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DYNAMICAL LOCALIZATION OF THE VIBRATIONAL QUANTUM NUMBER IN A PAUL TRAP[1]

M. El Ghafar[2], K. Riedel[3], Päivi Törmä[4], V. Savichev[5], E. Mayr, A. Zeiler, W. P. Schleich[6]
Abteilung für Quantenphysik, Universität Ulm, 89069 Ulm, Germany

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We have shown [1] that dynamical localization happens in the quantum motion of an ion in a Paul trap interacting with a standing wave laser field. The variable which shows dynamical localization is the vibrational quantum number of a reference oscillator, which leads to localization in both momentum and position. Here we discuss shortly the effect of decoherence.

1. Introduction

Dynamical localization is a peculiarity of quantum systems with classically chaotic counterparts. It is an analogue of Anderson localization [2] of electronic waves in one-dimensional disordered solids. Whereas in Anderson localization the disorder leads to randomness in the classical dynamics, in systems showing dynamical localization the classical dynamics is only quasirandom, i.e. deterministic but chaotic. In both cases, according to classical physics, diffusion in the dynamical variables is expected, while quantum theory predicts localization. Dynamical localization has been experimentally verified in two types of systems: as suppression of ionization of Rydberg atoms in microwave fields [3], and localization in the momentum distribution of an atom moving in a phase modulated standing wave [4]. We have shown that dynamical localization appears also in the quantum motion of an ion in a Paul trap interacting with a standing wave laser field [1].

For the analysis of dynamical localization the definition of suitable action-angle variables is essential. In the system considered here the Floquet theory of the time-dependent harmonic oscillator [5] simplifies the dynamics so that we are able to identify

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[2]E-mail address: elg@physik.uni-ulm.de
[3]E-mail address: rie@physik.uni-ulm.de
[4]E-mail address: toe@physik.uni-ulm.de
[5]E-mail address: sav@physik.uni-ulm.de
[6]E-mail address: schleich@physik.uni-ulm.de

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the vibrational quantum number of a reference oscillator as the variable which shows localization. In Section 2 we discuss this briefly, following the argumentation presented in [1].

Dynamical localization is a quantum coherence effect and extremely sensitive to dissipation. For this reason we have chosen the standing wave laser field to be far detuned from the atomic transition. In section 3 we discuss the role of decoherence.

2. Dynamical localization

The phenomenon of dynamical localization in the Paul trap emerges because (i) the Paul trap is an explicitly time dependent device, (ii) a standing laser wave provides a spatially periodic light potential for the center-of-mass motion, and (iii) the temperature of the ion is so low that its motion has to be treated quantum mechanically.

We consider the standard Paul trap set-up realized experimentally in many labs (e.g. [7]): a standing laser field of frequency ω_L and wave vector k aligned along the x-axis couples the internal states of a single two-level ion of mass m to the center-of-mass motion. The resulting dynamics of the ion follows from the time-dependent Schrödinger equation with the Hamiltonian

$$\hat{\tilde{H}} = \frac{\hat{\tilde{p}}^2}{2m} + \frac{1}{2}\frac{m\omega^2}{4}[a + 2q\cos(\omega\tilde{t})]\hat{\tilde{x}}^2 + \frac{1}{2}\hbar\omega_a\hat{\sigma}_z + \hbar\Omega_0\hat{\sigma}_x\cos(k\hat{\tilde{x}})\cos(\omega_L\tilde{t}). \quad (1)$$

Here the parameters a and q denote [8] the DC and AC voltages applied to the trap. The frequency of the AC field is ω. Here ω_a is the atomic transition frequency and Ω_0 is the Rabi-frequency.

In order to avoid decoherence we assume the detuning $\Delta = \omega_L - \omega_a$ to be large. After making the rotating wave approximation and introducing the dimensionless position $x \equiv 2k\tilde{x}$, time $t \equiv \omega\tilde{t}/2$ and momentum $p \equiv 4k\tilde{p}/m\omega$ we get the dimensionless Hamiltonian

$$\hat{H} \equiv \frac{16k^2}{m\omega^2}\hat{\tilde{H}} = \frac{1}{2}\hat{p}^2 + \frac{1}{2}(a + 2q\cos 2t)\hat{x}^2 + \Omega\cos(\hat{x}) \quad (2)$$

with the effective coupling constant $\Omega = 2\hbar k^2\Omega_0^2/m\omega^2\Delta$ [9]. The corresponding effective Planck constant is $k = 8k^2\hbar/m\omega$.

Applying the Floquet theory of the time-dependent harmonic oscillator [5], we find that the Hamiltonian can be written in terms of the action variable I and the angle variable θ of a time-independent reference harmonic oscillator of frequency ω_r: $H_I = \omega_r I + \Omega V(I,\theta,t)$. The new potential is $V(I,\theta,t) = |\epsilon(t)|^2\cos[|\epsilon(t)|\sqrt{2I/\omega_r}\sin\theta]$, where $\epsilon(t)$ is the solution of the classical Mathieu equation [5]. The reference oscillator frequency $\omega_r = \omega_s + \sum_{-\infty}^{\infty} nc_n$, where ω_s is the secular frequency and c_n are the Fourier coefficients of $\epsilon(t)$.

The Hamiltonian H_I suggests classical diffusion in I [10], implying that diffusion in position and momentum has a square-root, not linear, dependence on time – quantum mechanically we expect localization in I. We have confirmed this by wave packet simulations, and can thus conclude that the variable which shows dynamical localization

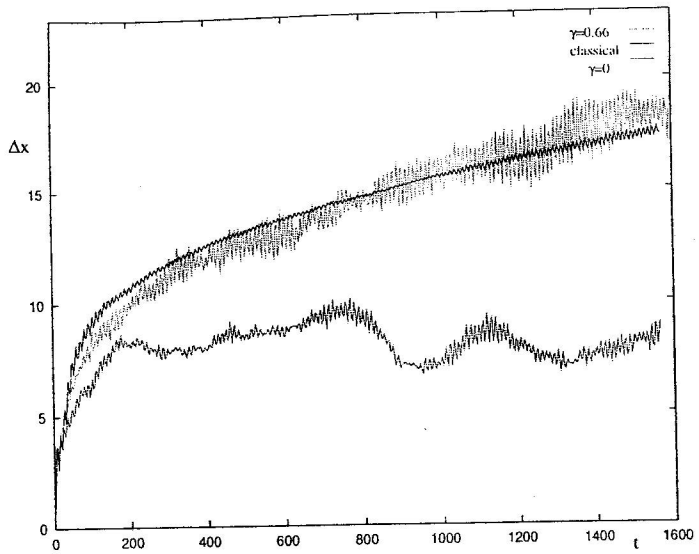

Fig. 1. Classical and quantum dynamics of a single ion moving under the influence of a standing laser field and a time dependent harmonic potential. The black lines show the time dependence of the width of the classical (upper curve) and quantum mechanical (lower curve) position distribution. In the on-resonance ($\Delta = 0$) case spontaneous emission ($\gamma/\omega = 0.66$) destroys the quantum coherence and the result follows (grey line) the classical result. The parameters used are $k = 0.3$, $a = 0$, $q = 0.4$ and $\Omega = 0.65$, and in the Monte Carlo simulation with spontaneous emission an average over 48 runs was made.

is the vibrational quantum number of the reference oscillator. We have shown that the relation between the localization length and the classical diffusion constant is of the same form as for the kicked rotator [6], which is the standard model system of dynamical localization.

3. The effect of decoherence

The observation of dynamical localization is possible with the present ion trap systems. We consider a dipole transition from the ground state of $^9Be^+$ with the decay rate $\gamma/2\pi \sim 20$MHz [7]. In order to achieve the relatively large value of $k \sim 0.3$ used in our simulations we need a small trap frequency such as $\omega/2\pi \sim 10$MHz. To be consistent with the assumption of far detuning, the term $\Omega_0/\Delta \equiv \epsilon$ in the dimensionless coupling $\Omega = (k\Delta)/(4\omega)\epsilon^2$ has to be small. For $\epsilon = 0.1$ and the detuning $\Delta/2\pi \sim 10$GHz we obtain $\Omega = 0.65$, as used in the simulations presented here.

In Fig. 1 we show results of the quantum and classical calculations for a wave packet and the corresponding classical ensemble having initially zero mean position and momentum. The classical position spread Δx grows due to the chaotic dynamics. The quantum spread follows this behaviour in the beginning, but then stabilizes. To show the effect of decoherence we present in Fig. 1 also the results of a Monte Carlo simulation

for the on-resonance case $\Delta = 0$ of Eq. (1); it is formally the same as the Hamiltonian for the far-detuned case, Eq. (2). In this case spontaneous emission destroys, via recoil kicks, the quantum coherence of the motion, and the pure state describing it becomes a mixed state; this reproduces the classical result, as expected. The destructive effect of decoherence can be avoided by large enough detuning.

4. Conclusions

We discussed dynamical localization of the motion of an ion stored in a Paul trap and interacting with a standing wave laser field [1]. We showed, as a check of consistency, that when the quantum coherence of the motional wave packet is destroyed by spontaneous emission, we obtain the same result as for a classical ensemble. The coherence necessary for the observation of dynamical localization can be preserved by detuning the laser field far off from the atomic transition.

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