; a, e)$ is located at

$$r_z = \frac{e^2}{2}; \tag{40}$$

turning points of uncharged particles are located at the sphere $r = r_z$. Extrema of $Z^{t}_{\pm}(r; a, e)$ are given by zeros of

$$\frac{\partial Z^{t}_{\pm}}{\partial r} = 1 - \frac{a^2}{r^2} \pm \frac{r^3(r-1) + a^2(r - a^2 - e^2)}{r^2\sqrt{\Delta(r^2 + a^2)}}, \tag{41}$$

i.e., by solutions of

$$B(r; a, e) \equiv \left(r^2 - a^2\right)^2 \left(r^2 + a^2\right)\Delta - \left[r^3(r-1) + a^2\left(r - a^2 - e^2\right)\right]^2 = 0. \tag{42}$$

For $r \to \infty$, there is $\partial Z^{t}_{+}/\partial r \to 2$, while $\partial Z^{t}_{-}/\partial r$ disappears as

$$\frac{\partial Z^{t}_{-}}{\partial r} = \frac{(1 - e^2)}{2r^2}. \tag{43}$$

Therefore, we can conclude that for $r \to \infty$ the function $Z^{t}_{-}(r; a, e) \to 1$ from above (below), in Kerr–Newman spacetimes with $e^2 < 1$ ($e^2 > 1$).

At those r's which are the solutions of (42), the *circular off-equatorial* ($\theta \neq \pi/2$) orbits can exist for particles with $E = m$, $\Phi = Q = 0$, and corresponding value of Z, given by (39). These circular orbits, forming a stable stationary spherical shell, can exist only in the field of some Kerr–Newman naked singularities; their characteristics will be given later. Now, we shall prove in a straightforward way that no such orbits can exist outside the event horizon of black holes. In the case of extreme black holes ($a^2 + e^2 = 1$) there is

$$B(r; a, e) = -a^2(r-1)^2(r^4 + 2r^3 + a^2r^2 + a^2 - a^4), \tag{44}$$

and we immediately see that $B(r; a, e) < 0$ at all $r > 0$ — the orbits cannot exist in the field of extreme black holes. In the case of non-extreme black holes ($a^2 + e^2 < 1$), it is clear that at $r \ge r_+$ the inequality

$$\begin{aligned} B(r; a, e) < -a^2[(r^4 + a^2r^2 - a^4)\Delta &+ 2r^3(r-1)(r - a^2 - e^2) \\ &+ a^2(r - a^2 - e^2)^2] < 0 \end{aligned} \tag{45}$$

holds — the orbits cannot exist outside the outer horizon $r_+ = 1 + (1 - a^2 - e^2)^{1/2}$. It will be shown later that such orbits cannot exist even under the inner horizons.

In a naked-singularity spacetime with parameters a, e given, the radii of stationary spherical shells are determined by the relation

$$\begin{aligned} e^2 &= e^2_{\mathrm{ex}\pm}(r; a) \\ &\equiv \frac{r^2 - a^2}{2a^4}\left[(r^2 + a^2)^2 - 2a^2r \pm \mathrm{sign}(r^2 - a^2)(r^2 + a^2)\sqrt{(r^2 + a^2)^2 - 4a^2r}\right] \end{aligned} \tag{46}$$

Denoting the radius of such a stationary shell as r_s, the specific charge of particles of the shell is given by $Z^{t}_{-}(r_s; a, e)$ or $Z^{t}_{+}(r_s; a, e)$. For $r \to \infty$ (and any value of the parameter

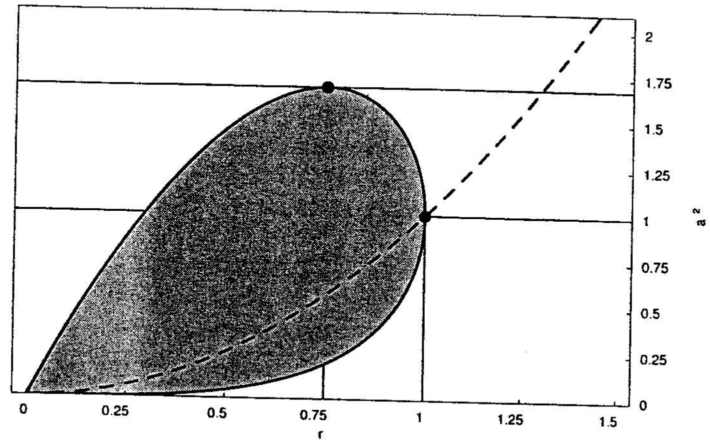

Fig. 1. The reality condition of the family of functions $e^2_{\mathrm{ex}\pm}$ is given by $a^2_{\mathrm{r(ex)}\pm}$ (solid lines). It is fulfilled outside the shaded region. The zero points of $e^2_{\mathrm{ex}\pm}$ are given by $a^2_{\mathrm{z(ex)}}$ (dotted line).

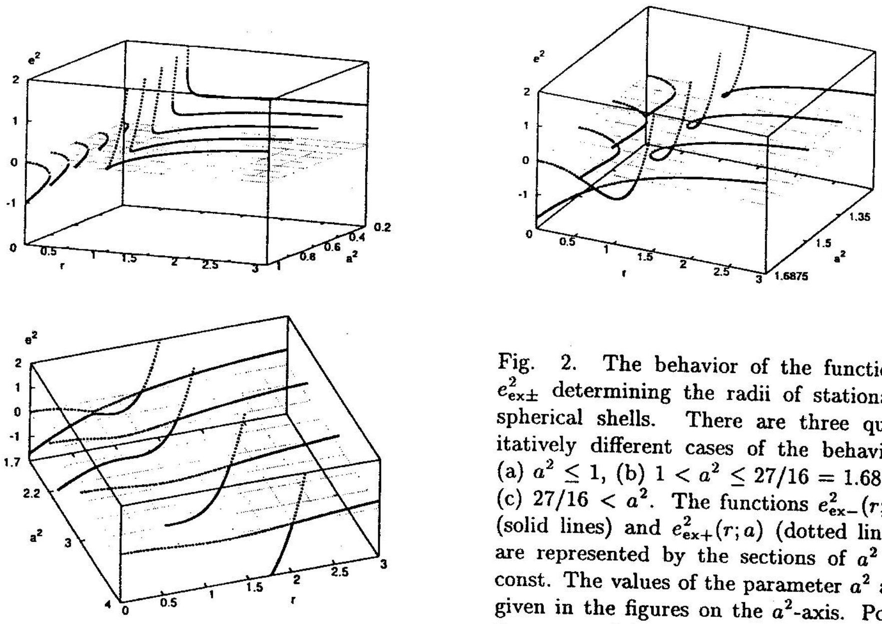

Fig. 2. The behavior of the functions $e^2_{ex\pm}$ determining the radii of stationary spherical shells. There are three qualitatively different cases of the behavior: (a) $a^2 \leq 1$, (b) $1 < a^2 \leq 27/16 = 1.6875$, (c) $27/16 < a^2$. The functions $e^2_{ex-}(r;a)$ (solid lines) and $e^2_{ex+}(r;a)$ (dotted lines) are represented by the sections of $a^2 =$ const. The values of the parameter a^2 are given in the figures on the a^2-axis. Positive parts of these functions are physically relevant only.

a), there is $e^2_{ex+}(r;a) \to \infty$, while $e^2_{ex-}(r;a) \to 1$. Zeros of the functions $e^2_{ex\pm}(r;a)$ are located at $r = 0$ and $r's$ expressed by the relation

$$a^2 = a^2_{z(ex)}(r) \equiv r^2. \tag{47}$$

The reality condition of this family of functions is

$$a^2 \geq a^2_{r(ex)+}(r), \tag{48}$$

$$a^2 \leq a^2_{r(ex)-}(r), \tag{49}$$

where

$$a^2_{r(ex)\pm} \equiv r(2 - r \pm 2\sqrt{(1-r)}). \tag{50}$$

The functions $a^2_{r(ex)\pm}$ are defined at $r \leq 1$. There is a maximum of $a^2_{r(ex)+}$ at $r = 3/4$, where $a^2_M = 27/16 = 1.6875$. If $a^2 \leq 1$, there are no zeros of $e^2_{ex\pm}(r;a)$ at $r = a$, because the function $a^2_{z(ex)}(r)$ lies in the region "forbidden" by the functions $a^2_{r(ex)\pm}$ (see Fig. 1).

A minimum of the function $e^2_{ex-}(r;a)$ is located at $r = 1$, if $a^2 < 1$. Then there is

$$e^2_{ex-}(r = 1; a) \equiv e^2_{min} = 1 - a^2, \tag{51}$$

and the function $e^2_{ex-}(r;a)$ determines maxima of the function $Z^t_-(r;a,e)$ at $r > 1$, and minima of $Z^t_+(r;a,e)$ at $r < 1$. The other minima of the function $Z^t_+(r;a,e)$ are determined by the function $e^2_{ex+}(r;a)$. The functions $e^2_{ex\pm}(r;a)$ are illustrated in Fig. 2a in the case $a^2 < 1$. Clearly, they must be positive in order to give physically relevant results. If $a^2 \geq 1$, there are no extrema of $e^2_{ex\pm}(r;a)$ in regions, where $e^2_{ex\pm}(r;a) \geq 0$. Then $e^2_{ex-}(r;a) > 0$ determines the maxima of the function $Z^t_-(r;a,e)$, while $e^2_{ex+}(r;a)$ determines the minima of $Z^t_+(r;a,e)$. For $1 < a^2 < 27/16$, the functions $e^2_{ex\pm}(r;a)$ are given in Fig. 2b, for $a^2 > 27/16$, they are given in Fig. 2c.

Now, it is clear from the behaviour of the functions $e^2_{ex\pm}(r;a)$ that no stationary spherical shells can exist even under the inner horizon of Kerr–Newman black holes.

The stationary shells can exist only if the charge parameter $Z > 1$. Particles with $Z \leq 1$ can fall from infinity; they will return back from a turning point given by the function $Z^t_-(r_s;a,e)$. Contrary to the case of Kerr naked singularities, no "radially" falling particles can reach the surface $r = 0$ of Kerr–Newman naked singularities. Uncharged particles have turning points at r_z given by (40). Particles with $Z < 0$ (attracted) have their turning points at $r_t < r_z$, and particles with $Z > 0$ (repulsed) have $r_z < r_t$ if $Z < e^2 + (4a^2/e^2)$ but $r_t < r_z$ if $Z > e^2 + (4a^2/e^2)$. For particles with $Z = 1$ the turning point is at $r = \alpha$, where

$$\alpha^2 = \frac{a^2 e^2}{1 - e^2}. \tag{52}$$

Now, the physical relevance of the effective potentials $Z^t_\pm(r_s;a,e)$ will be determined. We shall consider only particles in positive-root states, which have positive energy relative to local observers (e.g., the locally non-rotating observers). For such particles there is $dt/d\tau > 0$. Particles in negative-root states have negative energy relative to local observers, and $dt/d\tau < 0$; such particles constitute "Dirac's negative energy sea" (for details see [6]). Using equations (35) and (39) we arrive at

$$\Sigma \frac{dt}{d\tau} = -a^2 \sin^2\theta \mp \frac{(r^2 + a^2)^{3/2}}{\Delta^{1/2}}. \tag{53}$$

Therefore, the stationary shells determined by the function $Z^t_+(r_s;a,e)$ have $dt/d\tau < 0$, and their particles are in the negative-root states (unphysical from the point of view of classical physics). Physically relevant stationary shells with particles in the positive-root states are determined by $Z^t_-(r_s;a,e)$ giving $dt/d\tau > 0$. We conclude that the physically relevant stationary spherical shells in the positive-root states can exist only in the field of Kerr–Newman naked singularities with the parameter $e^2 < 1$. No such a shell can exist, if $e^2 > 1$; only shells in the negative-root states are possible in this case.

We illustrate the behaviour of the functions $Z^t_\pm(r;a,e)$ in the case of naked singularities with $e^2 < 1$, allowing existence of physically relevant stationary spherical shells, in Fig. 3. For comparison, we give the functions also in the cases of spacetimes not allowing the existence of such shells in Fig. 4: for naked singularities with $e^2 > 1$ (a), extreme black holes (b), and non-extreme holes (c). Further, we restrict our attention to the Kerr–Newman naked-singularity spacetimes with $e^2 < 1$. We give the charge

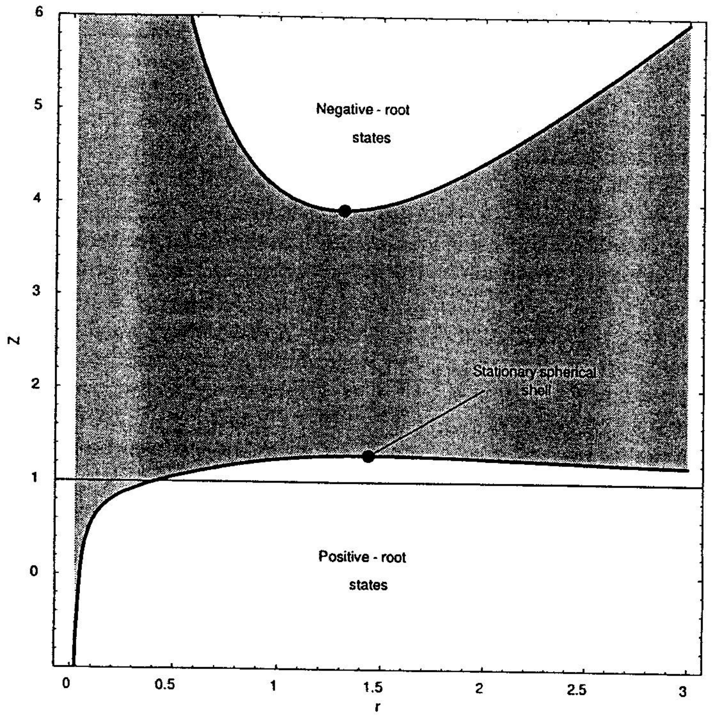

Fig. 3. The behavior of the effective potentials $Z^t_\pm$ determining radial motion in the field of the Kerr–Newman naked singularities with $e^2 < 1$, when physically relevant stable stationary spherical shells can exist. They are determined by the maximum of Z^t_-. The regions forbidden for the radial motion are shaded. The potentials are drawn for $a^2 = 1.69$, $e^2 = 0.09$.

parameter Z of the physically relevant spherical shells as a function of the parameters of these spacetimes in Fig. 5.

The azimuthal equation (34) implies the turning points of the ϕ-motion to be given by

$$Z = Z_\phi(r; e) \equiv 2 - \frac{e^2}{r}. \tag{54}$$

Common turning point of the radial and azimuthal motion exists just for uncharged particles at $r = r_z$. For particles of the stationary shells there is

$$\Sigma \frac{d\phi}{d\tau} = -a\left(1 - \sqrt{\frac{r^2 + a^2}{\Delta}}\right). \tag{55}$$

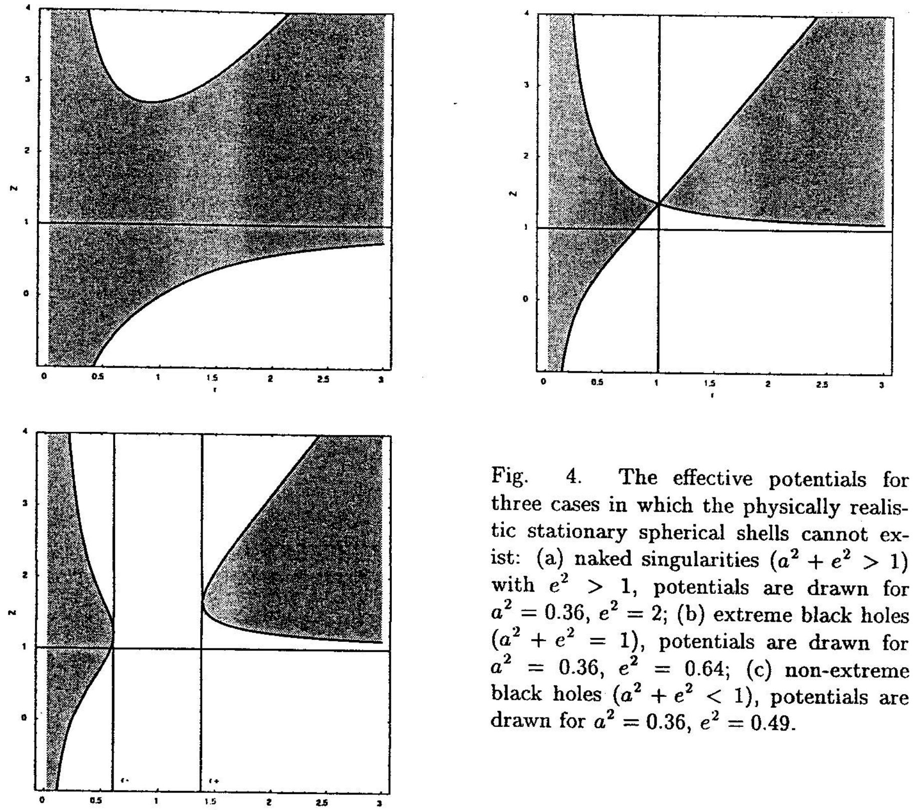

Fig. 4. The effective potentials for three cases in which the physically realistic stationary spherical shells cannot exist: (a) naked singularities ($a^2 + e^2 > 1$) with $e^2 > 1$, potentials are drawn for $a^2 = 0.36$, $e^2 = 2$; (b) extreme black holes ($a^2 + e^2 = 1$), potentials are drawn for $a^2 = 0.36$, $e^2 = 0.64$; (c) non-extreme black holes ($a^2 + e^2 < 1$), potentials are drawn for $a^2 = 0.36$, $e^2 = 0.49$.

The shells are corotating relative to distant static observers with the angular velocity given by the relation

$$\frac{d\phi}{dt} = \frac{a(r^2 + a^2 - \sqrt{\Delta})}{(r^2 + a^2)^{3/2} + a^2\sqrt{\Delta}\sin^2\theta}. \tag{56}$$

Rotation of the stationary shells is differential due to the latitudinal dependence of the angular velocity. We can simply convince ourselves that the shells are corotating relative to distant static observers.

We now give the velocity of the stationary shells relative to the family of locally non-rotating observers. The LNRF components of the velocity of charged test particles are given in Section 2. For particles of a stationary shell they read:

$$v^{(r)} = 0, \tag{57}$$
$$v^{(\theta)} = 0, \tag{58}$$

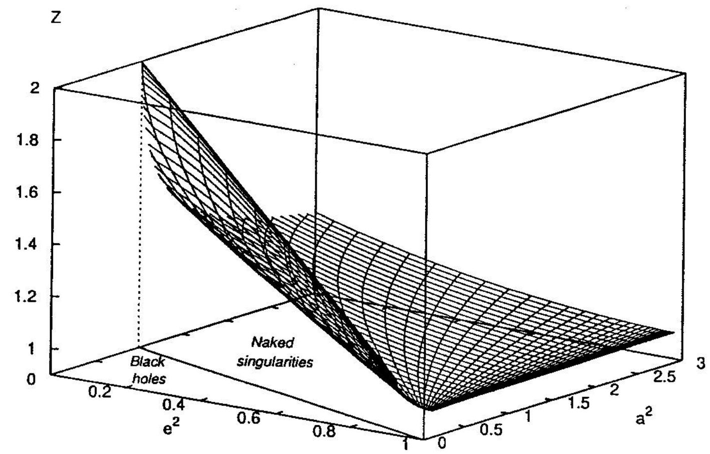

Fig. 5. The dependence of the charge parameter Z_s of the physically relevant stationary spherical shells on a^2 and e^2. The function $Z^t_-(r_s; a, e)$ is defined on the region $0 \leq e^2 \leq 1$, $a^2 + e^2 > 1$, i.e., on the subset of the region of naked singularities restricted by the condition $e^2 \leq 1$.

$$v^{(\phi)} = -\frac{\left[r^2 + a^2 - \sqrt{\Delta(r^2 + a^2)}\right] \sin\theta}{(r^2 + a^2)^{3/2} - a^2\sqrt{\Delta}\sin^2\theta}. \tag{59}$$

We can see that the stationary shells are counterrotating relative to the family of locally non-rotating observers.

Distant static observers can, in principle, determine radius of a stationary shell by using photons moving along the geodesics of principal null congruence (the outgoing PNC photons). The observers can distinguish such photons since these photons are characterized by a specific point in their plane of sky (see [12]).

The PNC photons move along θ = const surfaces with the constants of motion $\Phi = aE\sin^2\theta$, $Q = -a^2E^2\cos^4\theta$ (see [9]). Using Eqs (13) and (16), we find that an outgoing PNC photon radiated out at a given r_s and t will reach a distant observer at r^* and time t^* after the time interval $\Delta t_{\rm ph} \equiv t^* - t$ given by

$$\begin{aligned}\Delta t_{\rm ph} &= r^* - r_s \ln\frac{r^{*2} - 2r^* + a^2 + e^2}{r_s^2 - 2r_s + a^2 + e^2} + \\ &+ \frac{2 - e^2}{\sqrt{a^2 + e^2 - 1}}\left[\arctan\frac{r^* - 1}{\sqrt{a^2 + e^2 - 1}} - \arctan\frac{r_s - 1}{\sqrt{a^2 + e^2 - 1}}\right].\end{aligned} \tag{60}$$

The redshift z of the PNC photons as measured by distant observers gives information about the position r_s of a particle at the moment of emission of the PNC photon. The standard formula, $1 + z = (U^\alpha k_\alpha)_{\rm em}/(U^\alpha k_\alpha)_{\rm obs}$, where $U^\alpha_{\rm em}$, $U^\alpha_{\rm obs}$ are 4-velocities of the emitting particle and observer, $k^\alpha_{\rm em}$, $k^\alpha_{\rm obs}$ are 4-vectors tangent to the null geodesics at the moment of emission and reception of the photon, leads to the expression:

$$z = \frac{(2 - Z)r - e^2 \pm \sqrt{R/m^2}}{\Delta}. \tag{61}$$

The redshift is independent of the latitudinal coordinate of the particles of the shell, being determined by the parameters of the spacetime and the radius of the shell by the relation

$$z(r_s; a, e) = \sqrt{\frac{r_s^2 + a^2}{r_s^2 - 2r_s + a^2 + e^2}} - 1, \tag{62}$$

where $r_s = r_s(a, e)$ is given by the function $e^2_{\rm ex-}(r; a)$. We illustrate the dependence of the redshift on the parameters of the spacetimes in Fig. 6.

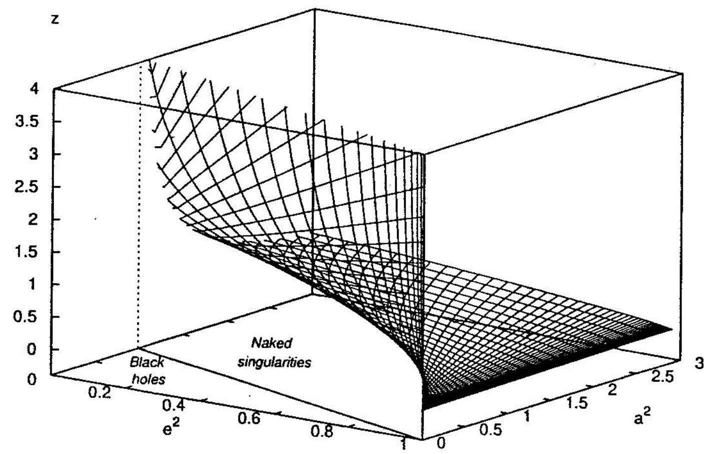

Fig. 6. The redshift $z(r_s; a, e)$ of the PNC photons radiated by the physically relevant stationary spherical shells as a function of parameters a^2 and e^2.

4 Conclusions

We have shown that a stationary shell of charged dust with constant specific charge can exist in Kerr–Newman naked-singularity spacetimes with $e^2 < 1$. The shell is corotating relative to distant static observers, but counterrotating relative to the family of locally non-rotating observers, and its rotation is differential. Distant static observers can obtain an information about the position of the shell by measuring the redshift of the outgoing PNC photons radiated by particles of the shell. The redshift will be finite and independent of the proper time of distant static observers. No such phenomenon can be observed in the case of Kerr–Newman black holes.

Acknowledgements This work has been supported by the GAČR Grant No. 202/96/020…

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