acta physica slovaca vol. 48 No. 1, 55 – 66 February 1998

QUANTUM SCATTERINGS ON STATIC SPHERICALLY SYMMETRIC GRAVITATIONAL BACKGROUND IN THE WEAK FIELD NEWTONIAN LIMIT

D. Tatomir[†], D. Radu[†], O. Mihalache[‡]

[†] *Department of Theoretical Physics, Faculty of Physics, "Al. I. Cuza" University, 11 Copou Blvd., Iasi 6600, Romania*

[‡] *Institute of Technical Physics, 47 D. Mangeron Blvd., Iasi 6000, Romania*

Received 28 January 1997, in final form 1 June 1997, accepted 23 October 1997

Following Gupta's linear approximation, the scattering of various spin particles (scalar mesons, neutrinos, photons, vector mesons, gravitinos, tensor mesons and gravitons) in the static gravitational field, described by Schwarzschild metric, is studied. Using S-matrix formalism in the external field approximation, the rules of Feynman type for diagrams are derived, and consequently, the corresponding cross sections are obtained. We shall notice that in the small angle approximation, the differential cross sections of massive Klein-Gordon, Dirac, Proca, Rarita-Schwinger and tensor particles have the same form, and in the ultrarelativistic case they coincide with those corresponding to the zero-rest-mass mesons, neutrinos, photons, gravitinos and gravitons. In other words, in this limit case the gravitational particle scattering does not depend on their spin, according to Vladimirov's ideas. As particularly important result, we point out that helicity is conserved by the scattering process of the electromagnetic waves, whereas for the gravitational waves helicity is not conserved.

1. Introduction

In the present paper, using S-matrix formalism [1] and Gupta's linear approximation [2][1]

$$\sqrt{-g}g^{ij} = \eta^{ij} - \chi y^{ij}, \tag{1}$$

the scattering of various spin particles -massive scalar mesons (in the Klein-Gordon -Fock formalism [3]), neutrinos (Dirac field), photons (Maxwell field, following Mitskevich's ideas [4]), massive vector mesons (Proca field), gravitinos (Rarita-Schwinger field [5]), gravitons and massive tensor mesons - in external gravitational field (static field,

[1] The 3-vector components are labelled by Greek indices, while the 4-vector ones carry Roman indices. The exceptions will be mentioned especially

described by Schwarzschild metric) is studied. Here g^{ij}, η^{ij}, and y^{ij} are the metric tensor, the Minkowski tensor - diag(1,-1,-1,-1) and the tensor of the weak gravitational field, respectively, $g = det(g_{ij})$, and $\chi = \sqrt{16\pi G}$ (in natural units), G being the Newton constant.

2.The gravitational particle scattering and corresponding effects

In order to describe the interaction between the gravitational field and other fields, to the expression of Einstein Lagrangian, the matter field Lagrangians written in the curved space (obtained by using the principle of minimal coupling [6]) are added:

$$\begin{aligned} L_{int} = \; & \sqrt{-g}\Big\{\Big[g^{ij}\phi^*_{;i}\phi_j - (m^2 + \tfrac{1}{4}R)\phi^*\phi\Big] \\ & + \tfrac{i}{4}\Big[\overline{\Psi}\tilde{\gamma}^j(1+\tilde{\gamma}^5)\Psi_{;j} - \overline{\Psi}_{;j}\tilde{\gamma}^j(1+\tilde{\gamma}^5)\Psi\Big] \\ & - \tfrac{1}{4}g^{ik}g^{jl}F_{ij}F_{kl} - \Big[\tfrac{1}{2}g^{ik}g^{jl}G^*_{ij}G_{kl} - \mu^2 g^{ij}B^*_iB_j\Big] \\ & + \tfrac{1}{2}\varepsilon^{ijkl}(\overline{\Psi}_i\tilde{\gamma}^5\tilde{\gamma}_j\Psi_{k;l} - \overline{\Psi}_{i;l}\tilde{\gamma}^5\tilde{\gamma}_j\Psi_k) \\ & + \tfrac{1}{2}g^{ik}g^{jl}(g^{mn}\Phi^*_{ij;m}\Phi_{kl;n} - M^2\Phi^*_{ij}\Phi_{kl})\Big\}. \end{aligned} \tag{2}$$

Here ϕ and ϕ^* are the operators of charged KG field (asterisk signifying hermitic conjugation), $F_{ij} = A_{j;i} - A_{i;j} \equiv A_{j,i} - A_{i,j}$ and A_i, $G_{ij} = B_{j;i} - B_{i;j} \equiv B_{j,i} - B_{i,j}$ and B_i are the tensor and potential of the Maxwell and charged Proca fields, respectively, Φ_{ij} are operators corresponding to the massive charged tensor field, Ψ and Ψ_i are the components of the massless Dirac and massless R.S. fields, respectively (with $\overline{\Psi} = \Psi^*\tilde{\gamma}^0$, $\overline{\Psi}_i = \Psi^*_i\tilde{\gamma}^0$), $\tilde{\gamma}^j$, $\tilde{\gamma}^5$ are the generalized Dirac matrices, ε^{ijkl} is the completely antisymmetric Levi-Civita tensor, R is the scalar curvature and m, μ and M are the masses of the corresponding particles. Also, commas and semicolons denote the partial and covariant derivatives, respectively.

In the Dirac and R.S, fields' case, using the vierbein formalism [4], the covariant derivatives of the Ψ, $\overline{\Psi}$ spinors and Ψ_i, $\overline{\Psi}_i$ spin-vectors are, respectively:

$$\Psi;j = \Psi_{,j} - \Gamma_j\Psi, \qquad \overline{\Psi}_{;j} = \overline{\Psi}_{,j} + \overline{\Psi}\Gamma_j,$$
$$\Psi_{i;j} = \Psi_{i,j} - \Gamma_j\Psi_i \qquad \overline{\Psi}_{i;j} = \overline{\Psi}_{i,j} + \overline{\Psi}_i\Gamma_j, \tag{3}$$

where

$$\begin{cases} \Gamma_j = \tfrac{1}{4}\tilde{\gamma}_{i;j}\tilde{\gamma}^i \\ \tilde{\gamma}_{i;j} = \tilde{\gamma}_{i,j} - \Gamma^k_{ij}\tilde{\gamma}_k, \end{cases} \tag{4}$$

are the Fock-Ivanenko spin coefficients of affine connection, Γ^k_{ij} being the Christoffel's symbols. The generalized Dirac matrices may also be expresed in terms of the usual ones, as:

$$\tilde{\gamma}^i = L^i(o)\gamma(o), \qquad \tilde{\gamma}_i = L_i(o)\gamma(o) \qquad (o = \overline{1,4}), \tag{5}$$

where the vierbein coefficients obey the following constraints:

$$L^i(o)L^j(o) = g^{ij}, \qquad L_i(o)L_j(o) = g_{ij}. \tag{6}$$

Following Mitskevich's ideas [4], the expression of the Maxwell field Lagrangian written in the curved space:

$$L_M = -\frac{1}{4}\sqrt{-g}F_{ik}F_{ik} - \frac{1}{2}\sqrt{-g}A^i_{;i}A^k_{;k}, \tag{7}$$

with

$$A^i_{;i} = 0, \tag{8}$$

becomes:

$$L_M = -\frac{1}{2}\sqrt{-g}(A_{i;k}A^{i;k} - A_iA_kR^{ik}) - \frac{1}{2}\big[\sqrt{-g}(A^i_{;i}A^k - A^k_{;i}A^i)\big]_{;k}. \tag{9}$$

that besides a divergence is

$$L_M = -\frac{1}{2}\sqrt{-g}g^{ik}g^{jl}(A_{i;j}A_{k;l} - R_{ij}A_kA_l), \tag{10}$$

where R_{ij} is the Ricci tensor.

Taking into account the de Donder-Fock gauge: $(\sqrt{-g}g^{ij})_{,j} = 0$, developing all quantities in series in terms of χ - according to (1) - , i.e.

$$\begin{aligned} &\sqrt{-g} = 1 - \tfrac{1}{2}\chi y\ldots, && y = y_i{}^i, \\ &g^{ij} = \eta^{ij} - \chi h^{ij} + \ldots, && g_{ij} = \eta_{ij} + \chi h_{ij} + \ldots, \\ &h^{ij} = y^{ij} - \tfrac{1}{2}\eta^{ij}y && \Gamma^k_{ij} = \tfrac{1}{2}\chi\,(h_i{}^k{}_{,j} + h_j{}^k{}_{,i} - h_{ij,}{}^k) + \ldots \\ &R_{ij} = \tfrac{1}{2}\chi h_{ij,k}{}^k + \ldots && R = -\tfrac{1}{2}\chi y_{,k}{}^k + \ldots \\ &\tilde{\gamma}^j = \gamma^j - \tfrac{1}{2}\chi\gamma_i h^{ij} + \ldots, && \tilde{\gamma}_j = \gamma_j + \tfrac{1}{2}\chi\gamma_i h^i{}_j + \ldots \end{aligned} \tag{11}$$

and passing to the flat space, the first-order interaction Lagrangians between the gravitational and matter fields have the form[2]

$$L^{(1)}_{K.G.} = -\chi\big[\phi^*_{,i}\phi_{,j}y_{ij} + \frac{1}{2}\phi^*\phi(m^2y - \frac{1}{4}y_{,ii})\big], \tag{12a}$$

$$L^{(1)}_D = -\frac{1}{8}\chi[\overline{\Psi}\gamma_i(1+\gamma_5)\Psi,j - \overline{\Psi}_{,j}\gamma_i(1+\gamma_5)\Psi]s_{ij}, \tag{12b}$$

$$L^{(1)}_M = -\frac{1}{2}\chi\big[A_iA_{j,k}(h_{ij,k} + h_{ik,j} - h_{jk,i}) + A_{i,j}A_{i,k}h_{jk} + A_{i,j}A_{k,j}y_{ik} + \frac{1}{2}A_iA_jh_{ij,kk}\big], \tag{12c}$$

[2] We specify that in relations (12c) and (12f) the scattering occours at small enough angles, when the covariant derivative may be replaced by usual one, according to Vladimirov's ideas [7].

$$L_P^{(1)} = -\chi(G_{ij}^* G_{ik} u_{jk} + \mu^2 B_i^* B_j y_{ij}), \tag{12d}$$

$$L_{R.S.}^{(1)} = -\frac{1}{4}\chi\varepsilon_{ijkl}\left(\overline{\Psi}_i\gamma_5\gamma_m\Psi_{j,l} - \overline{\Psi}_{i,l}\gamma_5\gamma_m\Psi_j\right)y_{km}, \tag{12e}$$

$$L_T^{(1)} = -\chi\left(\frac{1}{2}\Phi_{ij,k}^*\Phi_{ij,l}y_{kl} + \Phi_{ij,k}^*\Phi_{lj,k}h_{il} + M^2\Phi_{ij}^*\Phi_{kj}u_{ik}\right), \tag{12f}$$

where

$$s_{ij} = y_{ij} + \tfrac{1}{2}\delta_{ij}y, \quad h_{ij} = y_{ij} - \tfrac{1}{2}\delta_{ij}y,$$

$$u_{ij} = y_{ij} - \tfrac{1}{4}\delta_{ij}y, \quad y = y_{ii}, \tag{13}$$

γ_i, γ_5 being the usual Dirac matrices and δ_{ik} is the Kronecker symbol.

In order to study the scattering of gravitons in the classical gravitational field itself, following Gupta's ideas [1], the first-order self-coupling gravitational Lagrangian is given by

$$L_G^{(1)} = -\tfrac{1}{2}\chi h_{ij}t_{ij}$$

$$t_{ij} = \tfrac{1}{2}\left[y_{kl,i}y_{kl,j} - \tfrac{1}{2}y_{,i}y_{,j} - \tfrac{1}{2}\delta_{ij}\left(y_{kl,m}y_{kl,m} - \tfrac{1}{2}y_{,m}y_{,m}\right)\right], \tag{12g}$$

t_{ij} being the energy-momentum pseudotensor of the weak gravitational field.

As external field we consider the static gravitational field described by Schwarzschild metric, i.e.:

$$y_{ij}^{ext}(\vec{x}) = \delta_{i4}\delta_{j4}y(\vec{x}), \qquad y(\vec{x}) = \frac{\chi M_\Theta}{4\pi|\vec{x}|}, \tag{14}$$

where M_Θ is the central body mass (the source) that creates the field (for example, the Sun), $|\vec{x}|$ being the distance to this centre.

The processes are described by Feynman diagram presented in Fig. 1. Here, k and $e_i^{(a)}(\vec{k})[e_i^{(\lambda)}(\vec{k}), e_{ij}^{(t)}(\vec{k}), e_{kl}^{(a)}(\vec{k})]$, p and $e_j^{(b)}[e_j^{(b)}(\vec{p}), e_{ij}^{(u)}(\vec{p}), e_{kl}^{(b)}(\vec{p})]$ are the four-momenta and polarization vectors (or tensors) of the initial and final particles, respectively ($a, b = 1, 2$ for real photons or gravitons and $t, u = \overline{1,5}$ correspond to the massive tensor particles) whereas (r), (s) ($r, s = 1, 2$) denote the polarizations (spins) of the initial and final Dirac and R.S. massless particles, and q is the four-momentum of the virtual graviton ($\vec{q} = \vec{p} - \vec{k}$ and $q_0 = p_0 - k_0 = 0$ - the conservation of the energy).

According to the standard quantum field theory, considering for real gravitons $y = 0$ and choosing for photons and gravitons the gauge in which $e_4 = 0$ and $e_{4i} = 0$ (the transverse-traceless gauge), respectively, the parts of Lagrangians (12) - casted into the normal form - are[3]

$$N[L_{K.G.(x)}^{(1)}] = -\chi\left\{\phi_{,i}^{*(-)}(x)\phi_{,j}^{(+)}(x)y_{ij}^{ext}(x) + \frac{1}{2}\phi^{*(-)}(x)\phi^{(+)}(x)\left[m^2 y^{ext}(x) - \frac{1}{4}y_{,ii}^{ext}(x)\right]\right\}, \tag{15a}$$

$$N[L_D^{(1)}(x)] = -\frac{1}{8}\chi\left[\overline{\Psi}^{(-)}(x)\gamma_i(1+\gamma_5)\Psi_{,j}^{(+)}(x) - \overline{\Psi}_{,j}^{(-)}(x)\gamma_i(1+\gamma_5)\Psi^{(+)}(x)\right]s_{ij}^{ext}(x), \tag{15b}$$

[3] In formula (15c) the symmetrization with respect to the photon labels is considered

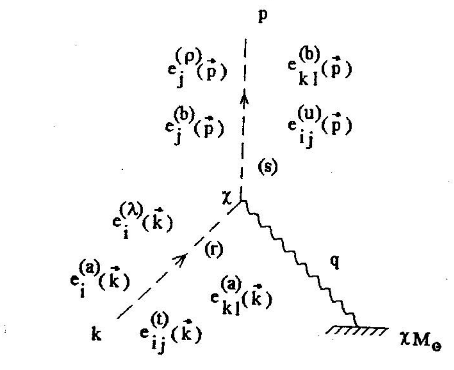

Fig. 1. The first-order Feynman vertex. The wavy line represents a graviton (external static

Fig. 1. The first-order Feynman vertex. The wavy line represents a graviton (external static gravitational field). The dashed line represent either scalar, spin-1/2, electromagnetic, vector, spin-3/2, tensor or gravitational quanta.

$$N[L_M^{(1)}(x)] = -\chi\left\{\tfrac{1}{2}\left[A_i^{(-)}(x)A_{j,k}^{(+)}(x) + A_{j,k}^{(-)}(x)A_i^{(+)}(x)\right]\right.$$
$$\times\left[h_{ij,k}^{ext}(x) + h_{ik,j}^{ext}(x) - h_{jk,i}^{ext}(x)\right]$$
$$\left. + A_{i,j}^{(-)}(x)A_{i,k}^{(+)}(x)h_{jk}^{ext}(x) + \tfrac{1}{2}A_i^{(-)}(x)A_j^{(+)}(x)h_{ij,kk}^{ext}(x)\right\}, \tag{15c}$$

$$N[L_P^{(1)}(x)] = -\chi\left[G_{ij}^{*(-)}(x)G_{ik}^{(+)}(x)u_{jk}^{ext}(x) + \mu^2 B_i^{*(-)}(x)B_j^{(+)}(x)y_{ij}^{ext}(x)\right], \tag{15d}$$

$$N[L_{R.S.}^{(1)}(x)] = -\frac{1}{4}\chi\varepsilon_{ijkl}\left[\overline{\Psi}_i^{(-)}(x)\gamma_5\gamma_m\Psi_{j,l}^{(+)}(x) - \overline{\Psi}_{i,l}^{(-)}(x)\gamma_5\gamma_m\Psi_j^{(+)}(x)\right]y_{km}^{ext}(x), \tag{15e}$$

$$N[L_T^{(1)}(x)] = -\chi\left[\tfrac{1}{2}\phi_{ij,k}^{*(-)}(x)\phi_{ij,l}^{(+)}y_{kl}^{ext}(x)\right.$$
$$\left. + \phi_{ij,k}^{*(-)}(x)\phi_{lj,k}^{(+)}(x)h_{il}^{ext}(x) + M^2\phi_{ij}^{*(-)}(x)\phi_{kj}^{(+)}(x)u_{ik}^{ext}(x)\right], \tag{15f}$$

$$N[L_G^{(1)}(x)] = -\frac{1}{2}\chi y_{kl,i}^{(-)}(x)y_{kl,j}^{(+)}y_{ij}^{ext}(x), \tag{15g}$$

where $(+)$ and $(-)$ denote the positive and negative frequency parts, corresponding to the annihilation and creation of particles in x, respectively.

Using the S-matrix formalism we deduce the rules of Feynman type for diagrams in the external gravitational field approximation, by which we can calculate the matrix elements $\langle p|S|k\rangle$ in the mentioned approximation.

Thus, taking into account the Fourier transform of the static external gravitational potential:

$$y(\vec{q}) = \frac{1}{(2\pi)^{3/2}} \int e^{-i\vec{q}.\vec{x}} y(\vec{x}) d^3x = \frac{\chi M_\Theta}{(2\pi)^{3/2}|\vec{q}|^2}, \tag{16}$$

and choosing for photons and gravitons the gauge in which

$$e_4^{(a)}(\vec{k}) = e_4^{(b)}(\vec{p}) = 0, \quad \vec{e}^{(a)}(\vec{k}).\vec{k} = \vec{e}^{(b)}(\vec{p}).\vec{p} = 0,$$
$$e_{4i}^{(a)}(\vec{k}) = e_{4j}^{(b)}(\vec{p}) = 0, \quad \vec{e}_{\alpha\beta}^{(a)}(\vec{k})k_\beta = \vec{e}_{\alpha\beta}^{(b)}(\vec{p})p_\beta = 0, \tag{17}$$

the matrix elements in the external field approximation, corresponding to the diagram in Fig.1 are, respectively:

$$\langle p|S|k\rangle_{K.G.} = -\frac{i\chi^2 M_\Theta}{2(2\pi)^2\sqrt{k_0p_0}} \int \frac{\delta(q_0)}{|\vec{q}|^2}\Big[p_ik_j\delta_{i4}\delta_{j4} + \tfrac{1}{2}(m^2 + \tfrac{1}{4}\delta_{i\alpha}\delta_{i\beta}q_\alpha q_\beta)\Big]\delta(\vec{p}-\vec{k}-\vec{q})d^3q = F_{K.G.}(k,p)\delta(q_0), \tag{18a}$$

$$\langle p|S|k\rangle_D = \frac{\chi^2 M_\Theta}{8(2\pi)^2} \int \frac{\delta(q_0)}{|\vec{q}|^2}\Big\{\Big[\overline{u}^{(s)}(\vec{p})\gamma_i(1+\gamma_5)u^{(r)}(\vec{k})k_j + p_j\overline{u}^{(s)}(\vec{p})\gamma_i(1+\gamma_5)u^{(r)}(\vec{k})\Big] \times(\delta_{i4}\delta_{j4} + \tfrac{1}{2}\delta_{ij}\Big\}\delta(\vec{p}-\vec{k}-\vec{q})d^3q = F_D(k,p)\delta(q_0), \tag{18b}$$

$$\langle p|S|k\rangle_M = \frac{i\chi^2 M_\Theta}{2(2\pi)^2\sqrt{k_0p_0}} \int \frac{\delta(q_0)}{|\vec{q}|^2}\Big\{\tfrac{1}{2}q_\alpha\Big[(\delta_{i4}\delta_{j4} - \tfrac{1}{2}\delta_{ij})\delta_{k\alpha} + (\delta_{i4}\delta_{k4} - \tfrac{1}{2}\delta_{ik})\delta_{j\alpha} - (\delta_{j4}\delta_{k4} - \tfrac{1}{2}\delta_{jk})\delta_{i\alpha}\Big]\Big[e_j^{(a)}(\vec{k})e_i^{(b)}(\vec{p})k_k - e_i^{(a)}(\vec{k})e_j^{(b)}(\vec{p})p_k\Big] - (\delta_{j4}\delta_{k4} - \tfrac{1}{2}\delta_{jk})e_i^{(a)}(\vec{k})e_i^{(b)}(\vec{p})k_kp_j + \tfrac{1}{2}(\delta_{i4}\delta_{j4} - \tfrac{1}{2}\delta_{ij}) \times\delta_{k\alpha}\delta_{k\beta}q_\alpha q_\beta e_j^{(a)}(\vec{k})e_i^{(b)}(\vec{p})\Big\}\delta(\vec{p}-\vec{k}-\vec{q})d^3q = F_M(k,p)\delta(q_0), \tag{18c}$$

$$\langle p|S|k\rangle_P = -\frac{i\chi^2 M_\Theta}{2(2\pi)^2\sqrt{k_0p_0}} \int \frac{\delta(q_0)}{|\vec{q}|^2}\Big\{\Big[e_k^{(\lambda)}(\vec{k})k_i - e_i^{(\lambda)}(\vec{k})k_k\Big]\Big[[e_j^{(\rho)}(\vec{p})p_i - e_i^{(\rho)}(\vec{p})p_j\Big] \times(\delta_{j4}\delta_{k4} - \tfrac{1}{4}\delta_{jk}) + \mu^2\delta_{i4}\delta_{j4}e_j^{(\lambda)}(\vec{k})e_i^{(\rho)}(\vec{p})\Big\} \delta(\vec{p}-\vec{k}-\vec{q})d^3q = F_P(k,p)\delta(q_0), \tag{18d}$$

$$\langle p|S|k\rangle_{R.S} = \frac{\chi^2 M_\Theta}{4(2\pi)^2} \int \frac{\delta(q_0)}{|\vec{q}|^2}\big[\delta_{k4}\delta_{m4}\varepsilon_{ijkl}(k_l + p_l) \times\overline{u}_i^{(s)}(\vec{p})\gamma_5\gamma_m u_j^{(r)}(\vec{k})\big]\delta(\vec{p}-\vec{k}-\vec{q})d^3q = F_{R.S}(k,p)\delta(q_0) \tag{18e}$$

$$\langle p|S|k\rangle_T = -\frac{i\chi^2 M_\Theta}{2(2\pi)^2\sqrt{k_0p_0}} \int \frac{\delta(q_0)}{|\vec{q}|^2}\Big[\tfrac{1}{2}e_{ij}^{(t)}(\vec{k})e_{ij}^{(u)}(\vec{p})p_kk_l\delta_{k4}\delta_{l4} + e_{lj}^{(t)}(\vec{k})e_{ij}^{(u)}(\vec{p})k_kp_k \times(\delta_{i4}\delta_{l4} - \tfrac{1}{2}\delta_{il}) + M^2e_{kj}^{(t)}(\vec{k})e_{ij}^{(u)}(\vec{p}) \times(\delta_{i4}\delta_{k4} - \tfrac{1}{4}\delta_{ik})\Big]\delta(\vec{p}-\vec{k}-\vec{q})d^3q = F_T(k,p)\delta(q_0), \tag{18f}$$

$$\langle p|S|k\rangle_G = -\frac{\chi^2 M_\Theta}{4(2\pi)^2\sqrt{k_0p_0}} \int \frac{\delta(q_0)}{|\vec{q}|^2}\delta_{i4}\delta_{j4}p_ik_je_{kl}^{(a)}(\vec{k})e_{kl}^{(b)}(\vec{p})\delta(\vec{p}-\vec{k}-\vec{q})d^3q = F_G(k,p)\delta(q_0) \tag{18g}$$

where u, $\overline{u}$ and u_i, $\overline{u}_i$ are the positive energy massless Dirac spinors and massless R.S. vector-spinors, respectively.

After calculation, we obtained

$$F_{K.G.}(k,p) = \frac{i\chi^2 M_\Theta}{8(2\pi)^2 k_0\vec{k}^2\sin^2\frac{\Theta}{2}}Q_{K.G.}(k,p) \tag{19a}$$

$$F_D(k,p) = \frac{\chi^2 M_\Theta}{32(2\pi)^2 k_0^2\sin^2\frac{\Theta}{2}}Q_D(k,p) \tag{19b}$$

$$F_M(k,p) = \frac{i\chi^2 M_\Theta}{8(2\pi)^2 k_0^3\sin^2\frac{\Theta}{2}}Q_M(k,p) \tag{19c}$$

$$F_P(k,p) = \frac{i\chi^2 M_\Theta}{8(2\pi)^2 k_0\vec{k}^2\sin^2\frac{\Theta}{2}}Q_P(k,p) \tag{19d}$$

$$F_{R.S.}(k,p) = \frac{\chi^2 M_\Theta}{16(2\pi)^2 k_0^2\sin^2\frac{\Theta}{2}}Q_{R.S.}(k,p) \tag{19e}$$

$$F_T(k,p) = \frac{i\chi^2 M_\Theta}{16(2\pi)^2 k_0\vec{k}^2\sin^2\frac{\Theta}{2}}Q_T(k,p) \tag{19f}$$

$$F_G(k,p) = \frac{\chi^2 M_\Theta}{16(2\pi)^2 k_0\sin^2\frac{\Theta}{2}}Q_G(k,p) \tag{19g}$$

where

$$Q_{K.G}(k,p) = \frac{1}{2}(k_0^2 + \vec{k}^2\cos^2\frac{\Theta}{2}), \tag{20a}$$

$$Q_D(k,p) = \overline{u}^{(s)}(\vec{p})\big[2ik_0\gamma_4(1+\gamma_5) + \frac{1}{2}(k_i + p_i)\gamma_i(1+\gamma_5)\big]u^{(r)}(\vec{k}), \tag{20b}$$

$$Q_M(k,p) = k_0^2\cos^2\frac{\Theta}{2}e_\alpha^{(a)}(\vec{k})e_\alpha^{(b)}(\vec{p}) - \frac{1}{2}e_\alpha^{(a)}(\vec{k})p_\alpha e_\beta^{(b)}(\vec{p})k_\beta, \tag{20c}$$

$$Q_P(k,p) = \left(\vec{k}^2\cos^2\frac{\Theta}{2} + \frac{\mu^2}{2}\right)e_i^{(\lambda)}(\vec{k})e_i^{(\rho)}(\vec{p}) + 2\vec{k}^2\sin^2\frac{\Theta}{2}e_4^{(\lambda)}(\vec{k})e_4^{(\rho)}(\vec{p})$$
$$+ik_0\left[e_i^{(\lambda)}(\vec{k})p_i e_4^{(\rho)}(\vec{p}) + e_4^{(\lambda)}(\vec{k})e_i^{(\rho)}(\vec{p})k_i\right] - \frac{1}{2}e_i^{(\lambda)}(\vec{k})p_i e_j^{(\rho)}(\vec{p})k_j, \tag{20d}$$

$$Q_{R.S.}(k,p) = \overline{u}_i^{(s)}(\vec{p})\left(\varepsilon_{ij4l}(k_l+p_l)\gamma_5\gamma_4\right)u_j^{(r)}(\vec{k}), \tag{20e}$$

$$Q_T(k,p) = \left(\vec{k}^2\cos\Theta + \frac{M^2}{2}\right)e_{ij}^{(t)}(\vec{k})e_{ij}^{(u)}(\vec{p}) + \vec{k}^2\sin^2\frac{\Theta}{2}e_{i4}^{(t)}(\vec{k})e_{i4}^{(u)}(\vec{p}), \tag{20f}$$

$$Q_G(k,p) = e_{\alpha\beta}^{(a)}(\vec{k})e_{\alpha\beta}^{(b)}(\vec{p}). \tag{20g}$$

The differential corss section results by averaging over the initial and summing over the final spin (polarization) states of particles [8]:

$$d\sigma = (2\pi)^2\left\langle\sum_{f.sp.}|F(k,p)|^2\right\rangle_{i.sp.} k_0^2 d\Omega, \tag{21}$$

where $d\Omega = 2\pi\sin\Theta d\Theta$ is the infinitesimal solid angle element in the direction of emeging particles. Thus, we have:

$$|F_{K.G.}|^2 = \left[\frac{\chi^2 M_\Theta}{8(2\pi)^2 k_0\vec{k}^2\sin^2\frac{\Theta}{2}}\right]^2 |Q_{K.G.}(k,p)|^2, \tag{22a}$$

$$\sum_{f.sp.}|F_D|^2 = \left[\frac{\chi^2 M_\Theta}{32(2\pi)^2 k_0^2\sin^2\frac{\Theta}{2}}\right]^2 \sum_{sp.}|Q_D(k,p)|^2, \tag{22b}$$

$$\left\langle\sum_{f.sp.}|F_M|^2\right\rangle_{i.sp.} = \left[\frac{\chi^2 M_\Theta}{8(2\pi)^2 k_0^3\sin^2\frac{\Theta}{2}}\right]^2 \frac{1}{2}\sum_{pol.}|Q_M(k,p)|^2, \tag{22c}$$

$$\left\langle\sum_{f.sp.}|F_P|^2\right\rangle_{i.sp.} = \left[\frac{\chi^2 M_\Theta}{8(2\pi)^2 k_0\vec{k}^2\sin^2\frac{\Theta}{2}}\right]^2 \frac{1}{3}\sum_{pol.}|Q_P(k,p)|^2, \tag{22d}$$

$$\left\langle\sum_{f.sp.}|F_{R.S.}|^2\right\rangle_{i.sp.} = \left[\frac{\chi^2 M_\Theta}{16(2\pi)^2 k_0^2\sin^2\frac{\Theta}{2}}\right]^2 \frac{1}{2}\sum_{pol.}|Q_{R.S.}(k,p)|^2, \tag{22e}$$

$$\left\langle\sum_{f.sp.}|F_T|^2\right\rangle_{i.sp.} = \left[\frac{\chi^2 M_\Theta}{16(2\pi)^2 k_0\vec{k}^2\sin^2\frac{\Theta}{2}}\right]^2 \frac{1}{5}\sum_{pol.}|Q_T(k,p)|^2, \tag{22f}$$

$$\left\langle\sum_{f.sp.}|F_G|^2\right\rangle_{i.sp.} = \left[\frac{\chi^2 M_\Theta}{16(2\pi)^2 k_0\sin^2\frac{\Theta}{2}}\right]^2 \frac{1}{2}\sum_{pol.}|Q_{K.G.}(k,p)|^2. \tag{22g}$$

On the one hand, for the calculus of the polarization sums over photon or spin-1 meson vectors and spin-2 meson or graviton tensors, the following completeness relations [8,9] are used:

$$\sum_{a=1}^{2} e_\alpha^{(a)}(\vec{k})e_\beta^{(a)}(\vec{k}) = d_{\alpha\beta} = \delta_{\alpha\beta} - \frac{k_\alpha k_\beta}{\vec{k}^2}, \tag{23c}$$

$$\sum_{\lambda=1}^{3} e_i^{(\lambda)}(\vec{k})e_j^{(\lambda)}(\vec{k}) = D_{ij} = \delta_{ij} + \frac{k_i k_j}{\mu^2}, \tag{23d}$$

$$\sum_{t=1}^{5} e_{ij}^{(t)}(\vec{k})e_{kl}^{(t)}(\vec{k}) = D'_{ik}D'_{jl} + D'_{il}D'_{jk} - \frac{2}{3}D'_{ij}D'_{kl}, \qquad D'_{ij} = \delta_{ij} + \frac{k_i k_j}{M^2}, \tag{23f}$$

$$\sum_{a=1}^{2} e_{\alpha\beta}^{(a)}(\vec{k})e_{\mu\nu}^{(a)}(\vec{k}) = d_{\alpha\mu}d_{\beta\nu} + d_{\alpha\nu}d_{\beta\mu} - d_{\alpha\beta}d_{\mu\nu}, \tag{23g}$$

respectively.

On the other hand, to evaluate the spin sums for positive energy massless Dirac spinors and R.S. spin-vectors, the following projection operators - written in the covariant form [8,10] are used:

$$P^+(\vec{k}) = \sum_{r=1}^{2} u^{(r)}(\vec{k})\overline{u}^{(r)}(\vec{k}) = \frac{\gamma_j k_j}{2ik_0}, \tag{23b}$$

$$P_{ij}^+(\vec{k}) = \sum_{r=1}^{2} u_i^{(r)}(\vec{k})\overline{u}_j^{(r)}(\vec{k}) == \frac{1}{2ik_0}\left(\delta_{ij}\gamma_l k_l + \frac{1}{2}\gamma_i\gamma_l\gamma_j k_l - \gamma_i k_j - \gamma_j k_i\right) = -\frac{\gamma_j\gamma_l\gamma_i k_l}{4ik_0}, \tag{23e}$$

obtaining

$$\sum_{sp.}|Q_D(k,p)|^2 = \frac{1}{8k_0^2}\mathrm{Sp}\left\{\left[4ik_0\gamma_4 + (k_i+p_i)\gamma_i\right](1+\gamma_5)\gamma_k p_k\left[4ik_0\gamma_4 + (k_j+p_j)\gamma_j\right]\gamma_l k_l\right\}, \tag{24b}$$

respectively:

$$\sum_{sp.}|Q_{R.S.}(k,p)|^2 = -\frac{1}{16k_0^2}\varepsilon_{ij4l}\varepsilon_{mn4k}p_r k_s(k_l+p_l)(k_k+p_k)\mathrm{Sp}(\gamma_5\gamma_4\gamma_m\gamma_r\gamma_i\gamma_5\gamma_4\gamma_j\gamma_s\gamma_n). \tag{24c}$$

After laborious calculus, the corresponding differential cross sections are given by [11-14]:

$$d\sigma_{K.G.} = (GM_\Theta)^2 \frac{d\Omega}{\sin^4\frac{\Theta}{2}}\left(\frac{1+w^2\cos^2\frac{\Theta}{2}}{2w^2}\right)^2, \qquad (25a)$$

$$d\sigma_D = (GM_\Theta)^2 \frac{d\Omega}{\sin^4\frac{\Theta}{2}}\cos^2\frac{\Theta}{2}, \qquad (25b)$$

$$d\sigma_M = (GM_\Theta)^2 \frac{d\Omega}{\sin^4\frac{\Theta}{2}}\cos^4\frac{\Theta}{2}, \qquad (25c)$$

$$d\sigma_P = (GM_\Theta)^2 \frac{d\Omega}{\sin^4\frac{\Theta}{2}}\left[\left(\frac{1+w^2}{2w^2}\right)^2 - \frac{2\sin^2\frac{\Theta}{2}}{3w^2}\left(2-w^2\sin^2\frac{\Theta}{2}\right)\right], \qquad (25d)$$

$$d\sigma_{R.S.} = (GM_\Theta)^2 \frac{d\Omega}{\sin^4\frac{\Theta}{2}}\cos^2\frac{\Theta}{2}, \qquad (25e)$$

$$d\sigma_T = (GM_\Theta)^2 \frac{d\Omega}{\sin^4\frac{\Theta}{2}}\Bigg\{\left(\frac{1+w^2}{2w^2}\right)^2 + \frac{2}{5}\left[\frac{1+w^2}{3w(1-w^2)^2}\right]^2\left[15(1-w^2)^3\right.$$
$$\left. +24w^2(1-w^2)^2\sin^2\frac{\Theta}{2} + 16w^4(1-w^2)\sin^4\frac{\Theta}{2} + 8w^6\sin^6\frac{\Theta}{2}\right]\sin^2\frac{\Theta}{2}\Bigg\}, \qquad (25f)$$

$$d\sigma_G = (GM_\Theta)^2 \frac{d\Omega}{\sin^4\frac{\Theta}{2}}\left(\cos^2\frac{\Theta}{2} + \frac{1}{8}\sin^4\Theta\right), \qquad (25g)$$

where we denoted by w the ratio $|\vec{k}|/k_0$ (with $k_0 = \sqrt{\vec{k}^2+m^2}$, $\sqrt{\vec{k}^2+\mu^2}$ and $\sqrt{\vec{k}^2+M^2}$, respectively), in agreement with results in [4,15-19], obtained in different manners[4]. In addition, it is worthwile to mention that the gravittional scattering of massless and massive R.S. particles is rarely studied in concrete applications [13,20] even it exists an outstanding general approach by the use of extended supergravities [5].

[4] Thus, the gravitational scattering of neutrinos is analysed by Boccaletti et al. [15] starting from Gupta's coupling between the energy-momentum tensor T_{ij} of matter field and the weak gravitational field h_{ij}. They showed that there is no difference between a two-component and four-component neutrinos.

The interaction between the grvitational and electromagnetic fields and calculation of the effect corresponding to one of the three crucial test of Einstein theory, *i.e.* the bending of a light beam in a gravitational field, were studied by Boccaletti et al. [16], Mitskevich [4] and Lotze [17],who used the interaction Lagrangian (28), equivalent with our one (10).

Concerning the scattering of massive vector particles on Schwarzschild background, our results generalise the Vladimirov's ones obtained at small angles in Sueckelberg formalism [7].

At last, the scattering of gravitons in external gravitational field were analysed by de Logi and Kovacs [18], Mitskevich [4] and Lotze [17], starting from Einstein Lagrangian proportional to scalar curvature R. Our results correspond to those of mentioned authors, apart from a factor $\cos^2\theta$, in agreement with Peter's results [19], obtained using a method based on Green's function formalism.

3. Discussion and conclusions

Taking into account the relations (25a,d,f) the differential cross-sections in the small angle approximation become:

$$d\sigma^{massive}_{small\Theta} = (GM_\Theta)^2 \frac{d\Omega}{\sin^4\frac{\Theta}{2}}\left(\frac{1+w^2}{2w^2}\right)^2 = d\sigma^{massive}_{Ruth.}, \qquad (26)$$

i.e., they are the differential cross-sections of Rutherford type. As we can see from (25d,f) the expression for $d\sigma^{massive}_{Ruth.}$ is exactly the differential cross-section for the massive scalar particle (for instance, scalar mesons); therefore we can interpret the second term in relations (25d,f) as being the spin contribution of the massive vector and tensor particles, respectively. In the same approximation the scattering of massless particles occours identically. Indeed we have

$$d\sigma^{massless}_{small\Theta} = (GM_\Theta)^2 \frac{d\Omega}{\sin^4\frac{\Theta}{2}} = d\sigma^{massless}_{Ruth.}. \qquad (27)$$

On the other hand, as we can see from relations (25), in the backward scattering limit case (*i.e.* for $\Theta = \pi$), the differential cross-sections for electromagnetic waves vanishes, whereas for gravitational ones it is not. According to de Logi and Kovacs [18], this fact means that the helicity of photons is conserved, while for gravitons the helicity is not conserved.

Comparing these results with those already obtained by us [11-14,20], we shall notice that in the small angle approximation, the differential cross-section of massive Klein-Gordon, Dirac, Proca, R.S. and tensor particles have the same form, and in the ultrarelativistic limit ($w \to 1$) they coincide with those corresponding to the massless Klein-Gordon particles, neutrinos, photons, massless R.S. particles (gravitinos) and gravitons. In other words, in this limit case the gravitational particle scattering is spin independent, in agreement with Vladimirov's results [7].

Finally, we can conclude that unlike the mentioned authors [4,16,17], who used the interaction Lagrangian

$$L_M = -\frac{1}{4}\sqrt{-g}g^{ik}g^{jl}F_{ij}F_{kl}, \qquad (28)$$

we have started from Lagrangian (10) (equivalent with (28), besides a divergence) which explicitly contains the covariant derivative. This fact seems to us more naturally, because in order to determine the appropriate first-order interaction Lagrangian, we have used the principle of minimal coupling in Quantum Gravity.

References

[1] N.D. Birrell, P.C.W. Davies: *Quantum Fields in Curved Space*, Cambridge University Press, 1984

[2] S. Gupta: *Proc. Roy. Soc* **A65** (1952) 161 : 608

[3] K. Staniukovich, V. Melnikov: *Gidrodinamika, polia i constanti v teorii gravitacii*, Moskva, Energoatomizdat, 1983 (in Russian)

[4] N. Mitskevich: *Fiziceskie polia v obscei teorii otnositelnnosti*, Izd. Nauka, Moskva, 1969 (in Russian)

[5] P. Van Nieuwenhuizen: *Physics Reports* **68** (1981) 189

[6] B. De Witt: *Phys. Rev.* **160** (1967) 1 113; *ibid* **162** (1967) 1195; 1239

[7] Iu. Vladimirov: Izd-vo V.U.Zov, *Fizika* nr. 2 (1963) 133; *ibid* nr. 6 (1964) 82

[8] L.H. Ryder: *Quantum field theory*, Cambridge University Press, 1965

[9] L. Schwinger: *Particles, Sources and Fields*, Addison-Wesley Publ. Comp., 1970 (printed in USA)

[10] B. De Witt: *Relativity, Groups and Topology*, Gordon and Breach, NY 1964

[11] D. Tatomir: *Romanian J. Phys* **39** (1994) 373

[12] D. Tatomir, D. Radu: *Romanian J. Phys* **39** (491) 194

[13] D. Tatomir, S. Oancea, V. Manta: *Tensor* **57** (1996) 44

[14] D. Radu, D. Tatomir: *I Nuovo Cimento* **111B** (1996) 1465

[15] D. Boccalatti, V. De Sabbata, P. Fortini, C. Gualdi: *Il Nuovo Cimento* **11B** (1972) 289

[16] D. Boccalatti, V. De Sabbata, C. Gualdi, P.Fortini: *Il Nuovo Cimento* **48** (1967) 58

[17] K. Lotze: *Acta Physica Polonica* **B9** (1978) 665; 677

[18] W. De Logi, S.J. Kovacs Jr.: *Phys. Rev.* **D16** (1977) 237

[19] P.C. Peters: *Phys. Rev.* **D13** (1976) 775

[20] D. Tatomir: in *10th International Conference in General Relativity and Gravitation*, Padova, 4-9 July, 1983, p. 140