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KERR–NEWMAN–ANTI-DE SITTER BLACK-HOLE SPACETIMES WITH A SURFACE OF DEGENERACY

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Black holes in spacetimes with a negative vacuum energy, i.e., with an attractive cosmological constant $\Lambda < 0$, are described by the Kerr–Newman–anti-de Sitter geometry. It is proposed that if the specific angular momentum of a black hole and the attractive cosmological constant are combined appropriately, the spacetime can be considered as consisting of causally disconnected regions with opposite signature of the metric tensor, corresponding to opposite character of the geometry outside the black-hole horizons and between the horizons, respectively. No photons and test particles can cross a surface of degeneracy at a constant latitudinal coordinate, which separates the causally disconnected regions. Differences of the properties of the motion of test particles in the separated regions are discussed. They are given by the different normalization condition of the equations of motion, i.e., motion in the region with the opposite signature is of "tachyonic" nature. It is demonstrated in the simplest case of uncharged particles moving along the axis of symmetry.

1 Introduction

In the framework of the cosmological inflationary paradigm, bubbles of false vacuum with large values (either positive or negative) of vacuum energy-density are frequently invoked. The bubbles with a positive vacuum energy are considered in the early, inflationary, stages of the evolution of the Universe, or in the models of creation of "baby" universes [1, 2]. The bubbles with a negative vacuum energy are investigated in connection with the hypothetical phase transition leading, according to some versions of the unification theories of interactions of elementary particles, to a bubble nucleation process in the present state of the Universe with zero (or a very small) energy-density of the vacuum state. Contrary to the bubbles with a positive vacuum energy, which collapse from the point of view of an external observer, the bubbles with a negative vacuum energy have to expand from the point of view of an external observer, covering

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the whole observable Universe, eventually, because their pressure is positive, i.e., higher than zero (or a very small negative) pressure of the present vacuum state.

The phase transition leading to a state with a significant negative vacuum energy is implied, e.g., by standard Coleman-Weinberg one-loop Higgs effective potential for the field $\phi^2 \equiv \phi^\dagger \phi$, which is given by [3]:

$$V = -\frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4 + B\phi^4 \log \frac{\phi^2}{M^2} \tag{1}$$

and

$$B = \frac{1}{1024\pi^2}(9g^4 + 6g^2g'^2 + 3g'^4 + 9\lambda^2 - 3g_Y^4), \tag{2}$$

where μ and λ have the standard meaning from the theory of scalar fields, while g, g' and g_Y are the SU(2) gauge coupling, the U(1) gauge coupling and the top quark Yukawa coupling, respectively. If $B < 0$, then the vacuum is unstable, and the Coleman-Weinberg effective potential has a negative minimum at a large value of the field ϕ. It can be shown that sufficiently large top quark mass could destabilize the standard-model vacuum [4]. However, if the top quark mass is less than 190 GeV, the lifetime of the unstable zero-energy vacuum is greater than the age of the Universe. The notion of the bubbles of negative-energy vacuum states is not restricted to the Coleman-Weinberg theory. For example, they appear frequently also in the framework of the Kaluza-Klein multidimensional theories [5, 6].

Therefore, it is interesting to investigate properties of black holes in spacetimes with a negative-energy vacuum, described by the Kerr–Newman–anti-de Sitter geometry with a negative cosmological constant ($\Lambda < 0$), corresponding to cosmological attraction. For completeness, dyon spacetimes carrying a non-zero magnetic monopole charge will be taken into account.

In Section 2, pseudosingularities of the Kerr–Newman–anti-de Sitter geometry are considered. Black-hole and naked-singularity spacetimes are separated in the space of the parameters of the geometry. Besides the pseudosingularities of the radial metric component g_{rr}, determining the horizons of the black-hole spacetimes, the Kerr–Newman-anti-de Sitter geometry can have also pseudosingularities of the latitudinal metric component $g_{\theta\theta}$, separating regions of the geometry with opposite signature of the metric tensor. The latitudinal pseudosingularities will be called *surfaces of degeneracy*. Although Carter [7] excludes such possibilities, it is interesting to look for a physical interpretation of the geometry even in this unusual situation. In this paper, attention will be focused on the properties of the black-hole spacetimes containing a surface of degeneracy. The properties can be conveniently understood and illustrated by the motion of test particles and photons. In Section 3, the equations of motion will be given. Character of the latitudinal motion, crucial for understanding of the nature of the spacetimes with a surface of degeneracy, will be summarized in Section 4. A physical interpretation of the black-hole spacetimes with a surface of degeneracy will be presented in Section 5. In Section 6, the differences between the black-hole spacetimes with and without a surface of degeneracy will be illustrated by the motion of uncharged

particles along the axis of symmetry of the geometry. Finally, in Section 7, some concluding remarks will be given, concerning the proposed interpretation of the spacetimes with a surface of degeneracy, and its relevance in the context of phase transitions into states with a negative vacuum energy-density.

2 The geometry and its pseudosingularities

In the standard Boyer-Lindquist coordinates (t, r, θ, ϕ) and the geometric units ($c = G = 1$), the Kerr–Newman–anti-de Sitter dyon geometry is determined by the line element

$$ds^2 = -\frac{\Delta_r}{I^2\rho^2}(dt - a\sin^2\theta\, d\phi)^2 + \frac{\Delta_\theta \sin^2\theta}{I^2\rho^2}\left[a\, dt - (r^2 + a^2)\, d\phi\right]^2 + \frac{\rho^2}{\Delta_r}dr^2 + \frac{\rho^2}{\Delta_\theta}d\theta^2, \tag{3}$$

where

$$\Delta_r = -\frac{1}{3}\Lambda r^2(r^2 + a^2) + r^2 - 2Mr + a^2 + e^2 + p^2, \tag{4}$$

$$\Delta_\theta = 1 + \frac{1}{3}\Lambda a^2 \cos^2\theta, \tag{5}$$

$$I = 1 + \frac{1}{3}\Lambda a^2, \tag{6}$$

$$\rho^2 = r^2 + a^2\cos^2\theta. \tag{7}$$

The electromagnetic field connected with the geometry is given by the electromagnetic potential

$$A_\mu = \frac{1}{I\varrho^2}\left\{(er + ap\cos\theta)\delta^t_\mu - \left[aer\sin^2\theta + (r^2 + a^2)p\cos\theta\right]\delta^\varphi_\mu\right\}. \tag{8}$$

The parameters of the spacetime and its electromagnetic field are: mass (M), specific angular momentum (a), electric charge (e), magnetic charge (p), cosmological constant (Λ). It is convenient to define a dimensionless cosmological parameter

$$y = \frac{1}{3}\Lambda M^2 \tag{9}$$

which has to be negative for the spacetimes under consideration. For simplicity we put $M = 1$ hereafter. Equivalently, also the coordinates t, r and parameters of the spacetime a, e, p are expressed in units of M and become dimensionless.

The event horizons of the spacetime are given by the pseudosingularities of the radial coordinate, determined by the condition $\Delta_r = 0$. In the asymptotically anti-de Sitter spacetimes there can exist two black-hole horizons, but no cosmological horizon behind which the spacetime is dynamic — contrary to the case of asymptotically de Sitter spacetimes with the repulsive cosmological constant $\Lambda > 0$.

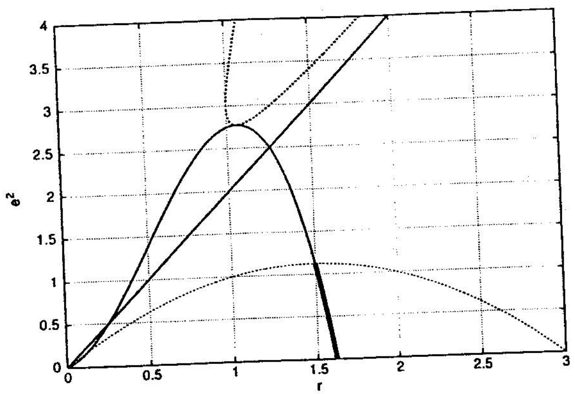

Fig. 1. Functions governing the character of event horizons in the Kerr–Newman spacetimes with a non-zero cosmological constant. The inflex points of the function $y_h(r;a,e)$ are given by the branch of the function $e^2_{ex(ex(h))-}(r)$ located at $r \geq 1.5$ and $e^2 \leq 9/8$ (emphasized by extra thick curve), where $a^2_{ex(h)+}(r;e) > 0$. The thick dotted line and the thin dotted line represent the curves $e^2_{z(ex(h))}(r)$ and $e^2_{r(ex(h))}(r)$, respectively.

The loci of the event horizons are determined by the condition

$$y = y_h(r;a,e) \equiv \frac{r^2 - 2r + a^2 + e^2}{r^2\,(r^2 + a^2)}. \tag{10}$$

(The magnetic charge p enters the Kerr–Newman-anti-de Sitter metric in the same way as the electric charge e, therefore a re-definition $e^2 + p^2 \to e^2$ can be introduced in the rest of this section.) The function $y_h(r;a,e)$ diverges at $r = 0$, while it goes to zero from above for $r \to \infty$. If $a^2 > 0$ and/or $e^2 > 0$, $y_h \to \infty$ for $r \to 0$. However, in the special case $a^2 = e^2 = 0$ there is $y_h \to -\infty$ for $r \to 0$. Therefore, in any Schwarzschild-anti-de Sitter geometry with $y < 0$ there is a black-hole horizon located at r_h determined by the relation

$$r_h = \left(-\frac{1}{y}\right)^{1/3}\left\{\left[1 + \left(1 - \frac{1}{27y}\right)^{1/2}\right]^{1/3} + \left[1 - \left(1 - \frac{1}{27y}\right)^{1/2}\right]^{1/3}\right\}. \tag{11}$$

Clearly, $r_h \to 2$ for $y \to 0$, while $r_h \to 0$ for $y \to -\infty$.

Due to the asymptotic behaviour of $y_h(r;a,e)$, event horizons exist in the Kerr–Newman spacetimes with an attractive cosmological constant, if the function $y_h(r;a,e)$ has a minimum at negative values of y. Therefore, we have to consider zeros of the function $y_h(r;a,e)$. They are determined by the relation

$$a^2 = a^2_{z(h)}(r;e) \equiv 2r - r^2 - e^2. \tag{12}$$

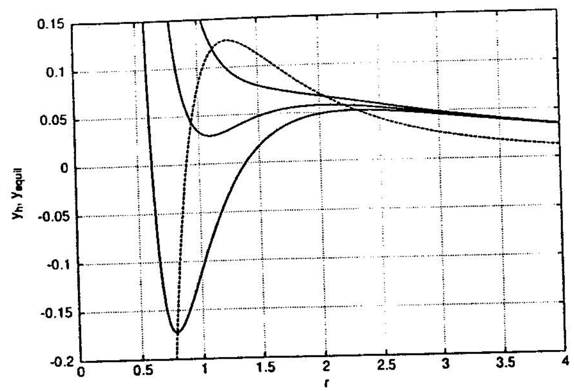

Fig. 2. Location of event horizons (solid lines) of the Kerr–Newman spacetimes with a non-zero cosmological constant is given by the function $y_h(r;a,e)$ in three typical situations with a fixed value of the charge parameter $e^2 = 0.49$ and three characteristic values of the rotation parameter $a^2 = 0.81$ (no extreme), $a^2 = 0.5625$ (two extrema at $y > 0$), and $a^2 = 0.36$ (a maximum at $y > 0$ and a minimum at $y < 0$). Only the third case gives Kerr–Newman-anti-de Sitter black-hole spacetimes. Location of equilibria of uncharged particles on the axis of symmetry of the spacetimes (dashed line) is given by the function $y_{equil}(r;a,e)$; it is presented in the third case only.

The function $a^2_{z(h)}(r;e)$ determines the horizons of the Kerr–Newman black holes with a zero cosmological constant. The maximum of this function is at $r = 1$, where $a^2 = 1 - e^2$. This corresponds to the extreme Kerr–Newman black holes. The Kerr–Newman-anti-de Sitter black holes can exist only for parameters a and e satisfying the condition

$$a^2 + e^2 < 1, \tag{13}$$

if the relation

$$a^2 < a^2_{z(h)}(r;e) \tag{14}$$

is satisfied. Then fixed values of a and e permit the existence of local extrema of the function $y_h(r;a,e)$ necessarily. The local extrema of the function $y_h(r;a,e)$ are determined (due to the condition $\partial y_h/\partial r = 0$) by the relation

$$\begin{aligned} a^2 &= a^2_{ex(h)\pm}(r;e) \\ &\equiv \frac{1}{2}\left\{-2r^2 + r - e^2 \pm \left[r^2(8r+1) - e^2\left(4r^2 + 2r - e^2\right)\right]^{1/2}\right\}. \end{aligned} \tag{15}$$

Since the functions $a^2_{ex(h)\pm}(r;e)$ govern the horizons of the Kerr–Newman–de Sitter geometry, we have to discuss their properties carefully. The reality condition of this family of functions is

$$e^2 \geq e^2_{r(ex(h))+}, \tag{16}$$

$$e^2 \leq e^2_{\mathrm{r(ex(h))-}}, \tag{17}$$

where

$$e^2_{\mathrm{r(ex(h)\pm)}}(r) \equiv r\left\{2r + 1 \pm 2[r(r-1)]^{1/2}\right\}. \tag{18}$$

Zero points of $a^2_{\mathrm{ex(h)\pm}}(r;e)$ are determined by the condition

$$e^2 = e^2_{\mathrm{z(ex(h))}}(r) \equiv \frac{(3-r)r}{2}, \tag{19}$$

and their extremal points are given by the relation

$$e^2 = e^2_{\mathrm{ex(ex(h))\pm}}(r) \equiv \frac{r}{8}\left(-16r^2 + 24r + 11 \pm |4r-1||4r-5|\right). \tag{20}$$

All the functions $e^2_{\mathrm{r(ex(h)\pm)}}(r)$, $e^2_{\mathrm{z(ex(h))}}(r)$, $e^2_{\mathrm{ex(ex(h))\pm}}(r)$ are illustrated in Fig. 1, which enables us to determine properties of the functions $a^2_{\mathrm{ex(h)\pm}}(r;e)$ and $y_{\mathrm{h}}(r;a,e)$. Three qualitatively different cases for the behaviour of the function $y_{\mathrm{h}}(r;a,e)$ are possible (see Fig. 2).

All the characteristic functions of a^2 and e^2 must be positive in order to give phys-
ically relevant results. It can be shown that $a^2_{\mathrm{ex(h)-}}(r;e) < 0$ for all $r, e^2 > 0$. On
the other hand, $a^2_{\mathrm{ex(h)+}}(r;e) > 0$ in regions where $e^2 < e^2_{\mathrm{z(ex(h))}}$. The relevant ex-
trema of $a^2_{\mathrm{ex(h)+}}(r;e)$ are given by $e^2_{\mathrm{ex(ex(h))-}}(r)$ at the branch lying under the curve
$e^2_{\mathrm{z(ex(h))}}$. Therefore, the relevant extrema of $a^2_{\mathrm{ex(h)+}}(r;e)$ exist for $e^2 < \frac{9}{8}$. In the
limiting case of $e = 0$, the function $a^2_{\mathrm{ex(h)+}}(r)$ has its maximum at $r_{\mathrm{crit}} = 1.61603$,
with a corresponding critical value of the rotation parameter $a^2_{\mathrm{crit}} = 1.21202$. If
$0 < e^2 < 9/8$, the critical value $a^2_{\mathrm{crit}}(e)$, governing an inflex point of $y_{\mathrm{h}}(r;a,e)$, is
determined by $e^2_{\mathrm{ex(ex(h))-}}(r)$. For $a^2 < a^2_{\mathrm{crit}}(e)$, the function $y_{\mathrm{h}}(r;a,e)$ has two local
extrema $y_{\mathrm{h(min)}}(a,e)$ and $y_{\mathrm{h(max)}}(a,e)$, determined by (15) and (10), with a given value
of the parameter e. For $a^2 = a^2_{\mathrm{crit}}(e)$, these extrema coincide at $y_{\mathrm{crit}}(e)$ which is the
limiting value for black-hole spacetimes with a fixed parameter e. The black-hole space-
times exist for $y_{\mathrm{h(min)}}(a,e) < y < y_{\mathrm{h(max)}}(a,e)$ (see Fig. 2). The Kerr–Newman–anti-de
Sitter black holes correspond to the range of parameters

$$y_{\mathrm{h(min)}}(a,e) < y < 0. \tag{21}$$

If $y = y_{\mathrm{h(min)}}(a,e)$, the geometry determines an extreme black hole, and for $y < y_{\mathrm{h(min)}}(a,e)$ it determines a naked singularity.

Distribution of black-hole and naked-singularity Kerr–Newman–anti-de Sitter space-
times in the parameter space is given by the function $y_{\mathrm{h(min)}}(a,e)$, and can be de-
termined by a numerical code. The results are given in Fig. 3. We can see that
black-hole spacetimes can exist for all values of attractive cosmological constant $y < 0$,
contrary to the case of repulsive cosmological constant $y > 0$ when black-hole space-
times must have $y \leq 2/27$, and the extremal value $y = 2/27$ corresponds to the extreme

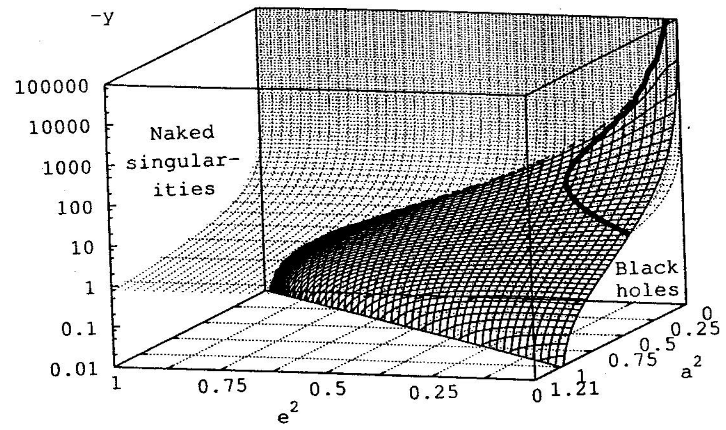

Fig. 3. Parameter space of the Kerr–Newman–anti-de Sitter spacetimes. The black-hole spacetimes can exist for any $y < 0$; therefore, we give the $(-y)$ axis in the logarithmic scale. The boundary between the black-hole and naked-singularity states is given by the function $y_{\mathrm{h(min)}}(a,e)$, determined by a numerical code. The light surface corresponds to the boundary $(y_{\mathrm{d}}(a))$ between the spacetimes with and without the surface of degeneracy.

Reissner-Nordström-de Sitter geometry with $e^2 = 9/8$ [8]. Distribution of the black-hole and naked-singularity Kerr–Newman–de Sitter spacetimes with a repulsive cosmological constant is given in [9].

The Kerr–Newman–anti-de Sitter spacetimes can have also pseudosingularities of the latitudinal coordinate. In spacetimes with $ya^2 < -1$, a surface of degeneracy, determined by the condition $\Delta_\theta = 0$, is located at a constant latitude

$$\theta_{\mathrm{d}} = \arccos\frac{1}{\sqrt{-ya^2}}. \tag{22}$$

The combinations of the parameters $ya^2 = -1$ must be excluded due to the fact that for $I = 0$ the geometry is not well defined. Regions of the parameter space, corresponding to the spacetimes with a surface of degeneracy, are given by the condition

$$y < y_{\mathrm{d}}(a) \equiv -\frac{1}{a^2}. \tag{23}$$

The function $y_{\mathrm{d}}(a)$ is also illustrated in Fig. 3. We shall see that the surfaces of degeneracy separate the spacetimes into two causally disconnected regions.

3 The equations of motion of test particles

The motion of charged test particles is determined by the Lorentz equation

$$m\frac{Du^\mu}{D\tau} = (\epsilon F^\mu{}_\nu + g\,{}^*F^\mu{}_\nu)u^\nu, \tag{24}$$

and the normalization condition

$$g_{\mu\nu}\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda} = -m^2. \tag{25}$$

The test particle has mass m and carries electric charge ϵ and magnetic monopole charge g; $u^\mu = dx^\mu/d\tau$ is its four-velocity, and τ is its proper time, related to the affine parameter λ by the relation $\tau = m\lambda$. For photons $m = 0$. The tensor of the electromagnetic field $F_{\mu\nu} = A_{\nu;\mu} - A_{\mu;\nu}$; the tensor $^*F_{\mu\nu}$ is dual to the tensor $F_{\mu\nu}$, and it is determined by the relation

$$^*F_{\mu\nu} = \epsilon_{\mu\nu\alpha\beta}F^{\alpha\beta}, \tag{26}$$

where $\epsilon_{\mu\nu\alpha\beta}$ is the completely antisymmetric Levi-Civita symbol. The dual tensor is determined by the vector potential (8) with interchanges $e \to p$, $p \to -e$. The motion of uncharged particles and photons is determined by the geodesic equations.

In the integrated and separated form, the motion of test particles and photons is determined by Carter's equations [7]:

$$\rho^2\frac{dr}{d\lambda} = \pm\sqrt{R(r)}, \tag{27}$$

$$\rho^2\frac{d\theta}{d\lambda} = \pm\sqrt{W(\theta)}, \tag{28}$$

$$\rho^2\frac{d\phi}{d\lambda} = -\frac{IP_\theta}{\Delta_\theta\sin^2\theta} + \frac{aIP_r}{\Delta_r}, \tag{29}$$

$$\rho^2\frac{dt}{d\lambda} = -\frac{aIP_\theta}{\Delta_\theta} + \frac{(r^2+a^2)IP_r}{\Delta_r}, \tag{30}$$

where

$$R = P_r^2 - \Delta_r(m^2r^2 + \mathcal{K}), \tag{31}$$

$$W = (\mathcal{K} - a^2m^2\cos^2\theta)\Delta_\theta - \left(\frac{P_\theta}{\sin\theta}\right)^2, \tag{32}$$

and

$$P_r = IE(r^2+a^2) - Ia\Phi - (\epsilon e - gp)r, \tag{33}$$

$$P_\theta = IaE\sin^2\theta - I\Phi + (\epsilon p - ge)\cos\theta. \tag{34}$$

The constants of the motion are: energy (E), connected with the stationarity of the geometry, the axial angular momentum (Φ), connected with the axial symmetry of the geometry, and the "total" angular momentum ($\mathcal{K}$), connected with the hidden symmetries of the geometry. Notice that E and Φ cannot be interpreted as energy and the axial angular momentum at infinity, since the spacetime is asymptotically anti-de Sitter.

It should be stressed that the radial motion is influenced by interactions of electric (or magnetic) charges of the source and the particle, while the latitudinal motion is influenced by their mixed electro-magnetic interactions.

4 The latitudinal motion

We shortly summarize properties of the latitudinal motion, which is influenced by the "electro-magnetic" interaction of the source and the test particle, characterized by the parameter

$$\beta = \frac{\epsilon p - ge}{IaE}. \tag{35}$$

The motion can be determined by the "effective potential" with respect to the constant of motion $K = \mathcal{K}/(aE)^2$, given by the relation

$$K_t = \frac{(1+ya^2)^2(\sin^2\theta - l - \beta\cos\theta)^2}{(1+ya^2\cos^2\theta)\sin^2\theta} + \frac{\cos^2\theta}{\gamma^2}, \tag{36}$$

where the impact parameter $l = \Phi/(aE)$ and the specific energy $\gamma = E/m$ have been introduced.

If there is no interaction between the electric and magnetic charges of the source and the particle ($\beta = 0$), the motion is symmetric relative to the equatorial plane ($\theta = \pi/2$) of the spacetime, and is allowed in this two-dimensional plane. For the "electro-magnetic" interaction switched on ($\beta \neq 0$), the motion is asymmetric relative to the equatorial plane, and is not allowed there.

If $y > 0$, the latitudinal motion is of the same character as in the well known case of Kerr–Newman spacetimes. However, in the considered case of Kerr–Newman–anti-de Sitter spacetimes ($y < 0$), a qualitative difference appears: beside the stable $\theta = \text{const}$ orbits located outside the equatorial plane, there exist unstable $\theta = \text{const}$ orbits outside the plane. A detailed analysis of the latitudinal motion is given in [10] for $\beta = 0$, and in [11] for $\beta \neq 0$. The discussion have to be separated into two parts: if $ya^2 \geq -1$, the motion is allowed in regions where $K \geq K_t$; if $ya^2 < -1$, the motion is allowed in regions where $K \geq K_t$ for $\theta > \theta_d$, and $K \leq K_t$ for $\theta < \theta_d$; of course, due to the plane symmetry of the spacetime, we assume $0 \leq \theta \leq \pi/2$. The typical behaviour of the effective potential is in all the characteristic cases illustrated in Fig. 4. It can be checked easily from the behaviour of the effective potential that the surface of degeneracy cannot be penetrated by test particles and photons. In the next Section it will be shown, using directly the latitudinal equations of motion.

5 An interpretation of the spacetimes containing a surface of degeneracy

As can be directly inferred from Fig. 3, the Kerr–Newman–anti-de Sitter spacetimes with a surface of degeneracy can correspond to both black-hole and naked-singularity spacetimes. In the discussion of their character, we can consider the region of latitudinal coordinates $0 \leq \theta \leq \pi/2$ only, due to the symmetry of the spacetimes relative to the equatorial plane.

The geometry (3) with $ya^2 < -1$ has the usual form with the signature +2 in the region of $\theta > \theta_d = \arccos(1/\sqrt{-ya^2})$. In the black-hole spacetimes, the geometry is static outside the horizons, and it is dynamic between the horizons — see Fig. 5a. The

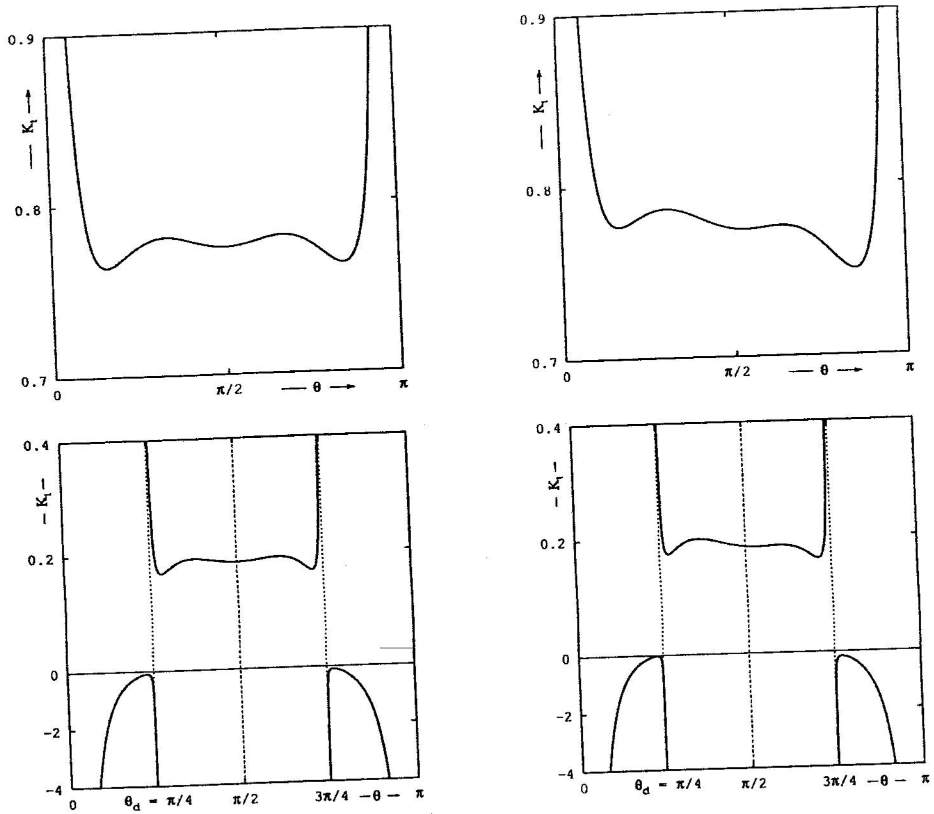

Fig. 4. The effective potential of the latitudinal motion in the Kerr–Newman–anti-de Sitter dyon spacetimes. The typical situations are illustrated for the spacetimes with or without the surface of degeneracy, and for uncharged or charged particles. For spacetimes without the surface of degeneracy, the effective potentials are given for $ya^2 = -0.2$, $\gamma^2 = 1.9$, $l = -0.1$ with (a) $\beta = 0$ and (b) $\beta = -0.007$. For spacetimes with the surface of degeneracy, the effective potentials are given for $ya^2 = -2$, $\gamma^2 = 2.5$, $l = 0.57$ with (c) $\beta = 0$ and (d) $\beta = -0.02$.

geometry is static at all radii in the naked-singularity spacetimes — see Fig. 5b. The motion of test particles is given by the equations (27)–(34) in this case.

On the other hand, in the region of $\theta < \theta_d$ the geometry (3) has the opposite signature -2, and the situation is just inverse to the situation in the region of $\theta > \theta_d$. The black-hole spacetimes are dynamic outside the horizons, while they are static between the horizons; the naked-singularity spacetimes are dynamic at all radii. According to the normalization condition (25), the equations of motion (27)–(34) govern tachyonic motion in the region of $\theta < \theta_d$. The constants of motion connected with the symmetries of the spacetime must have opposite signs due to the signature change: $p_\phi = -\Phi$,

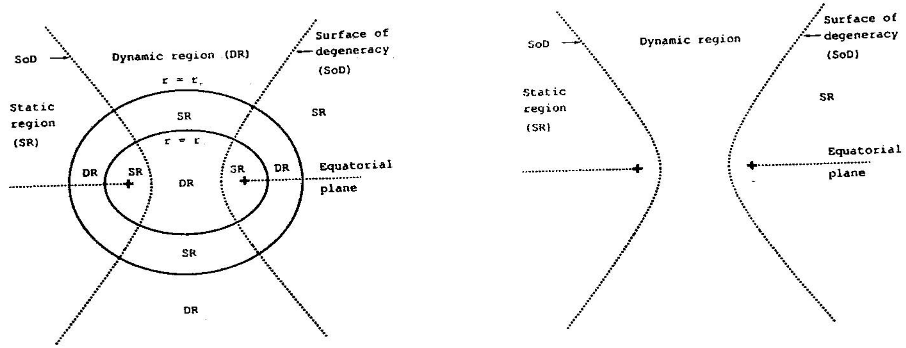

Fig. 5. The spacetimes with a surface of degeneracy. The sections of $t = \text{const}$ and $\phi = \text{const}$ of the spacetimes are represented in (a) a black-hole case, and (b) a naked-singularity case.

$p_t = E$. We can say that in the region of $\theta < \theta_d$ the motion of test particles will be determined by the equations of the tachyonic motion for the region of $\theta > \theta_d$ (but with opposite signs of the constants of motion Φ, E). Because of the normalization condition

$$g_{\mu\nu}\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda} = +m^2, \tag{37}$$

the motion will be governed by the equations (27)–(30) with

$$R = P_r^2 - \Delta_r(K - m^2 r^2), \tag{38}$$

$$W = (K + a^2 m^2 \cos^2\theta)\Delta_\theta - \left(\frac{P_\theta}{\sin\theta}\right)^2, \tag{39}$$

and

$$P_r = -IE(r^2 + a^2) + Ia\Phi - (\epsilon e - gp)r, \tag{40}$$

$$P_\theta = -IaE\sin^2\theta + I\Phi + (\epsilon p - ge)\cos\theta. \tag{41}$$

It is clear from equations (32) and (34) or (39) and (41) that no timelike and null geodesic or test-particle trajectories can cross the surface of degeneracy $\theta = \theta_d$, because $\Delta_\theta = 0$ there, and $(P_\theta/\sin\theta)^2$ enters the expressions for $W(\theta)$ with a negative sign. (It is an inverse situation to the case of the radial motion, when P_r^2 enters the expression for $R(r)$ with a positive sign, and the particles can cross the horizons where $\Delta_r = 0$.) In the special cases, when $P_\theta = 0$ at θ_d, the particles or photons approach the surface of degeneracy asymptotically [10]. Therefore, the Kerr–Newman–anti-de Sitter spacetimes with $ya^2 < -1$ can be considered as consisting of two causally separated regions.

6 Uncharged particles moving along the axis of symmetry

It is instructive to give an illustration of the different character of the motion in the black-hole spacetime with and without a surface of degeneracy in the simplest case of the motion of uncharged particles along the axis of symmetry. (More complicated case of the motion of charged particles will be treated separately [11].)

The equation of the latitudinal motion implies: $\theta = 0$, $\Phi = 0$ and

$$\mathcal{K} = \begin{cases} a^2m^2 & \text{if} \quad ya^2 > -1, \\ -a^2m^2 & \text{if} \quad ya^2 < -1. \end{cases} \tag{42}$$

The equation of the radial motion then implies for the motion along the axis of symmetry the relations

$$R(r) = \begin{cases} I^2E^2(r^2+a^2)^2 - \Delta_r m^2(r^2+a^2) & \text{if} \quad ya^2 > -1, \\ I^2E^2(r^2+a^2)^2 + \Delta_r m^2(r^2+a^2) & \text{if} \quad ya^2 < -1. \end{cases} \tag{43}$$

There are no turning points of the radial motion, if $\Delta_r < 0$ for $ya^2 > -1$, and if $\Delta_r > 0$ for $ya^2 < -1$.

It is convenient to introduce a new parameter of the specific energy by the relation

$$\tilde{\gamma} \equiv \frac{IE}{m}. \tag{44}$$

Then the turning points of the axial motion are given by the relation

$$\tilde{\gamma}^2 = V_{\text{eff}}(r; y, a, e), \tag{45}$$

where the effective potential must be positive, being determined by

$$V_{\text{eff}} \equiv \frac{\Delta_r}{r^2+a^2} \geq 0 \tag{46}$$

for spacetimes without the surface of degeneracy, while it is given by

$$V_{\text{eff}}^{(\text{d})} \equiv -\frac{\Delta_r}{r^2+a^2} \geq 0 \tag{47}$$

for spacetimes containing the surface of degeneracy. The motion is allowed in regions where $\tilde{\gamma}^2 \geq V_{\text{eff}}$, and $\tilde{\gamma}^2 \geq V_{\text{eff}}^{(\text{d})}$, respectively.

The effective potential has zeros at the black-hole horizons. It is well defined outside the horizons if $ya^2 > -1$; in this case $V_{\text{eff}} \to \infty$ for $r \to \infty$. It is well defined between the horizons, if $ya^2 < -1$, and there must be a maximum of the effective potential there.

The extrema of the effective potential, determining points of equilibrium positions on the axis of symmetry, are determined by the relation

$$y = y_{\text{equil}}(r; a, e) \equiv \frac{r^2 - e^2r - a^2}{r(r^2+a^2)^2}. \tag{48}$$

The function $y_{\text{equil}}(r; a, e)$ goes to zero from above for $r \to \infty$, while it diverges at $r = 0$ as $y_{\text{equil}} \to -\infty$. Because we restrict our attention to spacetimes with an attractive cosmological constant, we have to say that the function $y_{\text{equil}}(r; a, e)$ will be negative, if the condition

$$a^2 > a^2_{\text{z(equil)}}(r; e) \equiv r^2 - e^2r \tag{49}$$

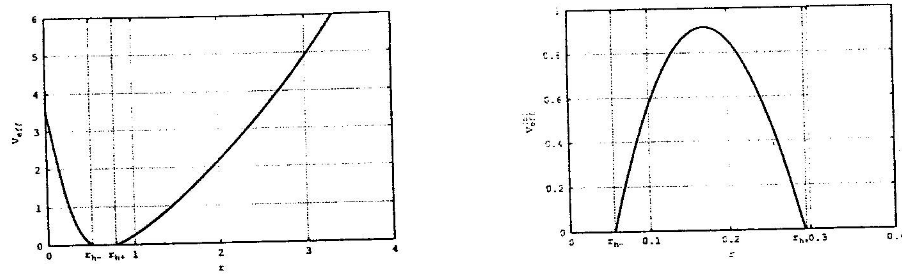

Fig. 6. The effective potential of the axial motion of uncharged particles in the Kerr–Newman–anti-de Sitter black-hole spacetimes. (a) For a spacetime without the surface of degeneracy, it is given for $y = -0.5$, $a^2 = 0.2$, $e^2 = 0.5$. (b) For a spacetime containing the surface of degeneracy it is given for $y = -25$, $a^2 = 0.1$, $e^2 = 0$. The extrema of the effective potential correspond to unstable equilibrium points on the axis.

is satisfied. The function $a^2_{\text{z(equil)}}(r; e)$ is positive at $r > e^2$, its extremum is located at $r = e^2/2$, i.e., out of the region where the function gives physically relevant results. Moreover, common points of the functions $a^2_{\text{z(equil)}}(r; e)$ and $a^2_{\text{z(h)}}(r; e)$ are located at $r = e^2/2$ and $r = 1$.

Extreme points of the function $y_{\text{equil}}(r; a, e)$ (given by the condition $\partial y_{\text{equil}}/\partial r = 0$) are determined by the relation

$$a^2 = a^2_{\text{ex(equil)}}(r; e) \equiv -3r^2 + 2r[r(3r - e^2)]^{1/2}. \tag{50}$$

The reality condition of the function $a^2_{\text{ex(equil)}}(r; e)$ is $r \geq e^2/3$, its zero point is at $r = 4e^2/3$, and its extreme point is again out of the region of physical relevance at $r = e^2/2$. Further, it is important to find common points of the functions $y_{\text{h}}(r; a, e)$ and $y_{\text{equil}}(r; a, e)$. They are determined by the relation

$$\begin{aligned} a^2 &= a^2_{\text{(h)(eq)}\pm}(r; e) \\ &\equiv \frac{1}{2}\left\{-2r^2 + r - e^2 \pm \left[r^2(8r+1) - e^2\left(4r^2 + 2r - e^2\right)\right]^{1/2}\right\}. \end{aligned} \tag{51}$$

Because there is

$$a^2_{\text{(h)(eq)}\pm}(r; e) = a^2_{\text{ex(h)}\pm}(r; e), \tag{52}$$

we can conclude that the function $y_{\text{equil}}(r; a, e)$ intersects the function $y_{\text{equil}}(r; a, e)$ just at its extreme points (see Fig.2). Now, we can easily convince ourselves that in the case of Kerr–Newman–anti-de Sitter black-hole spacetimes the equilibria positions of uncharged particles are impossible outside the horizons in spacetimes without the surface of degeneracy, while they exist between the horizons of spacetimes with the surface of degeneracy, being unstable with respect to radial perturbations.

The behaviour of the effective potential V_{eff} and $V_{\text{eff}}^{(\text{d})}$ is illustrated in Fig. 6. We see that the effective potentials in spacetimes with and without the surface of degeneracy can be considered as "complementary". However, we must stress that in the spacetimes with a surface of degeneracy there is no "mirror" symmetry between the causally disconnected regions $\theta < \theta_d$ and $\theta > \theta_d$, because of different character of the latitudinal motion in these regions.

7 Concluding remarks

The analysis of test-particle motion in the Kerr–Newman–anti-de Sitter geometry enables an unusual proposition that phase transitions into states with a negative vacuum energy, predicted by some versions of grandunified or multidimensional Kaluza-Klein theories of elementary particles, could lead to changes in the structure of rotating-black-hole spacetimes, connected with changes of signature of the metric tensor describing some regions of the spacetimes around their axis of symmetry. The regions with the changed signature of the metric tensor have to be causally disconnected from regions around the equatorial plane of the spacetimes, retaining the original signature of the metric tensor. In the region with changed signature, the equations of motion of test particles are of "tachyonic" character. The simplest case of the motion of uncharged particles along the axis of symmetry is discussed in the present paper, however, the proposed interpretation of the black-hole spacetimes containing a surface of degeneracy have to be further checked by an investigation of the properties of the motion outside the axis of symmetry.

Clearly, the "geometric" studies presented in this paper are not sufficient to verify the conjecture that rotating black holes can be described by the Kerr–Newman–anti-de Sitter geometry with a surface of degeneracy after a phase transition into a state with sufficiently low density of negative vacuum energy. They have to be extended by more sophisticated considerations, going behind the simple analysis of test particle motion, and taking into account interactions of the black hole with its environment.

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