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PHOTON ANTIBUNCHING VERSUS PHANTOM ANTIBUNCHING?[1]

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Photon antibunching defined by two-time correlation functions has hitherto, to our best knowledge, been considered to constitute a unique, well defined effect. We show explicitly that this is by no means the case. We analyze two of the most famous definitions showing that both antibunching and bunching effects according to one definition can be accompanied by arbitrary photon correlation effects according to another. As an example we discuss a model of parametric frequency conversion.

1. Introduction

Photon antibunching, apart from sub-Poissonian photon-number statistics and squeezing, is the foremost manifestation of the quantum nature of light. Since the classic experiments of Kimble, Dagenais and Mandel [1], antibunching has been in the forefront of both theoretical and experimental research of quantum opticians [2–9]. Singh [5], and Zhou and Mandel [6] have shown that antibunching need not be associated with sub-Poisson counting statistics and vice versa. However, it has been thought that definitions based on the unnormalized and normalized two-time correlation functions describe essentially the same effect. We show that these are two distinct phenomena which need not necessarily occur together.

2. Definitions

Photon antibunching for a single-mode radiation field is usually defined in two ways (see e.g. [1–7] and references therein). These definitions are interchangeably used in both theoretical and experimental research.

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2.1 Antibunching I

A first approach to antibunching is based on the unnormalized two-time light intensity correlation function (second-order correlation function, coincidence rate)

$$G^{(2)}(t,t+\tau) = \langle \mathcal{T} : \hat{n}(t)\hat{n}(t+\tau) :\rangle = \langle \hat{a}^\dagger(t)\hat{a}^\dagger(t+\tau)\hat{a}(t+\tau)\hat{a}(t)\rangle, \tag{1}$$

where the operator product is written in normal order and in time order. According to this definition (see e.g. Ref. [2]) antibunching occurs if the two-time light intensity correlation function $G^{(2)}(t,t+\tau)$ increases from its initial value at $\tau = 0$, i.e.,

$$\textbf{Def. 1}: \quad G^{(2)}(t,t+\tau) > G^{(2)}(t,t) \tag{2}$$

or, equivalently, for a well-behaved function $G^{(2)}(t,t+\tau)$, if its derivative

$$\Gamma(t) \equiv \Gamma^{(2)}(t) = \frac{\partial}{\partial\tau} G^{(2)}(t,t+\tau)\Big|_{\tau=0} > 0 \tag{3}$$

is positive. Similarly, bunching occurs for $\Gamma^{(2)}(t) < 0$, and unbunching occurs for a vanishing derivative $\Gamma^{(2)}(t) = 0$.

2.2 Antibunching II

According to another definition (see e.g. [1]), antibunching takes place if the two-time light intensity correlation function $g^{(2)}(t,t+\tau)$ increases from its initial value at $\tau = 0$, i.e.,

$$\textbf{Def. 2}: \quad g^{(2)}(t,t+\tau) > g^{(2)}(t,t) \tag{4}$$

in terms of the normalized version of function $G^{(2)}(t,t+\tau)$, viz. the normalized coincidence rate

$$g^{(2)}(t,t+\tau) \equiv \lambda(t,t+\tau) + 1 \equiv \frac{G^{(2)}(t,t+\tau)}{G^{(1)}(t)G^{(1)}(t+\tau)}, \tag{5}$$

for τ increasing from $\tau = 0$. In Eq. (5) the first-order correlation function $G^{(1)}(t) = \langle n(t)\rangle = \langle \hat{a}^\dagger(t)a(t)\rangle$ intervenes. Def. 2, for a well-behaved $g^{(2)}(t,t+\tau)$, can be given in terms of its positive derivative [7]

$$\gamma(t) \equiv \gamma^{(2)}(t) = \frac{\partial}{\partial\tau} g^{(2)}(t,t+\tau)\Big|_{\tau=0} > 0. \tag{6}$$

Again, bunching is said to exist for $\gamma^{(2)}(t) < 0$, and unbunching for $\gamma^{(2)}(t) = 0$.

2.3 Other definitions

Sometimes, different normalization of $G^{(2)}(t,t+\tau)$ is used in general case, which leads yet to another definition of photon antibunching [8]:

$$\textbf{Def. 3}: \quad \bar{g}^{(2)}(t,t+\tau) > \bar{g}^{(2)}(t,t), \quad \text{where} \quad \bar{g}^{(2)}(t,t+\tau) = \frac{G^{(2)}(t,t+\tau)}{\left[G^{(1)}(t)\right]^2}, \tag{7}$$

or, alternatively, for its derivative. It is clear that Def. 3, except for the cases $G^{(1)}(t) = 0$ and $G^{(1)}(t) \longrightarrow \infty$, is equivalent to Def. 1.

For completeness, we invoke the definition of antibunching based on the single-time correlation function or, equivalently, on the Mandel Q-parameter. The conditions

$$\textbf{Def. 4}: \quad Q(t) \equiv \langle n(t)\rangle \left(g^{(2)}(t,t) - 1\right) < 0 \quad \text{or} \quad g^{(2)}(t,t) < 1 \tag{8}$$

express sub-Poissonian photon-number statistics sometimes also called "antibunching" [3,4]. It is now well known, as was shown explicitly by Singh [5], and Zhou and Mandel [6] and others, that Def. 4 based on single-time correlation function is essentially different from Definitions 1–3 based on two-time correlation functions (or its single-time derivatives).

3. Comparison of antibunching I and II

Definitions 1 and 2 are equivalent for stationary fields for which $\langle n(t)\rangle = \langle n(t+\tau)\rangle$ holds. We claim, contrary to common belief, that Defs. 1 and 2 describe different effects for nonstationary fields. The difference is a result of mathematical properties of derivatives, namely $[f_1(\tau)/f_2(\tau)]'$ is not equal to $f_1'(\tau)/f_2(\tau)$ for any τ-dependent function f_2. As an example we discuss a simple model of parametric frequency conversion as described by the interaction Hamiltonian:

$$\hat{H}_{\rm int} = \hbar g \left(\hat{a}_1^\dagger \hat{a}_2 + \text{h.c.}\right). \tag{9}$$

The well known solutions for the first and second modes are, respectively,

$$\hat{a}_1(t) = \hat{a}_1\cos(gt) - i\hat{a}_2\sin(gt), \quad \hat{a}_2(t) = \hat{a}_2\cos(gt) - i\hat{a}_1\sin(gt). \tag{10}$$

Just for brevity we present formulas for the first mode only. Due to symmetry of solutions (11), expressions for the second mode are given by those for the first mode albeit with interchanged subscripts 1 and 2. For initial Fock states $|N_1\rangle$ and $|N_2\rangle$ we find

$$\begin{aligned}\Gamma_1(t) &= \frac{g}{2}\sin(2gt)\,\{N_2(N_2-1) - N_1(N_1-1) \\ &\quad - [N_1(N_1-1) - 4N_1N_2 + N_2(N_2-1)]\cos(2gt)\}, \\ \gamma_1(t) &= g\,\frac{N_1N_2}{\langle n_1(t)\rangle^3}\sin(2gt)\left[(N_1+1)\cos^2(gt) - (N_2+1)\sin^2(gt)\right],\end{aligned} \tag{11}$$

where the mean photon number in the first mode is $\langle n_1(t)\rangle = N_1\cos^2(gt) + N_2\sin^2(gt)$. Let us analyze a few simplest cases. For $N_1 = N_2 = 1$, Eqs. (12) reduce to $\Gamma_1(t) = \gamma_1(t) = g\sin(4gt)$, which implies that antibunching I and antibunching II occur together. For $N_1 = 2$, $N_2 = 0$, one obtains

$$\Gamma_1(t) = -2g\cos(gt)^2\sin(2gt), \quad \gamma_1(t) = 0, \tag{12}$$

which shows that (anti)bunching I is associated with unbunching II. Finally, for $N_1 = 2$, $N_2 = 1$ we have

$$\begin{aligned} \Gamma_1(t) &= g\sin(2gt)\,\{3\cos(2gt) - 1\}\,, \\ \gamma_1(t) &= 8g[3 + \cos(2gt)]^{-3}\sin(2gt)\,\{5\cos(2gt) + 1\}\,. \end{aligned} \quad (13)$$

It is clearly seen, that $\Gamma_1(t)$ and $\gamma_1(t)$ can have opposite signs due different expressions in curly brackets in Eqs. (14). We conclude that for properly chosen initial fields and evolution times, light can exhibit either (i) antibunching I concomitantly with antibunching II, (ii) bunching I concomitantly with bunching II; (iii) antibunching I concomitantly with bunching II; or (iv) vice versa – bunching I concomitantly with antibunching II. Due to a limited number of pages we cannot include other examples like, e.g., higher harmonics and subharmonics generation. A deeper analysis of the problem will be presented elsewhere [9].

4. Conclusion

We have compared the most famous definitions (Defs. 1 and 2) of photon antibunching. These definitions of photon antibunching have till now, to our best knowledge, been considered to describe a unique, well defined effect. Defs. 1 and 2 are equivalent for stationary fields. However, this is by no means the case for arbitrary fields. Def. 1 based on the unnormalized function $G^{(2)}(t, t+\tau)$ and Def. 2 in terms of $g^{(2)}(t, t+\tau)$ describe distinct quantum phenomena, and it seems to be highly important that these definitions shall not be confused. Therefore we address the question: *"Is the photon antibunching really the photon antibunching?"*

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References

[1] H.J. Kimble, M. Daganais, L. Mandel: *Phys. Rev. Lett.* **39** (1977) 691
[2] L. Mandel, E. Wolf: *Optical Coherence and Quantum Optics* (Cambridge Univ. Press, 1995) p. 712
[3] H. Paul: *Rev. Mod. Phys.* **54** (1982) 1061
[4] M.C. Teich, B.E.A. Saleh: *Progress in Optics* **26** (1988) 1
[5] S. Singh: *Opt. Commun.* **44** (1983) 254
[6] X.T. Zhou, L. Mandel: *Phys. Rev. A* **41** (1990) 475
[7] H.T. Dung, A.S. Shumovsky, N.N. Bogolubov, Jr.: *Opt. Commun.* **90** (1992) 322; E.I. Aliskenderov, H.T. Dung, L. Knöll: *Phys. Rev. A* **48** (1993) 1604
[8] J. Peřina: *Quantum Statistics of Linear and Nonlinear Optical Phenomena*, 2^{nd} ed. (Kluwer, Dordrecht, 1991)
[9] A. Miranowicz, J. Bajer, A. Ekert: in preparation.