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QUANTUM NOISE IN MICROLASERS[1]

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Numerical and approximative analytical methods are presented to solve the Heisenberg–Langevin equations pertaining to the microsphere laser system.

Micromasers and microlasers are attractive systems to study fundamental physics in the field of quantum optics. While the micromaser has been realized more than 10 years ago [1], much experimental effort is being currently devoted to operate its optical counterpart. Different systems have been conceived to observe genuine microlaser effects. For example, "thresholdless laser" was reported in multilayer semiconductor microcavities and in microdisks [2]. Single–atom laser action was demonstrated by using a miniature, high–Q Fabry–Pérot cavity [3].

A very promising candidate consists in using the whispering gallery modes of a high–Q, small–volume, fused–silica microsphere. Light in such modes is trapped near the surface by repeated internal reflections and travels in a circle around the sphere. Values of Q up to 10^9 can be achieved resulting in a cavity damping rate $\kappa \approx 1$ MHz for a mode having a very small volume about $300\mu\mathrm{m}^3$. The atoms, coupled to the mode, can be included in the material itself or be placed in the evanescent field of the resonator. Actual experiments [4] use neodymium–doped silica microsphere where Nd^{3+} ions provide the gain medium. Owing to the small mode volume, a considerably strong coupling, $g = 0.1 - 1$ MHz in terms of single–photon Rabi frequency, can be achieved.

The level structure of these rare–earth ions is rather complicated. However, for the study of the laser operation, the atomic scheme can be restricted to four relevant levels.

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The ions are pumped from the fundamental state into the uppermost level from which the system decays promptly to the upper level a of the laser transition. The lifetime of a is very long ($\Gamma_a^{-1} = 1$ ms) compared to that of the lower laser level b ($\Gamma_b^{-1} = 0.1\mu s$) which is emptied almost instantaneously by inelastic collisions with phonons. At room temperature the homogeneous linewidth Γ_{ab} of the laser transition is very broad, 1500 GHz. However, at 2 K the linewidth is expected to be reduced to a few MHz, thus we can expect that the dynamical effects of the atomic polarization may play a significant role.

The main goal of this paper is to investigate the effects of the spontaneous emission and of the other quantum noise sources on the performances of the microsphere laser. Novel behaviour is expected on the basis of semiclassical scaling parameters. The threshold pumping rate is $R_{th} = \kappa\Gamma_a\Gamma_{ab}/2g^2 \approx 10^4$ excited ion per second. Since the lifetime of the ions is in the order of millisecond, only a dozen of ions is sufficient to maintain the laser operation. The other scaling parameter is also unusual: the saturation intensity $I_{sat} = \Gamma_a\Gamma_{ab}\Gamma_b/2g^2(\Gamma_a + \Gamma_b) \approx 0.01$ photon is extremely low. One can then deduce that the contribution of the quantum fluctuations can be comparable to the stationary mean values themselves. We need thus a fully quantum–mechanical treatment to calculate the characteristics of the system. Neither the usual micromaser treatment nor the single–atom laser approach is appropriate for our purposes since the interaction between "many" atoms and a weak, few–photon field is lifetime–limited.

The Heisenberg–Langevin equations will be applied to describe a single–mode laser [5] since it relies exclusively on first principles. The following assumptions are made: (i) The cavity mode is on resonance with the $|a\rangle \leftrightarrow |b\rangle$ transition. (ii) We neglect the inhomogeneous broadening due to the matrix effect. (iii) The atoms are uniformly coupled to the field. (iv) The microsphere is heavily doped so that the number of accessible, non–excited ions is large enough to describe the Poissonian pumping process simply by an appearance rate R in the state a. Then the Heisenberg–Langevin dynamical equations for the operator variables can be written in the form [5]

$$\dot{a}(t) = gM(t) - \frac{\kappa}{2}a(t) + F_\kappa(t), \quad (1)$$

$$\dot{M}(t) = g\Big[N_a(t) - N_b(t)\Big]a(t) - \Gamma_{ab}M(t) + F_M(t), \quad (2)$$

$$\dot{N}_a(t) = R - g\Big[a^\dagger(t)M(t) + M^\dagger(t)a(t)\Big] - \Gamma_a N_a(t) + F_a(t), \quad (3)$$

$$\dot{N}_b(t) = g\Big[a^\dagger(t)M(t) + M^\dagger(t)a(t)\Big] - \Gamma_b N_b(t) + F_b(t), \quad (4)$$

where a, $a^\dagger$ are the boson operators of the field mode, M is the collective atomic polarization, N_a and N_b are the populations in levels a and b. The noise features are incorporated in the F reservoir operators. They obey the usual Langevin correlations:

$$\langle F_i(t)\rangle = 0\,, \qquad \langle F_i(t)F_j(t')\rangle = D_{ij}\delta(t-t') \quad (5)$$

where the non–vanishing diffusion coefficients are $D_{\kappa\kappa^\dagger} = \kappa$, $D_{aa} = \Gamma_a\langle N_a(t)\rangle + R$, $D_{bb} = \Gamma_b\langle N_b(t)\rangle$, $D_{M^\dagger M} = (2\Gamma_{ab} - \Gamma_a)\langle N_a(t)\rangle + R$, $D_{MM^\dagger} = (2\Gamma_{ab} - \Gamma_b)\langle N_b(t)\rangle$, $D_{bM} = \Gamma_b\langle M(t)\rangle$, $D_{Ma} = \Gamma_a\langle M(t)\rangle$.

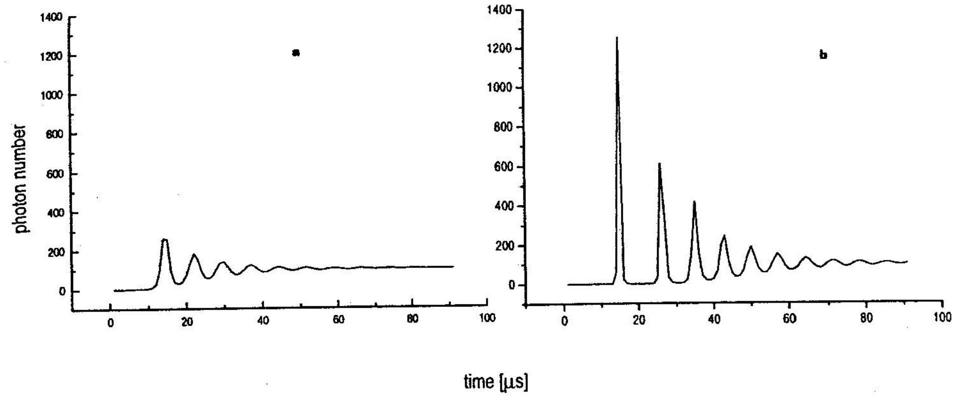

Fig. 1. Transient dynamics of the mean intensity is represented in accordance with (a) quantum (b) semiclassical theory. Parameters are: $g = 0.1$, $\kappa = 10$, $\Gamma_a = 0.001$, $\Gamma_b = 10$, $\Gamma_a b = 20$, all in μs^{-1}, pumping rate is $R/R_{th} = 100$.

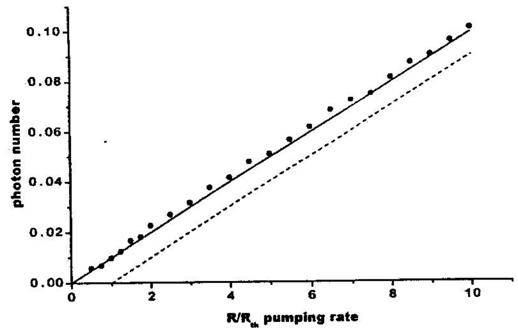

Fig. 2. The mean intensity is plotted as a function of the pumping rate. The pumping rate is normalized to the semiclassical threshold. The points correspond to the numerical results originated from the fully quantum model, while the approximative analytic model provides the straight line. The dashed line shows the semiclassical behaviour predicting a threshold. Parameters are: $g = 1$, $\kappa = 1$, $\Gamma_a = 0.001$, $\Gamma_b = 10$, $\Gamma_{ab} = 20$, all in μs^{-1}.

Eqs. (1)–(4) can be solved numerically on the following manner. First one replaces the operator variables by c–numbers using a chosen order. The Langevin noise operators are represented by c–number random variables acting as a random force. Their correlations must be calculated on taking into account the chosen ordering. The set of differential equations containing c–number variables are integrated by using a double Monte–Carlo method. A statistical set of random initial conditions has to be generated that simulates the phase–space distribution of the initial state. After this, the equations are integrated from each random initial condition. The evolution is, like a Brownian motion, stochastical due to the random forces. The ensemble average of any physical quantity composed of the system variables can then be derived by a statistical averaging. The essential virtue of this method is that the computational effort is independent of the actual dimension of the Hilbert space involved in the process.

To illustrate our method we plotted the transient dynamics of the microsphere in Fig. 1a. The plot in Fig. 1b is the solution of the Maxwell–Bloch semiclassical equations that we get simply by eliminating the Langevin noise terms in the Eqs. (1)–(4), i.e., by taking the mean of both the left and right–hand size. Although the stationary mean intensities are equal, much more accentuated oscillations of relaxation are predicted by the semiclassical model. Consequently, the quantum fluctuations considerably modify the means themselves.

A closer look of the fluctuations associated with the different system variables permits one to set up a hierarchy among them. Typically, in microlasers the relative fluctuations of the population inversion are much smaller than that of the photon number. When this condition is met, an approximative analytic solution can be found. The population inversion is replaced by its mean value in Eq. (2). Thus the steady–state photon number and atomic polarization as a function of the inversion can be deduced from Eqs. (1) and (2). These solutions are finally used in Eqs. (3) and (4) to determine the population inversion on a self–consistent way. This approach allows one to provide a complete statistical description in the stationary regime for any value of the pumping rate.

The most evident manifestation of spontaneous emission in lasers is the presence of a small radiation field in the mode even below threshold. When the spontaneous emission is forced to occur into the laser mode "thresholdless" behaviour can occur [2]. In Fig. 2 a typical mean intensity versus pumping rate plot shows this phenomenon in the case of the microsphere. This figure serves to check the approximative analytic method. The analytic curve fits well on the numerically calculated "exact" values.

In conclusion, using a rigorous quantum model, we calculated some characteristics of the microsphere laser. The predicted features associated with the quantum noise are hoped to be observable in experiments under progress in our laboratory.

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