acta physica slovaca vol. 47 No. 3/4, 277 – 280 June 1997

REALIZATION OF NONLINEAR OSCILLATORS WITH A TRAPPED ION[1]

G. Drobný
Institute of Physics, Slovak Academy of Sciences, Dúbravská cesta 9, 842 28 Bratislava, Slovakia; e-mail: drobny@savba.sk

B. Hladký
Department of Optics, Comenius University, Mlynská dolina, 84215 Bratislava, Slovakia; e-mail: hladky@fmph.uniba.sk

Received 25 April 1997, accepted 12 May 1997

We consider a trapped ion with a quantized center–of–mass motion in 2D trap potential. With external laser fields the effective (non)linear coupling of two orthogonal vibrational modes can be established via stimulated Raman transition. Nonclassical vibrational states such as squeezed states or two–mode entangled states (Schrödinger cat–like states) can be generated. When the vibrational modes are entangled with internal energy levels the Greenberger–Horne–Zeilinger (GHZ) states can be prepared.

The Jaynes–Cummings model (JCM) [1] which describes an interaction of one two–level atom with a quantized cavity field belongs to the fundamentals of quantum optics. The successful experimental realization of the JCM is associated with Rydberg atoms in a high-Q microwave cavity. The quantum effects such as the collapse–revival behavior and preparation of Schrödinger cat–like states have been demonstrated [2].

Recent experimental developments in laser cooling and ion trapping [3] have enabled to realize a formal analogue of the JCM in which the cavity field mode is replaced by a quantized vibrational mode of the center–of–mass motion of a trapped ion [4, 5, 6]. There are two virtues in experiments with trapped ions. Firstly, dissipative effects which are inevitable from cavity damping in the microwave or optical domain can be significantly suppressed for the ion motion. Secondly, the motion of the trapped ion can be well controlled by proper sequences of laser pulses tuned either to the atomic electronic transition or to resolved vibrational sidebands. These aspects make trapped ions candidates for quantum computing [7]. It is worth noticing that the most prominent nonclassical vibrational states of a trapped ion (coherent, squeezed, Fock and Schrödinger cat–like states) have been successfully prepared in laboratory [5].

[1]Presented at the Fifth Central-European Workshop on Quantum Optics, Prague, Czech Republic, April 25 - 28, 1997

0323-0465/97

The formal analogy of the vibrational mode of a trapped ion with the cavity field mode can be extended to multi–mode systems. In particular, schemes for preparing correlated two–mode Schrödinger cat states, the Bell and SU(2) states of vibrational motion in a two–dimensional trap have been proposed [8].

In this paper we consider a trapped ion with a quantized center–of–mass motion in 2D trap potential. Irradiating the ion with two external laser fields, which are tuned to well-resolved vibrational sidebands to stimulate Raman transition between internal energy levels, the effective linear or nonlinear coupling of two orthogonal vibrational modes can be established. It is shown how to realize with a trapped ion also analogues of other fundamental elements of quantum optics - nonlinear optical processes (multiwave mixings) such as the degenerate two–photon down conversion.

Consider a quantized center–of–mass motion of an ion which is confined in a 2D harmonic potential characterized by the trap frequencies ν_x and ν_y in two orthogonal directions x and y. The ion is irradiated by two external laser fields with frequencies ω_x, ω_y along the x and y axes. The laser fields stimulate transitions between three internal energy levels a, b, c in Λ configuration (with the upper c level). In dipole approximation the interaction Hamiltonian for the system under consideration can be written in the form:

$$\begin{aligned}\hat{H}_{int} &= \frac{1}{2}\hbar\Omega_x\left[e^{-i\omega_x t}\hat{D}_x(i\epsilon_x)|c\rangle\langle a| + e^{i\omega_x t}\hat{D}_x(-i\epsilon_x)|a\rangle\langle c|\right] \\ &+ \frac{1}{2}\hbar\Omega_y\left[e^{-i\omega_y t}\hat{D}_y(i\epsilon_y)|c\rangle\langle b| + e^{i\omega_y t}\hat{D}_y(-i\epsilon_y)|b\rangle\langle c|\right]. \qquad (1)\end{aligned}$$

The absorption (emission) of energy from (to) external laser field in q direction ($q = x, y$) is accompanied by momentum exchange between the field and the ion which is described in (1) by the displacement operator $\hat{D}_q(i\epsilon_q) = \exp[i\epsilon_q(\hat{a}_q^\dagger + \hat{a}_q)]$; $\hat{a}_q^\dagger$, $\hat{a}_q$ are the creation and annihilation operators of vibrational quanta in a given direction. The parameter ϵ_q is defined as $\epsilon_q^2 = E_q^{(r)}/(\hbar\nu_q)$ where $E_q^{(r)}$ is the classical recoil energy of the ion; Ω_q is a Rabi frequency of the laser–driven transition and $|i\rangle\langle j|$ is an atomic transition operator $(i, j = a, b, c)$.

In the Lamb-Dickě limit $\epsilon_x \approx \epsilon_y \ll 1$ only resonant processes can be taken into account. The upper c level can be adiabatically eliminated under resonance conditions on the frequencies of the laser fields:

$$E_a/\hbar + \omega_x + m\nu_x = E_b/\hbar + \omega_y + n\nu_y \qquad (2)$$

(i.e., m and n trap quanta are involved in the stimulated Raman transition between internal energy levels a and b) and off–resonance conditions for the transitions from levels a and b to the upper level c (with detuning $\Delta \gg m\nu_x, n\nu_y$):

$$E_c - E_a = \hbar\omega_x + m\hbar\nu_x + \hbar\Delta, \qquad E_c - E_b = \hbar\omega_y + n\hbar\nu_y + \hbar\Delta. \qquad (3)$$

The effective interaction Hamiltonian in the rotating–wave-approximation and the interaction picture reads:

$$\hat{H}_{eff} = -\left[\frac{\hbar(-1)^n(i\epsilon)^{m+n}\Omega_x\Omega_y}{4m!n!\Delta}\hat{a}_x^m\hat{a}_y^{\dagger n}|b\rangle\langle a| + h.c.\right]. \qquad (4)$$

For example, for $m = n = 1$ one gets $\hat{H}_{eff} = \hbar\lambda(\hat{a}_x\hat{a}_y^\dagger|b\rangle\langle a| + \hat{a}_x^\dagger\hat{a}_y|a\rangle\langle b|)$. This exactly solvable interaction Hamiltonian is known in the cavity QED within a context of two-photon transitions of a two-level atom interacting with a bichromatic field in a cavity as well for an ion in 1D trap with a resonator [9]. The mutual entanglement (correlation) of two vibrational modes with ionic internal degrees of freedom is established. When the vibrational modes x, y are initially prepared in coherent states $|\beta\rangle_x, |\gamma\rangle_y$, the dynamics of the vibrational modes x, y in phase space is characterized by revival times $t_R^{(x)} \approx \frac{2\pi\beta}{\lambda\gamma}$, $t_R^{(y)} \approx \frac{2\pi\gamma}{\lambda\beta}$ (i.e., the initial quasidistributions are partially restored). If $\beta \approx \gamma$ the revival times are independent of the coherent amplitudes and a collapse–revival structure of the atomic inversion can be observed. Similarly to the JCM, the quasidistributions bifurcate into two components. Nevertheless, the vibrational modes are entangled with the internal energy levels. To disentangle them, conditional measurements on the internal energy levels can be performed [10]. For example, the internal state of the ion can be determined by driving transition from the level b to an auxiliary level d and observing the fluorescence signal. No signal (no interaction with probing field) means the undisturbed ion in the level a. One can thus prepare *pure* two–mode correlated states of the vibrational system. At half of the revival time $t_R^{(x)} \approx t_R^{(y)}$ one finds a structure close to a pure two–mode Schrödinger cat–like state which consists of four components. Each particular vibrational mode is in a two–component mixture (the components are mutually rotated by π in the phase space).

It is worth noticing that GHZ states of importance in tests of quantum mechanics can be generated in a rather simple way when the vibrational modes are still entangled with internal energy levels. Preparing the initial state with one trap quantum in x mode, i.e., $|\psi(0)\rangle = |1\rangle_x|0\rangle_y|a\rangle$, one gets at quarter of the Rabi cycle $\lambda t = \pi/4$ the GHZ state $|\psi\rangle = \frac{1}{\sqrt{2}}(|1\rangle_x|0\rangle_y|a\rangle - i|0\rangle_x|1\rangle_y|b\rangle)$. If instead of (2) the resonance condition $E_a/\hbar + \omega_x + \nu_x = E_b/\hbar + \omega_y - \nu_y$ is assumed, one can start with both vibrational modes in the vacuum and the ion in the level b to produce a GHZ state.

In the degenerate case, when the level b is identical with the level a all the atomic operators can be eliminated (the c level is far of resonance) and the effective interaction Hamiltonian takes the form

$$\hat{H}_{eff}^{(xy)} = -\left[\frac{\hbar(-1)^n(i\epsilon)^{m+n}\Omega_x\Omega_y}{4m!n!\Delta}\hat{a}_x^m\hat{a}_y^{\dagger n} + h.c.\right]. \qquad (5)$$

For $m = n = 1$ the linear coupler is obtained. Nontrivial case - the nonlinear coupling of the bosonic modes - is realized when one of the lasers is tuned to the second vibrational sideband. For example $m = 1$, $n = 2$ leads to two–phonon down conversion governed by the interaction Hamiltonian $\hat{H}_{eff}^{(xy)} = \hbar\lambda_2(\hat{a}_x\hat{a}_y^{\dagger 2} + \hat{a}_x^\dagger\hat{a}_y^2)$. One can thus simulate the degenerate three–wave mixing [11] even on the long time–scale. It is well–known that starting with coherently excited x and empty y mode the squeezed vacuum in the latter is produced. Besides of that at the time of the maximal depletion of the x mode the oscillations in photon number distributions of both modes can be observed. In the y mode a two–component mixture is established with a remarkable interference pattern between the components (peaks) of the Wigner function [12].

quantity composed of the system variables can then be derived by a statistical averaging. The essential virtue of this method is that the computational effort is independent of the actual dimension of the Hilbert space involved in the process.

To illustrate our method we plotted the transient dynamics of the microsphere in Fig. 1a. The plot in Fig. 1b is the solution of the Maxwell–Bloch semiclassical equations that we get simply by eliminating the Langevin noise terms in the Eqs. (1)–(4), i.e., by taking the mean of both the left and right–hand size. Although the stationary mean intensities are equal, much more accentuated oscillations of relaxation are predicted by the semiclassical model. Consequently, the quantum fluctuations considerably modify the means themselves.

A closer look of the fluctuations associated with the different system variables permits one to set up a hierarchy among them. Typically, in microlasers the relative fluctuations of the population inversion are much smaller than that of the photon number. When this condition is met, an approximative analytic solution can be found. The population inversion is replaced by its mean value in Eq. (2). Thus the steady–state photon number and atomic polarization as a function of the inversion can be deduced from Eqs. (1) and (2). These solutions are finally used in Eqs. (3) and (4) to determine the population inversion on a self–consistent way. This approach allows one to provide a complete statistical description in the stationary regime for any value of the pumping rate.

The most evident manifestation of spontaneous emission in lasers is the presence of a small radiation field in the mode even below threshold. When the spontaneous emission is forced to occur into the laser mode "thresholdless" behaviour can occur [2]. In Fig. 2 a typical mean intensity versus pumping rate plot shows this phenomenon in the case of the microsphere. This figure serves to check the approximative analytic method. The analytic curve fits well on the numerically calculated "exact" values.

In conclusion, using a rigorous quantum model, we calculated some characteristics of the microsphere laser. The predicted features associated with the quantum noise are hoped to be observable in experiments under progress in our laboratory.

Acknowledgements One of us (P. D.) acknowledges the support by the National Scientific Research Fund of Hungary (OTKA) under Contracts No. F017380, T017386, T023777.

References

[1] D. Meschede, H. Walther, G. Müller: *Phys. Rev. Lett.* **54** (1985) 551; M. Brune, J.M. Raimond, P. Goy, L. Davidovich, S. Haroche: *Phys. Rev. Lett.* **59** (1987) 1899

[2] Y. Yamamoto, R.E. Slusher: *Phys. Today* **46** (1993) 66 and references therein

[3] K. An, J.J. Childs, R.R. Dasari, M. Feld: *Phys. Rev. Lett.* **73** (1994) 3375

[4] V. Sandoghdar, F. Treussart, J. Hare, V. Lefèvre–Seguin, J.M. Raimond, S. Haroche: *Phys. Rev. A* **54** (1996) R1777

[5] C. Benkert, M.O. Scully, J. Bergou, L. Davidovich, M. Hillery, M. Orszag: *Phys. Rev. A* **41** (1990) 2756; M.I. Kolobov, L. Davidovich, E. Giacobino, C. Fabre: *Phys. Rev. A* **47** (1993) 1431; L. Davidovich: *Rev. Mod. Phys.* **68** (1996) 127