acta physica slovaca vol. 47 No. 3/4, 307 – 310 June 1997

REVIVALS IN THE OFF-RESONANT JAYNES-CUMMINGS MODEL WITH A KERR MEDIUM[1]

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Received 25 April 1997, accepted 12 May 1997

On the example of the Jaynes-Cummings model with a Kerr medium and the initially coherent cavity field mode we show that the revival period of the oscillations may be strongly dependent on whether the initial mean photon-number is integer or non-integer.

The RWA Hamiltonian of the generalized Jaynes-Cummings model (JCM) with a Kerr medium [1,2] has the form ($\hbar = 1$)

$$H = \omega_f a^\dagger a + \omega_{\rm at} S_z + g\left(a^\dagger S_- + a S_+\right) + \chi a^{\dagger 2} a^2, \tag{1}$$

where ω_f denotes the frequency of the field mode while $\omega_{\rm at}$ is the atomic transition frequency. The symbol χ is the third-order susceptibility representing the nonlinearity of a Kerr medium, modelled here as an anharmonic oscillator.

At the initially inverted atom the atomic inversion $\langle S_z(t)\rangle$ evolves as follows:

$$\langle S_z(t)\rangle = \frac{1}{2} - 2\sum_{n=0} \frac{(n+1)g^2}{\Omega_n^2} P_n \left(1 - \cos\Omega_n t\right), \quad \Omega_n = 2\sqrt{(n\chi - \Delta/2)^2 + (n+1)g^2}, \tag{2}$$

where Ω_n denotes the Rabi frequency and P_n is the Poissonian distribution.

At a special choice of the detuning $\Delta = \omega_{\rm at} - \omega_f$

$$\Delta = 2\bar{n}\chi + g^2/\chi \tag{3}$$

the Rabi frequency has a minimum at the initial mean photon-number $\bar{n}$ [3]. Then, instead of the revivals resembling those manifested by the standard JCM, the nonlinear JCM exhibits superstructures [3]. We will consider in detail just this case solely.

[1] Presented at the **Fifth Central-European Workshop on Quantum Optics**, Prague, Czech Republic, April 25 - 28, 1997

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0323-0465/97

Let us treat the Rabi frequency as a continuous function and expand it around the point of its minimum

$$\Omega_n=\Omega_{\bar n}+\Omega_{\bar n}^{(2)}\,(n-\bar n)^2+\ldots,\qquad \Omega_{\bar n}=2g\sqrt{g^2/4\chi^2+\bar n+1},\qquad \Omega_{\bar n}^{(2)}=2\chi^2/\Omega_{\bar n}. \tag{4}$$

In fact, this expansion contains only even powers of $n-\bar n$. The first term of the above expansion is responsible for rapid oscillations of the model while the remaining terms are responsible for their envelope. Since the first nonvanishing derivative of the frequency is the second-order one, we can now speak of the second-order revivals, to distinguish them from the revivals exhibited by the standard JCM.

The first revival occurs if, at least the most heavily weighted terms of the series (2) acquire the phase difference equal to 2π. Let us consider first an integer $\bar n$. With respect to the symmetry properties of the expanded frequency (4) the cosines in the series (2), corresponding to $n=\bar n-I$ and $n=\bar n+I$ (I - an arbitrary integer), are always in phase. The revival period may be estimated as

$$(\Omega_{\bar n-1}-\Omega_{\bar n})\,T_R=(\Omega_{\bar n+1}-\Omega_{\bar n})\,T_R\approx\Omega_{\bar n}^{(2)}\,T_R=2\,\pi, \tag{5}$$

and in accordance with [3] reads

$$T_R^{\text{integer}}=\frac{2\pi}{\Omega_{\bar n}^{(2)}}=\frac{\pi}{\chi^2}\Omega_{\bar n}=\frac{2\pi g}{\chi^2}\sqrt{g^2/4\chi^2+\bar n+1}. \tag{6}$$

In turn, if $\bar n$ is a half-integer the terms corresponding to $\bar n-I/2$ and $\bar n+I/2$ are always in phase and the revival period is calculated as follows:

$$\left(\Omega_{\bar n-\frac32}-\Omega_{\bar n-\frac12}\right)T_R=\left(\Omega_{\bar n+\frac32}-\Omega_{\bar n+\frac12}\right)T_R\approx 2\Omega_{\bar n}^{(2)}\,T_R=2\,\pi, \tag{7}$$

and its general form is half of that for an integer $\bar n$

$$T_R^{\text{half-integer}}=\frac{\pi}{\Omega_{\bar n}^{(2)}}=\frac{\pi g}{\chi^2}\sqrt{g^2/4\chi^2+\bar n+1}. \tag{8}$$

The atomic inversion versus the scaled time is presented for $\bar n=5$ and 5.5 ($\chi=0.1g$, $\Delta=10g$ [3]) in the lower two graphs in Fig. 1. Due to such a choice of the time scale, the first revival for a half-integer $\bar n$ occurs at $gt/T_R=0.5$.

Let now $\bar n=N+f$; $f\in(0,1)$, but $f\neq 1/2$. N is an integer ≥ 1. There are no couples of terms oscillating in phase during the whole evolution. In comparison with the nearest neighbouring integer or half-integer $\bar n$, almost twice the number of terms have to reach a phase difference 2π and the revival period is expected to be longer. The terms corresponding to $\bar n-f-1$, $\bar n-f$ and $\bar n-f+1$ have the greatest weights. The first two of them become phased at

$$T_R^{(1)}=\frac{2\pi}{\Omega_{\bar n}^{(2)}(1+2f)},\quad\text{while the latter at}\quad T_R^{(2)}=\frac{2\pi}{\Omega_{\bar n}^{(2)}|1-2f|}.$$

To have only one formula valid either for $f<1/2$ or $f>1/2$, the absolute value of the denominator for $f>1/2$ has been introduced. Both times are equal for $f=0$, i.e.,

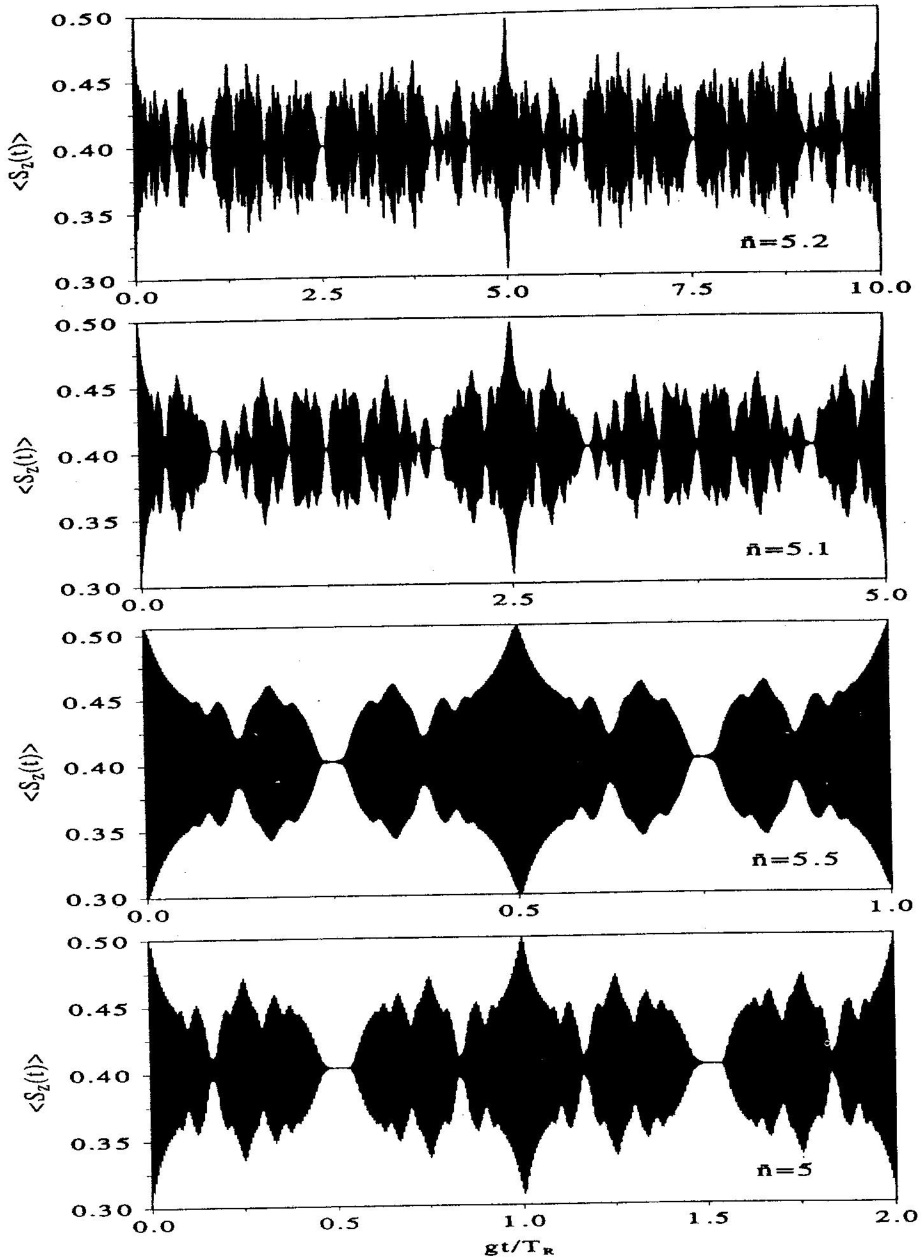

Fig. 1. Time evolution of the atomic inversion for $\bar n$ = 5, 5.5, 5.1 and 5.2 from the bottom to the top of the figure, respectively. The time is scaled by the quantity $T_R=2\pi g[g^2/(4\chi^2)+\bar n+1]^{1/2}/\chi^2$.

if $\bar{n}$ is an integer. For a fractional f (but, obviously, $f \neq 1/2$), the first revival will occur at $T_R = T_R^{(1)} \times I_1 = T_R^{(2)} \times I_2$, where $I_{1,2}$ are integers as small as possible. As a consequence, we obtain the following condition: $I_1/I_2 = (1+2f)/|1-2f|$. For instance, for $f=0.1$ the above condition leads to $I_1/I_2 = 3/2$, i.e., $I_1=3$, $I_2=2$ and the revival time for the model with $\bar{n} = N+0.1$ is equal to

$$T_R = \frac{5\pi g}{\chi^2}\sqrt{g^2/4\chi^2 + \bar{n} + 1}. \tag{9}$$

Its general form is multiplied by 2.5 in comparison with that for $\bar{n}=N$. The same form of the revival period holds for $f=0.3, 0.7$ and $f=0.9$.

In turn, for $f=0.2, 0.4, 0.6, 0.8$ one finds that

$$T_R = \frac{10\pi g}{\chi^2}\sqrt{g^2/4\chi^2 + \bar{n} + 1}. \tag{10}$$

This revival period is multiplied by 5 in comparison with that for $\bar{n}=N$.

Since the revival periods are longer, the pictures of the superstructures for $\bar{n}=5.1$ and 5.2 are more complicated (Fig. 1). As the number of digits after the decimal point grows in $\bar{n}$, the revivals periods grows as well; the superstructures get increasingly complex and become more and more blurred to some extent. The formulas (9) and (10) are also valid for $\bar{n}=f<1$ except $\bar{n}=0.1$ and 0.2. Then one has to consider quantum beats between the two meaningful terms only: $n=0$ and $n=1$. As a consequence, for these $\bar{n}$'s the revival period is multiplied by 5/4 and 5/3, respectively in comparison with that for $\bar{n}=N$.

We have recently shown that the Dicke model of an assemblage of A two-level atoms, coupled in an ideal cavity to a single-mode Fock field, also exhibits superstructures [4]. Contrary to the photon distribution mechanism discussed here, the origin of superstructures in the Dicke model is related to the collectivity of the system. The revival period of these collective revivals is strongly related to the parity of n in the weak-field domain ($n<A$), and to the parity of A in the strong-field domain ($A<n$). For instance, for a given A and $n<A$ the revival period is almost twice shorter for even n than for the nearest neighbouring odd n. The weak-field domain has its counterpart in the spontaneous emission of a partially inverted atomic system [5].

Acknowledgements This work was sponsored in part by the Program 2 PO3B 73 13 of the Polish Committee for Scientific Research.

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