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QUANTUM STATE RECONSTRUCTION WITH MULTIPORTS[1]

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The recently proposed arrangement with multiports [1] to spread a light beam over several detectors is analyzed. The use of high efficiency avalanche photodiodes for reconstructing the full quantum state of the system and the problem of measuring relative phase information in the two mode case are discussed.

1. Introduction

The measurement of the properties of nonclassical light modes is nontrivial [2]. Typical photodetectors are either of low quantum efficiency or do not yield the desired resolution in the photon statistics. Two types of photodetectors have in practice very high quantum efficiency: linear-response photodiodes and avalanche photodiodes. However, neither of them is appropriate even for directly measuring nonclassical oscillations in the photon statistics. Photodiodes are appropriate to measure strong signals, without one-photon resolution. In optical homodyne tomography they were successfully employed for indirectly measuring photon statistics and reconstructing a density matrix [3]. Avalanche photodiodes are sensitive for single photons, but are saturated and the one or more photon events cannot be distinguished. This suggests to detect weak signals with them.

The idea to spread the incoming light beam over several detectors with the help of passive optical multiports [4] (photon chopping) turned out to yield an arrangement

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providing, in principle, the full information about the quantum state if a well defined reference field is used [1]. Especially the photon statistics is measurable using avalanche photodiode type detectors [5], while for the phase information the discrimination of the single photon events is required. In the weak field limit (the photon numbers are compared to the size of the multiport) the method becomes exact, otherwise a cut-off has to be introduced.

We derive a first order correction which, in principle, offers the possibility of reconstructing also the phase information with avalanche photodiode type detectors for large enough multiports. The photon chopping method can also be generalized for the case of two spatially separable modes [6]. If interference between the modes is possible then relative phase information can be gained without external reference beams.

2. One mode state reconstruction with realistic detectors

We consider a fully symmetrical multiport with an unknown signal and a reference beam at two inputs. The corresponding quantum states for the signal and reference read

$$|\psi\rangle = \sum_{n=0}^{\infty} c_n |n\rangle\,, \qquad |\alpha\rangle = \sum_{n=0}^{\infty} \alpha_n |n\rangle, \tag{1}$$

respectively, where $|n\rangle$ are the Fock states. The multiport transformation leads to a linear combination of the known coefficients α_n of the reference state and the unknown coefficients c_n. The probability to measure $q_1, q_2, ..., q_N$ photons at the outputs reads

$$|\langle q_1, \ldots, q_N|\Psi_{out}\rangle|^2 = \frac{1}{N^n}\left|\sum_{s=0}^{n} c_{n-s}A(s,n) \sum_{\sum l_i = s} \binom{+}{-} \frac{\sqrt{q_1!\ldots q_N!}}{(q_1-l_1)!\ldots l_1!\ldots}\right|^2, \tag{2}$$

where we introduced the shorthand notation $\binom{+}{-} \equiv (-1)^{\sum_{i>N/2} l_i}$ and used the notation $n = \sum q_i$ (for the indices l_i it holds $0 \le l_i \le q_i$); $A(s,n) = \alpha_s \sqrt{(n-s)!s!}$. Although the multiport is fully symmetric (i.e., a single photon entering any one of the inputs appears with equal probability at any output) the concrete realization of the multiport allows to distinguish the outputs when interference effects occur due to more then one input fields. This is hidden here in the $\binom{+}{-}$ factor, which corresponds to the negative elements of the transformation matrix.

In case of one photon resolution linear detectors (like photomultipliers), all these probabilities can be measured and the inversion of the transformation is possible (applying an appropriate cut-off), yielding the complex c_n [1]. The absolute value of the coefficients $|c_n|$ can be determined without a reference beam. In order to gain phase information a reference beam in a state with known $|\alpha_n|$ coefficients, e.g. a weak laser beam, should be applied.

For avalanche photodiode type detectors the connection between the measured probabilities and the unknown coefficients is more complicated, since all probabilities with exactly n nonzero q_i coefficients should be summed up and an additional index p has

to be introduced in order to distinguish detectors corresponding to the negative sign in the transformation.

The weight of the terms in the expansion related to higher photon numbers decreases like $\frac{1}{N}$, and collecting them one arrives at a series in the inverse power of the size of the device N

$$w_{m,p} = \binom{N/2}{p}\binom{N/2}{m-p}\frac{1}{N^m} \times \left[S_{m,p} + \frac{1}{N}\left(\frac{1}{2}(m-p)S_{m+1,p} + \frac{1}{2}pS'_{m+1,p-1}\right) + \mathcal{O}\left(\frac{1}{N^2}\right)\right]. \tag{3}$$

The unknown coefficients are contained in the S and S' "interference" terms

$$S_{n,p} = \left|\sum_{s=0}^{n} c_{n-s}A(s,n)C(s,p,n)\right|^2, \tag{4}$$

$$S'_{m,p} = \left|\sum_{s=0}^{m} c_{m-s}A(s,m)C(s,p,m) - 4\sum_{s=1}^{m} c_{m-s}A(s,m)C(s-1,p,m-2)\right|^2. \tag{5}$$

Here we introduced the following notation for the combinatorial factors

$$C(s,p,n) \equiv \sum_{i=Max\{0,p-(n-s)\}}^{Min\{s,p\}} (-1)^i \binom{p}{i}\binom{n-p}{s-i}. \tag{6}$$

The unknown c_n can then be determined from the measured probabilities $w_{m,p}$ recursively, starting from $m = 1$. As already $w_{1,p}$ contains two unknown phases, more different reference beams needed to find them uniquely.

3. Two mode fields

The generalization for two spatially separable modes can be straightforwardly carried out. The unknown two-mode state

$$|\Psi\rangle = \sum_{n,n'=0}^{\infty} c_{nn'}|n,n'\rangle \tag{7}$$

fed into two distinct multiports can be treated similarly to the one mode case. Reference beams applied separately to the two modes allow also here the reconstruction of the full state. Especially the matrix of $|c_{nn'}|^2$ may be found without reference beams with avalanche photodiode type detectors (for more details see [6]).

An interesting feature of the two mode case is that interference between the two modes may be possible. The same arrangement as in the previous section and instead of the reference using the second mode leads to the measured probability

$$w_{n,p} = \binom{N/2}{p}\binom{N/2}{n-p}\frac{1}{N^n}S_{n,p}. \tag{8}$$

where the "interference factor" $S_{n,p}$

$$S_{n,p} = \left| \sum_{s=0}^{n} c_{n-s,s} \sqrt{(n-s)!s!} C(s,p,n) \right|^2 \tag{9}$$

contains now the $c_{nn'}$ two-mode coefficients, i.e., information about the phase difference $\phi_{n-s,s} - \phi_{n-s',s'}$. The probability $w_{n,p}$ is calculated here only up to the zeroth order, corresponding to selecting the one photon events, but an expansion similar to the previous section can be simply carried out.

4. Discussion

Employing multiports in measurements is promising especially for weak fields. The size of the multiports is practically limited due to losses introduced by the large number of optical elements.

High efficiency indirect photodetection with multiports could actually substitute direct photoncounting. It may find application in other state reconstruction schemes, as it is proposed by Opatrný *et al.* [7] who combine unbalanced homodyne detection with a multiport photon number measurement for full state reconstruction. Also state preparation is possible with them, schemes for Fock state [8] and Schrödinger cat like state [9] preparation are proposed.

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