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HIGHER-ORDER DISPLACED KERR STATES[1]

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We discuss quantum states generated in a Mach-Zehnder interferometer with a Kerr medium in one of its arms. These states are referred to as *displaced Kerr states*. Contrary to the former approaches, we describe a nonlinear Kerr medium not only by the third-order (two-photon) but also by higher-order (k-photon [$k = 3, 4, ...$]) nonlinear oscillators. We refer to the states generated in the system with higher optical Kerr nonlinearity as *higher-order displaced Kerr states* or *multi-photon displaced Kerr states*. We investigate the quantum-statistical properties of these states showing their dependence on the degree of the Kerr nonlinearity.

Systems with nonlinear Kerr-like media were the subject of numerous papers (see e.g. [1-11] and references cited therein) and were discussed from various points of view. For instance, systems involving Kerr media can, under some assumptions, exhibit self-squeezing properties [1,2,7], and can lead to n-photon states [3,6,8,9] or to Schrödinger cats [11] generation. These systems can also be a very interesting subject for classical and quantum chaos investigations [5,10].

In this paper we shall concentrate on systems very similar to that discussed by Wilson-Gordon *et al.* [4] namely, on the Mach-Zehnder interferometer with a nonlinear Kerr-like medium in one of its arms. Our system differs from that of Ref. [4] in that we assume an arbitrary degree of the nonlinearity λ_l. The states generated in the model of Ref. [4] have been referred to as displaced Kerr states, whereas the states discussed in this paper shall be referred to as *higher-order displaced Kerr states* (HDKS) or *multi-photon displaced Kerr states*. Our states

$$|\psi_{\rm HDK}\rangle = \widehat{U}_{\rm disp}\widehat{U}_{\rm Kerr}\,|\alpha\rangle\,, \tag{1}$$

can be generated from the usual coherent states $|\alpha\rangle$ by application of the standard Glauber displacement operator

$$\widehat{U}_{\rm disp} = \exp(\xi\widehat{a}^{\dagger} - \xi^{*}\widehat{a}) \tag{2}$$

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and the Kerr evolution operator defined in a rather general manner as

$$\widehat{U}_{\mathrm{Kerr}} = \exp\left(i\frac{\lambda_l}{l!}\widehat{a}^{\dagger l}\widehat{a}^l\right), \tag{3}$$

where the parameter $\lambda_l = l!\chi_l L/v$ depends on the coupling constant χ_l, length L of the Kerr medium and phase velocity v. The coupling constant χ_l describes the nonlinearity of the Kerr medium and is related to the $(2l-1)$-order susceptibility $\chi^{(2l-1)}$. Obviously, the parameter l determines the degree of the nonlinearity and hence, the order of HDKS. For these states we have derived an appropriate formula for the expansion in the number-states basis. We have performed calculations applying the procedure proposed in Ref. [4] and obtained the following result:

$$|\psi_{\mathrm{HDK}}\rangle = \sum_{n=0}^{\infty} b_n |n\rangle , \tag{4}$$

$$b_n = (n!)^{-1/2} e^{-|\xi|^2/2-|\alpha|^2/2} \sum_{k=0}^{\infty} e^{i\phi(n-k)} \alpha^k \exp\left[i\frac{\lambda_l}{l!}\prod_{r=0}^{l-1}(k-r)\right] \tag{5}$$
$$\times \sum_{j=0}^{\min(n,k)} (-1)^{k-j} |\xi|^{n+k-2j} \binom{n}{j} \frac{1}{(k-j)!} ,$$

We have applied in Eq. (5) the following representation for the complex displacement parameter $\xi = |\xi| exp(i\phi)$. This result resembles that derived by Wilson-Gordon et al. [4]. It is seen that the formula (5) is rather complicated. Nevertheless it can be easily handled numerically and be helpful in numerical calculations for various parameters describing the quantum properties of the field. For instance, the mean values of the operators $\langle\widehat{a}\rangle$ and $\langle\widehat{a}^2\rangle$ that are necessary to find many field parameters can be calculated and expressed by the amplitudes b_n

$$\langle\widehat{a}\rangle = \sum_{n=0}^{\infty}\sqrt{n+1}b_n{}^* b_{n+1} \quad \langle\widehat{a}^2\rangle = \sum_{n=0}^{\infty}\sqrt{(n+1)(n+2)}b_n{}^* b_{n+2} .$$

Among the many parameters characterizing the field we have found the Mandel Q parameter ($Q = \left(\langle(\Delta\widehat{n})^2\rangle - \langle\widehat{n}\rangle\right) / \langle\widehat{n}\rangle$) [12] describing the statistics of the photons. Thus, Fig. 1 shows the value of Q as a function of the Kerr nonlinearity parameter λ_l for various values of l. It is seen from Figs. 1a-c that the evolution of the value of Q starts from the zero for all of the cases shown here and Q decreases to its first minimum. This shows that the photon statistics is sub-Poissonian for small values of λ_l. For greater values of the nonlinearity parameter λ_l the value of Q increases and becomes greater than zero (the statistics becomes super-Poissonian). Of course, one should keep in mind that Figs. 1a-c show only the beginning of the evolution of Q and Fig. 1a does not exhibit the changes in the value of Q as the parameter λ_l becomes greater and greater. The plots of Figs. 1a-c indicate that the evolution of Q depends on the degree of the nonlinearity of the Kerr-like medium. It is visible that the first minimum of Q becomes sharper as the degree l of the nonlinearity increases. Moreover, the depth of the minima varies depending on the degree of the nonlinearity too. From Figs. 1a-c we see that the values of these minima are equal to ~ -0.82, -0.71 and -0.57 for $l = 2, 3, 4$, respectively.

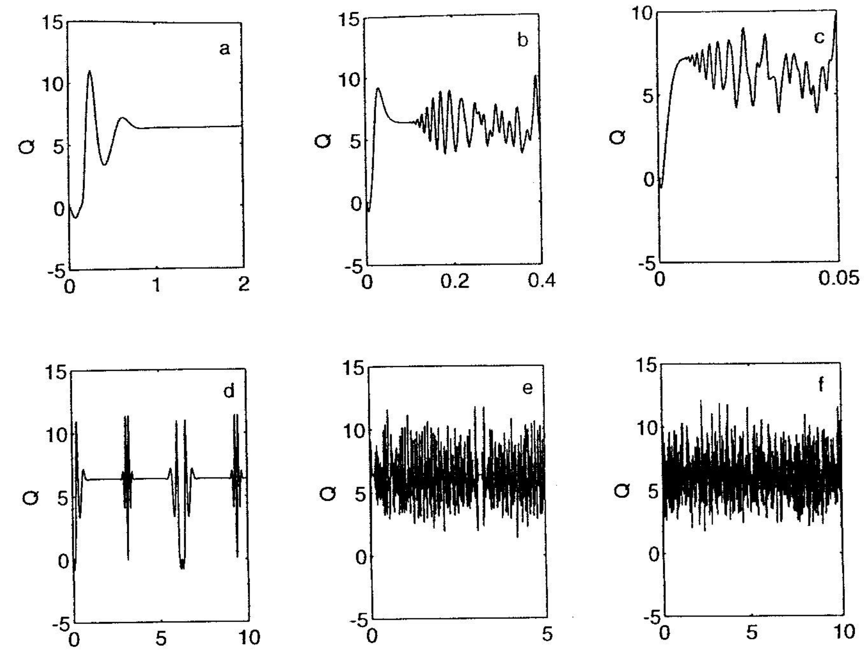

Fig. 1. Mandel Q parameter as a function of the parameter λ_l for various l (1a, 1d: $l = 2$; 1b, 1e: $l = 3$; 1c, 1f: $l = 4$). The parameters $\alpha = 4$ and $\xi = 2$.

Fig. 1b shows the value of the Mandel Q parameter for a longer time-scale (the parameter λ_l can be treated as generalized time). We see that for $l = 2$ the parameter Q behaves periodically with a period equal to 2π. We observe initial oscillations that are rapidly decreases to a constant value. When the value of λ_l becomes closer to π the oscillations revive. However, they strongly decrease again. Moreover, during these oscillations (for $\lambda_l \sim \pi$) the parameter Q does not reach negative values. This periodical behavior does not, however, result obviously from the complicated Eq. (5). We shall now apply an alternative procedure to derive the periodicity in question along more direct lines. Namely, we base on the operator solution in the Heisenberg picture. We define the annihilation operator corresponding to the HDKS as:

$$\widehat{a}(\xi,\lambda_l) = \widehat{U}_{\mathrm{Kerr}}(\lambda_l)\widehat{U}_{\mathrm{disp}}(\xi)\,\widehat{a}\,\widehat{U}^{\dagger}_{\mathrm{disp}}(\xi)\widehat{U}^{\dagger}_{\mathrm{Kerr}}(\lambda_l) \tag{6}$$

and after some algebra we find the following simple solution $\widehat{a}(\xi,\lambda_l)$:

$$\widehat{a}(\xi,\lambda_l) = \exp\left(-i\frac{\lambda_l}{(l-2)!l}\prod_{k=0}^{l-2}(\widehat{n}-k)\right)\widehat{a} - \xi . \tag{7}$$

Nevertheless, we need the expectation values of the operators that will allow us to find the various parameters describing the quantum properties of the field rather than the operator solutions. For instance, assuming that the field was initially in the coherent state $|\alpha\rangle$ the mean value of the annihilation operator $\widehat{a}(\xi,\lambda_l)$ can be expressed as:

$$\langle\widehat{a}(\xi,\lambda_l)\rangle = \alpha \sum_{m,j=0}^{\infty} e^{-|\alpha|^2}\frac{|\alpha|^{2m}x^j}{m!j!}\{m(m-1)\cdots[m-j(l-1)+1]\}-\xi, \quad (8)$$

where $x=-i\lambda_l/[(l-2)!l]$. Analogously, one can obtain the solution corresponding to any combination of the creation and annihilation operators. It is visible that formula (9) is rather complicated. Nevertheless, for the case of $l=2$ it becomes much simpler and can be written in the following form:

$$\langle\widehat{a}(\xi,\lambda_2)\rangle = \exp\left[-|\alpha|^2\left(1-e^{i\lambda_2}\right)\right]\alpha-\xi \quad (9)$$

Obviously, it is also possible to obtain the analytical solutions for any combinations of the creation and annihilation operators using this method. One can see that $\langle\widehat{a}(\xi,\lambda_2)\rangle$ is periodic with period equal to 2π. This fact explains the periodic behavior of Q shown in Fig. 1b.

However, for $l>2$ the dynamics of the parameter Q becomes more complicated and is obscured by oscillations that become dominant as the value of λ_l increases. Moreover, the frequencies of these oscillations become greater and greater with increasing degree of the Kerr nonlinearity. In consequence, the evolution of the Mandel Q parameter becomes very difficult to interpret.

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