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CORRELATION FUNCTIONS AND SUSCEPTIBILITIES OF PHOTONIC BAND GAP RESERVOIRS

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We investigate quantum statistical properties of photonic band gap reservoirs in terms of correlation functions and susceptibilities in time and spectral domains. Typical features are oscillations of the time-dependent correlation functions and susceptibilities. This is because photonic bad gap reservoirs are intrinsically non-Markovian reservoirs. The results help us to understand better how intrinsic quantum-statistical properties of a reservoir influence dynamics of an atom interacting with this reservoir.

1. Introduction

Boundary conditions influence time and spectral properties of the electromagnetic field. This well-known fact has a great importance in optics and generally in electromagnetism. Specific examples are resonators used in laser technique and cavity electrodynamics. In quantum optics high-Q microcavities are used for single-atom experiments when an atom can interact in a coherent way with an electromagnetic (EM) field which has its mode structure totally different from those in free space [1, 2]. In particular, interaction of an (effectively) two-level atom with a single-mode cavity field was observed [3] in the region of microwaves (with the wavelength about 1 cm). In 1987 Yablonovitch and John independently proposed that certain periodic dielectric structures can present forbidden frequency gaps (or pseudogaps in partially disordered structures) for transverse modes [4, 5]. Such periodic structures were named "photonic band structures" or "photonic crystals", in analogy with electronic crystals which also have a (forbidden) gap for electronic energy. For true photonic crystals the basic property of blocking EM wave propagation must be fulfilled for all waves within some frequency range, i.e. for all wavevector and polarization directions. That means a strong modification of the density of modes (DOM) and corresponding change in propagation of light and its

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interactions with atoms or molecules within such a medium. In this way spontaneous emission and its inhibition or enhancement appears possibly as a most repealing phenomenon demonstrating potential of the photonic-band structures (PBS). A new class of quantum electronic devices can be potentially constructed utilizing the advantages offered by modification of spontaneous decay rate. This includes semiconductor devices - lasers, with strongly decreased lasing threshold and with decreased losses due to undesirable transitions, in particular photon emission from the electron-hole recombination process [6]. Other interesting phenomena given by the nature of photonic band structures are for example lossless energy transport via the resonant dipole-dipole energy exchanges between atoms immersed in the medium [7, 8].

If the number of degrees of freedom of the EM field is large then this field is often called reservoir. From the point of view of statistical properties we can distinguish between Markovian and non-Markovian reservoirs. The example of a Markovian reservoir is that of the free-space vacuum radiation field. The field of such a reservoir has very short autocorrelation time, i.e. a present value of the field is not correlated to a value in the past. This is not a property of the (macroscopic) EM field in PBS. This fact is due to the periodicity of the structure which causes that the field is back-reflected on the boundaries of the structure and it gives rise to significant memory effects. Assume we have an atom in one of its excited unperturbed levels. We let such an atom to interact with the vacuum state radiation field (vacuum-state reservoir), for simplicity. Then the atomic expectation values can evolve in various qualitative ways depending on the properties of the reservoir into which it was inserted [9, 10]. The vacuum field in free space has different statistical properties as the vacuum in a PBS. The classification of reservoirs as Markovian and non-Markovian ones has then important influence also on the level of practical calculations. Markovian properties enables in some cases to obtain equations of motion [11] (master equation or quantum Langevin equation) for atomic variables while the properties of the reservoir field are included in these equations only via stationary characteristic functions. On the other hand, problems including non-Markovian reservoirs require special treatment and this can be a very involved problem.

Our intent is to highlight intrinsic quantum statistical properties of PB reservoirs. The properties analyzed in this work are of special interest because they are in a close connection with a wide class of non-trivial phenomena that occur in PBS.

Section 2 briefly reviews definitions of reservoir characteristic functions used in this paper. In sections 3 and 4 we evaluate shapes of two-time correlation functions and susceptibilities of the vacuum reservoirs. We explicitly show the differences between free-space case and the ones with the DOM modified by a presence of a gap. These characteristic functions (or functions very similar to them) enter the usual master equation for the motion of the atomic variables. In the last section we discuss the results. Thorough all the paper we work with simplified qualitative models of photonic band structures. We neglect all directional-dependent effects, two different polarizations of light and vector character of the EM field. This simplifications are due to a complexity of expressions in realistic cases. However, the results of this work are qualitatively acceptable for the aspects under investigation.

2. Brief review of characteristic functions

In the Markovian theory of radiative processes [11, 12] the properties of the reservoir enter equations via a two-time characteristic function - a complex correlation function

$$g(t',t'') = \mathrm{Tr}_R[\rho_R E(t')E(t'')] \;\equiv\; \langle E(t')E(t'')\rangle_R \tag{1}$$

where $E(t)$ is the reservoir electric field operator in Heisenberg picture

$$E(t) = i\sum_\omega \mathcal{E}_\omega a_\omega e^{-i\omega t} + \text{h.c.} \tag{2}$$

with

$$\mathcal{E}_\omega = \sqrt{\frac{\hbar\omega}{2\epsilon_0 V}}\,. \tag{3}$$

V is the quantization volume and ϵ_0 is the dielectric permitivity of the vacuum. The symbol $\sum_\omega$ is a symbolic sum over the transverse modes. In this work we assume that electromagnetic waves propagate in a simplified photonic crystals with an isotropic dispersion relation. Then the symbolic sums $\sum_\omega$ have a meaning of integrals over the modes according to the rule

$$\sum_\omega \to \int_0^\infty d\omega\rho(\omega) \tag{4}$$

with $\rho(\omega)d\omega$ being the number of the transverse EM modes with frequencies from the interval $[\omega,\omega+d\omega]$. The expression (2) for the electric-field operator should also include a position-dependent factors (like $\exp(ikx)$ in free space) depending on the geometry of the environment and on the dielectric-permitivity function [13, 14]. To simplify the notation and to purify the influence of effects of the different pattern of the DOM in PBS in comparison with the free-space case we take these factors equal to unity.

Since we assume that the reservoir is in a stationary state (particularly vacuum state in the case of the spontaneous emission), the function g depends only on the difference $\tau = t' - t''$. It can be decomposed into its real and imaginary parts:

$$g(\tau) = \frac{1}{2}\langle E(\tau)E(0) + E(0)E(\tau)\rangle_R + \frac{i}{2}\langle [E(\tau),E(0)]/i\rangle_R\,. \tag{5}$$

The first one is the two-time symmetric autocorrelation function

$$C(\tau) = \mathrm{Re}\; g(\tau) = \frac{1}{2}\langle E(\tau)E(0) + E(0)E(\tau)\rangle_R \tag{6}$$

where

$$\langle E(\tau)E(0)\rangle^* = \langle E(0)E(\tau)\rangle \tag{7}$$

and the two time moments are chosen to be 0 and τ. The second function is related to the two-time susceptibility function (again real) which is defined [12]

$$\chi(\tau) = \frac{2}{\hbar}\theta(\tau)\mathrm{Im}\; g(-\tau) = \frac{i}{\hbar}\theta(\tau)\langle [E(0),E(-\tau)]\rangle_R\,. \tag{8}$$

Spectral components (Fourier transforms) of the correlation function and susceptibility can be introduced:

$$\hat{C}(\omega) = \int_{-\infty}^{\infty} C(\tau)e^{i\omega\tau}d\tau \ , \ \ \hat{\chi}(\omega) = \int_{-\infty}^{\infty} \chi(\tau)e^{i\omega\tau} \ . \tag{9}$$

The function (6) tells us how the (free) field is correlated in two time moments. If this function is proportional to $\delta(\tau)$ then the field in one time moment is totally independent of the field in some other moment. The Fourier transform (9) gives the energy spectrum of the field fluctuations. The susceptibility function expresses the ability of the reservoir to be polarized by an interaction with other system, here a two-level atom [2]. It is a linear susceptibility, i.e. it describes small deviations of the reservoir state for short times from the beginning of the interaction. Since it is closely related to the imaginary part of the complex correlation function (1) it also contains information how the field is correlated in two different moments. The real part of the susceptibility spectrum (i.e. the Fourier transform) gives the linear response in phase with the small system while the imaginary part gives the response out of phase [12] in the same meaning as it is in a case of a simple damped harmonic oscillator subjected to a periodic external force.

Higher-order complex correlation functions of the field can be introduced including different times in each of the field operators and even different space points [15]. In general, normally ordered complex correlation function dependent on $M+N$ space-time points $(n) \equiv (\mathbf{r}_n, t_n)$ is defined

$$\Gamma^{(N,M)} = \langle E_{i_1}^{(-)}(1)E_{i_2}^{(-)}(2)...E_{i_N}^{(-)}(N)...E_{j_M}^{(+)}(M')...E_{j_2}^{(+)}(2')...E_{j_1}^{(+)}(1')\rangle_R \tag{10}$$

where $E^{(\pm)}$ are the positive- and negative-frequency parts of the field operators. The subscripts at E s are the Cartesian components from which only one is considered in this work. The function (1) written in this notation reads

$$g(\tau) = \langle E^{(+)}(t')E^{(-)}(t'')\rangle + \langle E^{(-)}(t')E^{(+)}(t'')\rangle + \langle E^{(+)}(t')E^{(+)}(t'')\rangle + \langle E^{(-)}(t')E^{(-)}(t'')\rangle_R \ . \tag{11}$$

If the reservoir is in a stationary state then the last two terms of this expression vanish.

In next sections we treat the properties of this two functions in the case of a model reservoir corresponding to the simple qualitative model of the photonic crystal with the field in the vacuum state.

3. Symmetric correlation function

The expressions for the symmetric correlation function and its Fourier transform reduce to the following sums (understood as integrals in the continuous-mode-spectrum

[2] Let us have an unperturbed system whose observable F has an equilibrium value $\langle F\rangle_{eq}$. If this system is a subject of a small perturbation proportional to some $\lambda(t)$, then to first order in the perturbation

$$\langle F\rangle = \langle F\rangle_{eq} + \int_{-\infty}^{\infty} dt'\chi(t-t')\lambda(t') \ .$$

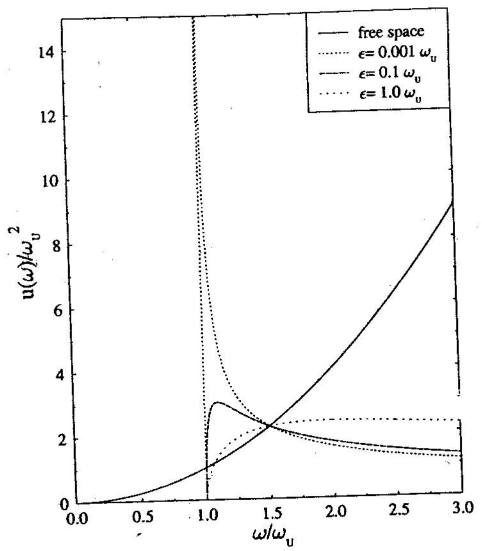

Fig. 1. The profiles of the models of DOM used for the results shown on next figures. The free-space dependence is proportional to the square of the mode frequency. Other curves are models of PBS given by (21) for various edge-smoothing parameters ϵ. We have chosen the same value for all the dependences at $\omega = 1.5\omega_U \equiv \omega_R$. The peak values derived from (21) are placed at $\omega/\omega_U = 1 + \epsilon/\omega_U$ and are equal to $K/(2\sqrt{\epsilon\omega_U^2})$ where K is a constant given by our choice of the frequency ω_R at which the DOM are equal. $K = (\omega_R - \omega_U + \epsilon)\omega_R^2/\sqrt{\omega_R - \omega_U}$.

case) for the vacuum-state reservoir.

$$C(\tau) = \sum_{\omega} \mathcal{E}_{\omega}^2 \cos(\omega\tau) \tag{12}$$

and

$$\hat{C}(\omega) = \pi \sum_{\omega'} \mathcal{E}_{\omega'}^2 [\delta(\omega - \omega') + \delta(\omega + \omega')] \ . \tag{13}$$

Up till now we did not specify whether we deal with discrete or continuous mode spectrum. In this section and sections 4 and 5 we apply expressions to continuous or quasi-continuous reservoir spectra. Then it is convenient to convert the sums over transverse modes into integrals over continuum of modes following (4). We write expression for the DOM as follows:

$$\rho(\omega) = \mathcal{K}u(\omega) \tag{14}$$

where $\mathcal{K}$ is given by

$$\mathcal{K} = 4\pi V/(2\pi c)^3 \ . \tag{15}$$

In free space we have the DOM

$$\rho^{\text{free}}(\omega) = \mathcal{K}u^{\text{free}}(\omega) \tag{16}$$

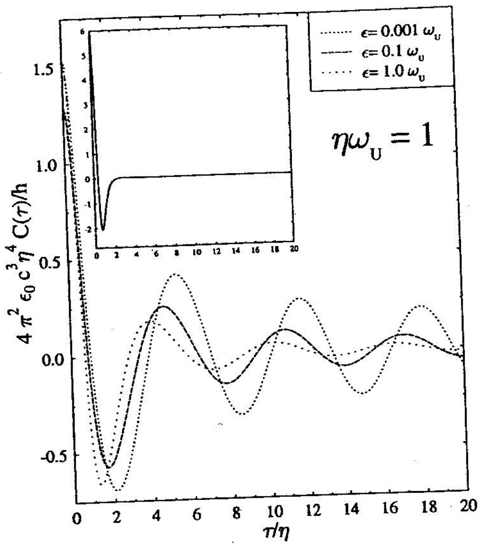

Fig. 2. The symmetric correlation functions of the vacuum-state reservoirs calculated for the same models of DOM as on Fig. 1. The free-space curve is plotted in the inset using the same units as other curves. The correlation functions reach a maximum (finite, due to the factor η) values at the zero time delay. The oscillating behaviour is characteristic for the models with the abrupt gap edge causing non-Markovian character of the reservoir. The frequency of the oscillations is approximately equal to the edge frequency ω_U. Its origin is in the refraction of the light from the boundaries in the periodic dielectric structure. The symmetric correlation functions in this figure are even in τ. We see sharper is the edge, stronger are the oscillations in the pattern of the correlation function and lower is the peak at the zero time delay. The second zero point of the free-space function is located at $(1+\sqrt{2})\eta$ (free-space correlation time).

with $u^{\rm free}(\omega) = \omega^2\theta(\omega)$. For non-free-space cases we take always the same constant K given by (15). All modifications of DOM from the free space density will be given by the function $u(\omega)$.

For the DOM given by a function $u(\omega)$ we get from (12) and (13)

$$C(\tau) = \frac{\hbar}{4\pi^2\epsilon_0 c^3}\int_0^\infty \omega u(\omega)\cos(\omega\tau)d\omega \qquad (17)$$

and

$$\hat{C}(\omega) = \frac{\hbar}{4\pi\epsilon_0 c^3}|\omega|u(|\omega|)\,. \qquad (18)$$

Utilizing the above expressions the free-space correlation function is obtained when we introduced the exponential factor $\exp(-\eta\omega)$ into the integral over frequencies to avoid

divergent integrals [12] [3].

$$C^{\rm free}(\tau;\eta) = \frac{3}{2}\frac{\hbar}{\pi^2\epsilon_0 c^3}\frac{\eta^4 - 6\eta^2\tau^2 + \tau^4}{(\eta^2+\tau^2)^4}\,. \qquad (19)$$

We see τ^{-4} dependence of the correlation function. The correlation time can be considered as $\tau_c = (1+\sqrt{2})\eta$. This is the larger zero-point of the function $C^{\rm free}(\tau;\eta)$ (see Fig. 2). The spectral density of vacuum fluctuations can be obtained by Fourier transforming this function or by straight calculation from (13). We get

$$\hat{C}^{\rm free}(\omega) = \frac{\hbar}{4\pi\epsilon_0 c^3}|\omega|^3 \qquad (20)$$

where we have omitted the factor $\exp(-\eta\omega)$ in the last expression. A realistic photonic band structure exhibits complicated dispersion relation between the mode frequency and the wavevector. In particular, frequency of boundaries of a forbidden frequency region depends on a direction of wavevector and on the wave polarization. As a consequence, the corresponding DOM is also a complicated anisotropic function. In this paper we neglect all directional and polarization effects and concentrate on the qualitative nature of time- and frequency-dependence of the characteristic functions. This makes possible to accept a simple isotropic model of DOM approximately corresponding to the so called "effective-mass" approximation of the DOM [16, 17, 18, 10]

$$u^{\rm PBS}(\omega) = K\frac{\sqrt{\omega-\omega_U}}{\omega-\omega_U+\epsilon}\theta(\omega-\omega_U) \qquad (21)$$

where ω_U is the upper bound of the gap, K is a suitable constant and ϵ is a smoothing factor. The illustration of this model of the DOM is given in Fig. 1. The modes on the left-hand side of the gap are neglected in this model since the gap is assumed to be sufficiently large. The true mode density far above the gap is also replaced by other dependence which is relatively easy to use in calculations and does not lead to divergences. We can understand the approximations in this model via the fact that if a two-level system interacting with the reservoir has its transition eigenfrequency ω_A near to ω_U and the natural linewidth $\Gamma << \omega_A$, it predominantly interacts with the reservoir modes near the upper edge ω_U of the gap. This is the reason why the DOM can be considered effectively as given by (21) [17]. Then the model (21) is acceptable for a qualitative description of the isotropic DOM function. In Fig. 1 we have chosen the same DOM for the frequency $\omega = 1.5\omega_U \equiv \omega_R$. Using Eq. (17) and the model (21) we get

$$C^{\rm PBS}(\tau;\eta) = \frac{\hbar K}{4\pi^2\epsilon_0 c^3}\int_{\omega_U}^\infty \frac{\omega\sqrt{\omega-\omega_U}}{\omega-\omega_U+\epsilon}\cos(\omega\tau)e^{-\eta\omega}d\omega\,. \qquad (22)$$

We plot this function in Fig. 2 for several values of cut-off smoothing parameters ϵ, together with the free-space correlation function given by (19). We see oscillations in

[3]This procedure can be used since we do not calculate a concrete real physical system but a model system where qualitative aspects are not modified by the introduction of such a "cut-off". All aspect treated in this paper are cut-off independent, does not matter even on a shape of the cut-off function. See [12].

the pattern of time-dependent correlation function. These oscillations are not present in the case of free-field reservoir and they are extended for the times much larger than is the free-space correlation time. This has important consequences on the behaviour of an atom interacting with such a reservoir, if the transition of the interest is tuned not in the smooth part of the reservoir. Reversible energy exchanges becomes more probably in such an interaction and the reservoir can not be assumed Markovian in a calculation of the atom dynamics. The frequency of the oscillations in the pattern of the correlation function is equal to the band-edge frequency ω_U. The amplitude of the oscillations depends on the edge-smoothing parameter ϵ. Sharper is the edge, higher is the amplitude of the oscillations, although this dependence can be week. The frequency of the oscillations gives the period of the oscillations. The value of the oscillation period can be clearly physically interpreted using a simple one-dimensional model [19] of the PBS with periodic dielectric layers. Let the lattice period of the structure be denoted by Λ. Moreover, we can simplify the model of the lattice (without neglecting any of its features important for the model (21) of PBS considered in this paper) taking limits of a large index of refraction of the dielectric layers and putting the widths of the layers to very small values. It is so-called delta-function Kronig-Penney model known from solid-state physics. It was used, for example, in works [17, 19]. Solving the Maxwell equations in such a structure one gets a dispersion relations having gaps in the frequencies of the modes propagating perpendicularly to the layers. The upper edge of the first gap is located at $\omega_U = c\pi/\Lambda$. This gives the period of the oscillations in the pattern of the correlation function (22) equal to $2\Lambda/c$. We interpret this time as the one needed for light to travel the distance from a particular space point to the nearest dielectric layer and back. Then the pattern of the autocorrelation function of the electric field has also oscillating character with the period near to $2\pi/\omega_U = 2\Lambda/c$. The spectrum of the correlation function (vacuum fluctuations) is given by explicit formula (18).

4. Reservoir susceptibility

In this section we derive formulas for the susceptibility of free space and PBS reservoirs for which we derived correlation functions in the previous section. We start from expressions (8) and (9), assuming the electric field operator is given by (2) and the reservoir is in the vacuum state. We get

$$\chi(\tau) = \frac{2}{\hbar}\sum_{\omega} \mathcal{E}_\omega^2 \theta(\tau)\sin(\omega\tau) \qquad (23)$$

and

$$\hat{\chi}(\omega) = -\frac{1}{\hbar}\sum_{\omega'} \mathcal{E}_{\omega'}^2 \left[P\frac{1}{-\omega'-\omega} + P\frac{1}{-\omega'+\omega} - i\pi\delta(-\omega'+\omega) + i\pi\delta(-\omega'-\omega)\right] . \qquad (24)$$

After conversion to integrals and separating into real and imaginary parts these sums become

$$\chi(\tau) = \frac{\theta(\tau)}{2\pi^2\epsilon_0 c^3}\int_0^\infty u(\omega)\omega\sin(\omega\tau)e^{-\eta\omega}d\omega\ , \qquad (25)$$

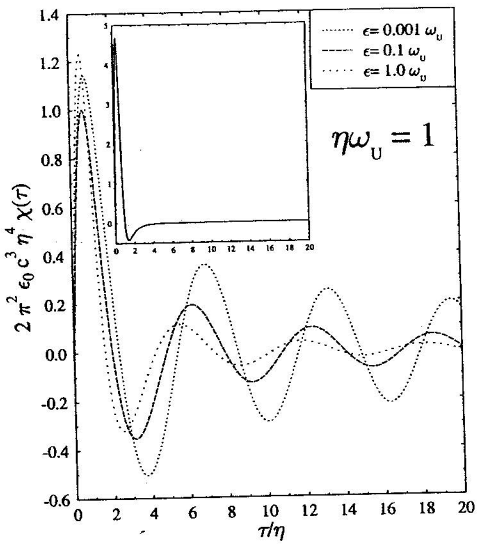

Fig. 3. The linear susceptibilities of the vacuum-state reservoirs of the parameters the same as on Fig. 1. The functions are odd in τ. The oscillations of the dependences has the same character as that of the symmetric correlation functions on Fig. 2.

$$\mathrm{Re}[\hat{\chi}(\omega;\eta)] = \frac{1}{4\pi^2\epsilon_0 c^3}P\int_0^\infty \frac{\omega' u(\omega')e^{-\eta\omega'}}{\omega+\omega'}d\omega' - \frac{1}{4\pi^2\epsilon_0 c^3}P\int_0^\infty \frac{\omega' u(\omega')e^{-\eta\omega'}}{\omega-\omega'}d\omega' \qquad (26)$$

and

$$\mathrm{Im}[\hat{\chi}(\omega;\eta)] = \frac{1}{4\pi\epsilon_0 c^3}\,\omega u(|\omega|)e^{-\eta|\omega|}\ . \qquad (27)$$

Now we substitute expression (21) into (25), (26) and (27) which gives us the time-dependent susceptibility function of the actual reservoir. Calculation for the free space $[u^{\mathrm{free}}(\omega) = \omega^2\theta(\omega)]$ gives

$$\chi^{\mathrm{free}}(\tau;\eta) = \frac{12}{\pi^2\epsilon_0 c^3}\frac{\eta\tau\,(\eta^2-\tau^2)}{(\eta^2+\tau^2)^4}\theta(\tau)\ . \qquad (28)$$

For spectral components of $\chi(\tau;\eta)$ we obtained (26) and (27). For free space this turns into

$$\mathrm{Re}[\hat{\chi}^{\mathrm{free}}(\omega;\eta)] = \frac{1}{4\pi^2\epsilon_0 c^3}P\int_0^\infty \frac{\omega'^3 e^{-\eta\omega'}}{\omega+\omega'}d\omega' - \frac{1}{4\pi^2\epsilon_0 c^3}P\int_0^\infty \frac{\omega'^3 e^{-\eta\omega'}}{\omega-\omega'}d\omega' \qquad (29)$$

and

$$\mathrm{Im}[\hat{\chi}^{\mathrm{free}}(\omega;\eta)] = \frac{1}{4\pi\epsilon_0 c^3}\,\omega|\omega|^2 e^{-\eta|\omega|}\ . \qquad (30)$$

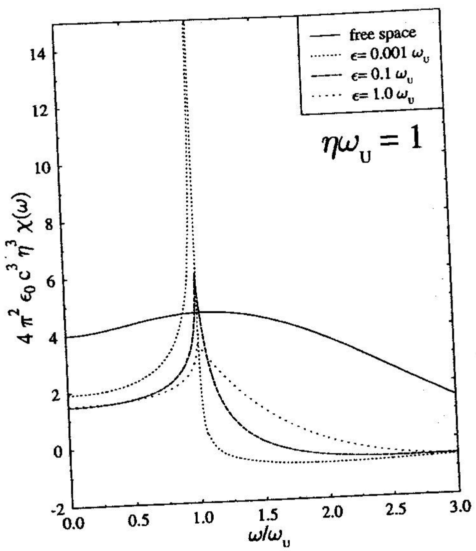

Fig. 4. The real part of the reservoir-susceptibility spectra. The linear susceptibility has non-vanishing values even in the gap, clearly demonstrating that linear-response theory is not sufficient to describe reservoir polarization due to the interaction with an atom. We use parameters as for the previous figures. The functions are even in ω.

The calculation of the principal parts see in the appendix. We obtain

$$\mathrm{Re}\big[\hat{\chi}^{\mathrm{free}}(\omega;\eta)\big] = \frac{1}{4\pi^2\epsilon_0 c^3}\left\{\frac{4+2\eta^2\omega^2}{\eta^3} + \omega^3\big[e^{\eta\omega}\mathrm{Ei}(-\eta\omega) - e^{-\eta\omega}\mathrm{Ei}(\eta\omega)\big]\right\} . \quad (31)$$

Due to the properties of the exponential integral function we have [20] $\mathrm{Re}\big[\hat{\chi}^{\mathrm{free}}(0;\eta)\big] = (\pi^2\epsilon_0 c^3\eta^3)^{-1}$. For the model (21) of the DOM we substitute (21) into (25), (26) and (27). We obtain the following expressions.

$$\chi^{\mathrm{PBS}}(\tau;\eta) = \frac{K}{2\pi^2\epsilon_0 c^3}\int_{\omega_U}^{\infty} \frac{\omega\sqrt{\omega-\omega_U}}{\omega-\omega_U+\epsilon}\sin(\omega\tau)e^{-\eta\omega}\,d\omega \quad (32)$$

and

$$\mathrm{Re}\big[\hat{\chi}^{\mathrm{PBS}}(\omega;\eta)\big] = \frac{K}{4\pi^2\epsilon_0 c^3}P\int_0^{\infty} \frac{\omega'\sqrt{\omega'-\omega_U}\,\theta(\omega'-\omega_U)}{(\omega+\omega')(\omega'-\omega_U+\epsilon)}e^{-\eta\omega'}\,d\omega' - \quad (33)$$

$$\frac{K}{4\pi^2\epsilon_0 c^3}P\int_0^{\infty} \frac{\omega'\sqrt{\omega'-\omega_U}\,\theta(\omega'-\omega_U)}{(\omega-\omega')(\omega'-\omega_U+\epsilon)}e^{-\eta\omega'}\,d\omega' . \quad (34)$$

The imaginary part $\mathrm{Im}\big[\hat{\chi}^{\mathrm{PBS}}(\omega;\eta)\big]$ is simply given by (27), where we substitute (21) for $u(\omega)$.

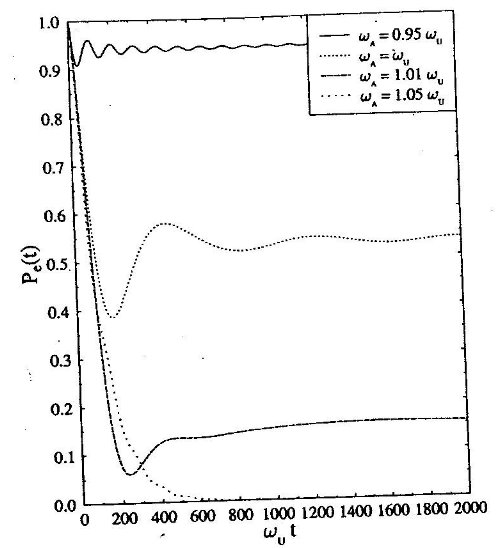

Fig. 5. Time-dependent probability that the atom remains in its exited state during the process of the spontaneous emission into the PBG reservoir. The curves are plotted according to the theory of Kofman *et al.* [18] for parameters $\epsilon = 0.001\omega_U$ and $C = 0.001\omega_U^{3/2}$ (expressing coupling strength). The dependences are far from exponential decay which occurs if the atom interacts with the free-space vacuum or even with PBG vacuum far in the allowed band. See also Fig. 1 and (21).

Let us discuss the reservoir susceptibility function. In time domain, it is a real function nonzero only for positive time difference τ. For free space it is given by (28). We plot the time-domain susceptibilities on Fig. 3 for the same parameters as in Fig. 2. Oscillating curves characterize essential memory properties of the PBS reservoir, similarly as the symmetric correlation function. The character of the oscillation is the same as for the symmetric correlation function considered in the previous section. As for spectral components of $\chi_R(\tau)$, we have non-vanishing both real and imaginary parts given by (31) and (30) for free space and (34), (27) with (21) for the model of PBS. On Fig. 4 we see quite different behaviour of real parts of the spectral susceptibilities for free space and PBS. Especially near the (upper) edge of the band gap, where the DOM nearly diverges and towards to the gap vanishes abruptly. However, the real part of the reservoir susceptibility is nonzero even in the gap. If we apply this result to calculate the reservoir response due to the interaction with an atom we obtain non-physical results. The problem is that the linear response of the reservoir takes place only for very short times from the beginning of the interaction. In general, we have for the e.g. electric field created by the interaction (in some fixed space point)

$$\langle E(t)\rangle = \langle E^L(t) + E^{NL}(t)\rangle \quad (35)$$

where $\langle E^L \rangle$ is the part of the field created due to the linear response and $\langle E^{NL} \rangle$ is the part created by the nonlinear response. If we make Fourier decomposition of the field $\langle E^L(t) \rangle$, we can obtain its spectral components also in the gap as given by the nonzero spectrum of the linear susceptibility. However, $\langle E^L \rangle$ is not a complete physical field. The complete physical field is only the complete field $\langle E \rangle$ (and vacuum fluctuations but they have zero expectation value) and this complete physical field has no spectral components in the gap. The field $\langle E^L \rangle$ is complete only for very short times but in this case the Fourier decomposition taken by an integral over large time ($\to \infty$) has only mathematical meaning.

5. Discussion and conclusions

This paper provides a view on the quantum-statistical properties of non-Markovian photonic band gap (PBG) reservoirs. These properties have direct consequence on interactions of atoms with the reservoirs. We calculated two-time symmetric correlation functions and susceptibilities of the PBG reservoirs in both the time and frequency domains. The comparison of the PBG and free-space correlation functions and susceptibilities clearly shows the main difference - the oscillations in the time-dependent pattern in the case of PBG. These oscillations are responsible for the finite-memory effects. The oscillations occur at the frequency of band edges which is given by the periodicity of the PB structure. Non-Markovian statistical properties of the PBS EM field have direct consequence on the interactions of atoms or molecules with this field [18]. We illustrate this statement on the example of spontaneous emission from a two-level atom. If the reservoir is the free-space one, the well-known Weisskopf-Wigner exponential decay takes place [21]. On the other hand, if the same atom emits into the PBG reservoir, the decay can have non-exponential character and can also be incomplete [18], depending on the frequency of the atomic transition and the position of the bandgap edge. Non-exponential decay occurs if the atom frequency is tuned around the band gap edge where the DOM is not a smooth function of the frequency. Fig. 5 shows the decay into the reservoir with the DOM given by (21) for various values of the atom transition frequency ω_A. These dependences are calculated according to the theory of Kofman *et al.* [18]. We can see that the intrinsic non-Markovian properties of the reservoir causes that the atom decays in the way totally different from a decay into a flat Markovian reservoir such as free space.

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Appendix

Here is the calculation of the integral

$$I_-(\omega;\eta) = P\int_0^\infty \frac{\omega'^3 e^{-\eta\omega'}}{\omega-\omega'}d\omega' \, . \tag{A1}$$

First we calculate the primitive function

$$\mathcal{I}_-(\omega,\omega',\eta) = \int \frac{\omega'^3 e^{-\eta\omega'}}{\omega-\omega'}d\omega' = \frac{\frac{2+\eta\omega+\eta^2\omega^2}{\eta^3} + \frac{(2+\eta\omega)\omega'}{\eta^2} + \frac{\omega'^2}{\eta}}{e^{\eta\omega'}} - \frac{\omega^3 \mathrm{Ei}(\eta\omega-\eta\omega')}{e^{\eta\omega}} \tag{A2}$$

where $\mathrm{Ei}(x)$ is the exponential integral function. Some of the properties of this function [20]:

$$\lim_{x\to 0}\mathrm{Ei}(x) = -\infty, \quad \lim_{x\to\infty}\mathrm{Ei}(x) = \infty, \quad \lim_{x\to-\infty}\mathrm{Ei}(x) = 0 \, . \tag{A3}$$

$\mathrm{Ei}(x)$ is neither even nor odd function, but it is even in the infinitesimal neighborhood of $x = 0$. Using (A2) we get for (A1)

$$I_-(\omega;\eta) = \lim_{\omega'\to\infty}\left[\mathcal{I}_-(\omega,\omega';\eta) - \mathcal{I}_-(\omega,\omega+\epsilon;\eta) + \mathcal{I}_-(\omega,\omega-\epsilon;\eta) - \mathcal{I}_-(\omega,0;\eta)\right] \, . \tag{A4}$$

The particular primitive functions are

$$\lim_{\omega'\to\infty}\mathcal{I}_-(\omega,\omega';\eta) = -\frac{\omega^3}{e^{\eta\omega}}\lim_{\omega'\to-\infty}\mathrm{Ei}(-\eta\omega') = 0 \tag{A5}$$

and

$$\mathcal{I}_-(\omega,\omega\pm\epsilon;\eta) = e^{-\eta\omega}\left[\frac{2+\eta\omega+\eta^2\omega^2}{\eta^3} + \frac{(2+\eta\omega)\omega}{\eta^2} + \frac{\omega^2}{\eta} - \omega^3\mathrm{Ei}(\mp\eta\epsilon)\right] \tag{A6}$$

(assuming that ϵ and η are very small positive values). Further,

$$\mathcal{I}_-(\omega,0;\eta) = \frac{2+\eta\omega+\eta^2\omega^2}{\eta^3} - \frac{\omega^3\mathrm{Ei}(\eta\omega)}{e^{\eta\omega}} \, . \tag{A7}$$

Collecting formulas (A4)-(A7) we obtain

$$I_-(\omega;\eta) = -\mathcal{I}_-(\omega,0;\eta) = \frac{2+\eta\omega+\eta^2\omega^2}{\eta^3} - \frac{\omega^3\mathrm{Ei}(\eta\omega)}{e^{\eta\omega}} \, . \tag{A8}$$

Now we can immediately write the expression for the second principal part [see (29)]:

$$I_+(\omega;\eta) = P\int_0^\infty \frac{\omega'^3 e^{-\eta\omega'}}{\omega+\omega'}d\omega' = -I_-(-\omega;\eta) \, . \tag{A9}$$

Having this we can write formula for the real part of the free-space susceptibility:

$$\mathrm{Re}\left[\chi_R^{\mathrm{free}}(\omega;\eta)\right] = \frac{1}{4\pi^2\epsilon_0 c^3}\left[I_-(\omega;\eta) - I_+(\omega;\eta)\right] =$$

$$\frac{1}{4\pi^2\epsilon_0 c^3}\left\{\frac{4+2\eta^2\omega^2}{\eta^3}+\omega^3\left[e^{\eta\omega}\mathrm{Ei}(-\eta\omega)-e^{-\eta\omega}\mathrm{Ei}(\eta\omega)\right]\right\}. \qquad (A10)$$

This is formula (31).

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