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PHASE ESTIMATION IN WEAK FIELDS[1]

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An objective theory of phase shift estimation is formulated within quantum mechanics by using estimation theory. While common analyses yield artificial discrete spectra of measured phase, Bayesian analysis allows us to preserve continuous character of a measured quantity even in quantum domain. Further, it can interpret arbitrary weak detected data in an unbiased way, and phase information, although small, can be acquired as soon as one measurement is performed. The formalism was applied on estimation of unknown phase in neutron interferometry operating near the quantum limit, when only a few particles are registered, and in the regime of strong fields as well.

1. Introduction

Although disputes about existence or non-existence of a proper phase operator date till the times of Dirac [1], the measurability of phase, or phase shift at least, is unquestionable. However, this measurement is often analyzed wrongly and discrepancies appear especially in the case of weak input fields ($\bar{N} \approx 1$), when only a few particles (photons, neutrons, etc.) interfere. This contribution describes flaws of the currently used analyses and proposes a method which does not suffer from them. The method is demonstrated on the Mach-Zehnder interferometer and was tested on a neutron interferometer with a Poissonian source [2] but its principle is applicable for other setups and non-classical sources as well.

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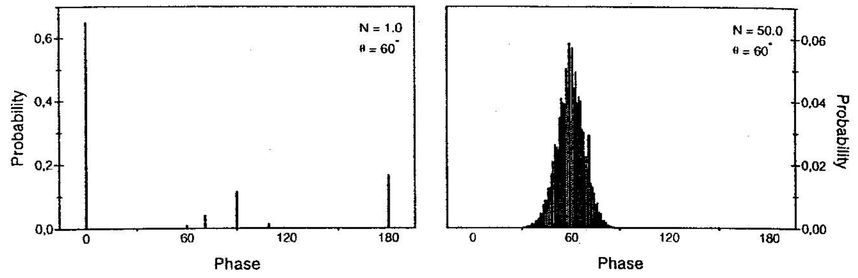

Fig. 1. Spectrum of possible results of the point estimation. Calculated for $\mathbf{D} = (N^o, N^h)$, visibilities $V^o = V^h = 1$, true phase shift $\theta = 60°$ and $\bar{N}_{in} = 1.0$ (left) or $\bar{N}_{in} = 50.0$ (right).

2. Point estimation

The common analysis of phase shift measurement ranges among point estimation methods [3, 4, 5]. Since phase shift cannot be measured directly, at first phase sensitive data $\mathbf{D}$ are *detected*. In our particular case, we used either particle numbers detected at both interferometer's outputs, or particle numbers detected at one output for several phase shift values along interference pattern. Afterwards a certain phase value is ***estimated*** from these data. The first weakness of this approach appears here because the correspondence ***phase shift*** $\longleftrightarrow$ *detected data* is probabilistic due to quantum uncertainty, while point estimation requires a strict one-to-one correspondence. Semi-classical approximation, maximum-likelihood method, or interference-pattern fitting are used but the contradiction remains: the estimated variable, phase shift, is continuous [6], whereas only a discrete spectrum of values can be obtained (see Fig. 1). Although a clever circumvention was proposed [5, 7], it gives no help for actual measurement analyses.

Detection and *estimation* are repeated many times, and phase estimates ϕ_i from individual measurements are averaged yielding the mean value and measurement error. The averaging is a second weak point: non-Gaussian distributed numbers (for weak inputs) are averaged regardless of their unequal information value. This insensitiveness results in strongly biased estimations for the case of weak input fields (see Fig. 2).

3. Bayes' (interval) estimation

The detection step is the same as in the previous case. But then the estimation evaluates not a discrete value but a continuous *likelihood function* [8]. The term *Bayes* indicates the use of Bayes' theorem about probability inversion [9]. Knowing probabilities $P(\mathbf{D}|\theta)$ of detection data $\mathbf{D}$ when phase shift θ is true, the likelihood can be written as

$$P(\phi|\mathbf{D}_i) = p(\phi)P(\mathbf{D}_i|\phi)/C_{\mathbf{D}_i}. \quad (1)$$

$C_{\mathbf{D}_i}$ is a normalization constant and $p(\phi)$ represents our *a priori* information about phase shift (often $1/2\pi$). These steps are repeated over and over and the information

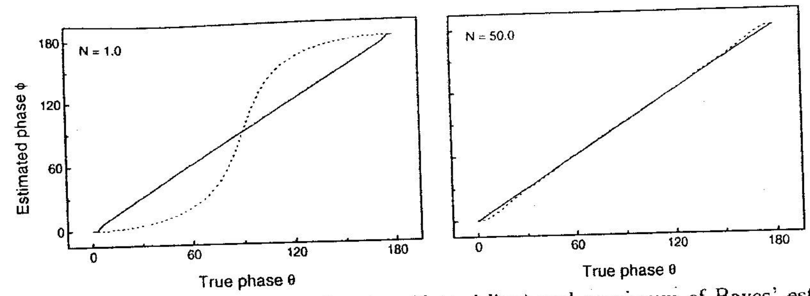

Fig. 2. Mean value of the point estimation (dotted line) and maximum of Bayes' estimation (solid line). Calculated for the same parameters as Fig. 1.

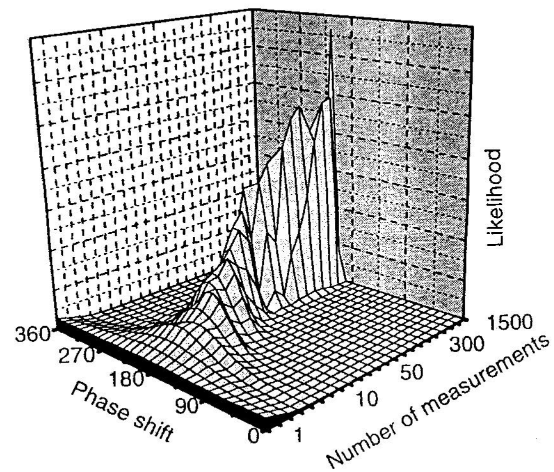

Fig. 3. Improvement of Bayes' phase estimation with increasing number of measurements (and thereby particles) used for the evaluation. Measured for $\mathbf{D} = (N_1^o \dots N_9^o)$, $V^o = 0.34$, true phase shift $\theta = 212°$ and $\bar{N}_{in} = 0.5$.

is accumulated in form of the product

$$P(\phi|\mathbf{D}) \propto \prod_i P(\phi|\mathbf{D}_i). \quad (2)$$

New repetitions make the resulting distribution sharper and make it tend to the true phase value (Fig. 3). The continuous character of interval estimation overcomes the first flaw of the point estimation and Fig. 2 shows that this distribution is unbiased as well.

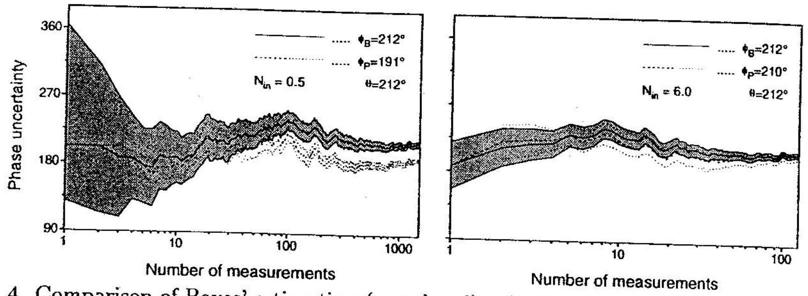

Fig. 4. Comparison of Bayes' estimation (gray band) with the point estimation (dotted band). 68% confidence intervals are plotted against the number of measurements used for evaluation. Measured for $\mathbf{D} = (N_1^o \ldots N_9^o)$, $V^o = 0.34$, true phase shift $\theta = 212°$ and $\bar{N}_{in} = 0.5$ (left) or $\bar{N}_{in} = 6.0$ (right).

4. Discussion

The theory was tested experimentally at the Atominstitut in Vienna with its 250kW TRIGA reactor neutron source on a perfect crystal neutron interferometer. It was verified that our proposal, Bayes' estimation, corrects defects of the common analyses — discrete spectra and biased estimates in the low particle number limit (see Fig. 4). The flaws of the point estimation disappear for higher particle numbers and then both the methods tend to the same results. In addition, Bayes' estimation allows us to state a phase prediction right after *one* measurement is performed. The prediction is probabilistic, not certain, in accordance with the probabilistic character of quantum mechanics.

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