acta physica slovaca vol. 47 No. 3/4, 267 – 272 June 1997

CONDITIONAL SCHRÖDINGER CAT-LIKE OUTPUT STATES OF A BEAM SPLITTER[1]

M. Dakna[2], T. Opatrný[3], L. Knöll, D.-G. Welsch
Friedrich-Schiller-Universität Jena, Theoretisch-Physikalisches Institut, Max-Wien-Platz 1, D-07743 Jena, Germany

Received 25 April 1997, accepted 12 May 1997

We show that when a squeezed vacuum and an ordinary vacuum are combined by a beam splitter and in one of the output channels the number of photons is measured, then conditional quantum states in the other output channel can be obtained which reveal all the properties of superpositions of macroscopically distinguishable states. The component states are very close to squeezed coherent states and approach coherent states for sufficiently large numbers of detected photons. We also consider the problem of producing the superposition states under the conditions of realistic photocounting, assuming multichannel detection with highly efficient avalanche photodiodes. We show that for properly chosen parameters quantum interferences can still be found even for a realistic arrangement of detectors.

1. Introduction

A fundamental difference between classical and quantum mechanics is that the latter allows – according to the superposition principle – for the phenomenon of interference of macroscopically distinguishable states as it is the case for Schrödinger cat-like states [1]. The experimental test has been a challenge since the early days of quantum mechanics. Recent progress in quantum optics has provided new possibilities for observing such superposition states of the translational motion of a trapped ion [2]. A number of proposals have been made to prepare a single-mode radiation field in a Schrödinger cat-like state (see, e.g., [3] and references therein). A promising way may be state reduction via conditional quantum measurements. According to the basic-theoretical concepts of the quantum-mechanical measurement process we know that when a quantity of one part of a correlated two-part system is measured, then the state of the other subsystem

[1]Presented at the **Fifth Central-European Workshop on Quantum Optics**, Prague, Czech Republic, April 25 - 28, 1997

[2]E-mail address: dakna@tpi.uni-jena.de

[3]Permanent address: Department of Theoretical Physics, Palacký University, Svobody 26, 77146 Olomouc, Czech Republic

collapses to a particular state. Inspired by this fact various ingenious schemes for producing conditional states of the type of Schrödinger cats have been proposed [4] and the experimental feasibilities with current technologies have recently been discussed [5]. In this paper we propose a scheme for generating Schrödinger cat-like states which is based on conditional measurement on a beam splitter with nonclassical input [6]. It is shown that when a squeezed vacuum enters one of the input channels (the second input channel being unused) and the photon number of the mode in one of the output channels is measured, then the conditional state of the mode in the other output channel exhibits all the typical features of a Schrödinger cat-like state.

The paper is organized as follows. In Section 2 we calculate the conditional output states and study their properties. The effect of realistic detection is discussed in Section 3. A summary and some concluding remarks are given in Section 4.

2. Conditional output states

It is well known that the input–output relations of radiation at a lossless beam splitter can be treated within the SU(2) algebra. In the Schrödinger picture, the output-density operator $\hat{\varrho}_{\rm out}$ can be obtained from the input-density operator $\hat{\varrho}_{\rm in}$ as $\hat{\varrho}_{\rm out} = \hat{V}^\dagger \hat{\varrho}_{\rm in} \hat{V}$. Ignoring irrelevant phase shifts, the operator $\hat{V}$ can be given by $\hat{V} = e^{-2i\theta \hat{L}_2}$, with $\hat{L}_2 = \frac{1}{2i}(\hat{a}_1^\dagger \hat{a}_2 - \hat{a}_2^\dagger \hat{a}_1)$ [7]. Let us assume that the input-state density operator is

$$\hat{\varrho}_{\rm in} = \hat{\varrho}_{\rm in1} \otimes |{\rm vac}_2\rangle\langle{\rm vac}_2|. \tag{1}$$

Then the output-state density operator can be given by [8]

$$\hat{\varrho}_{\rm out} = \sum_{n_2=0}^{\infty} \sum_{m_2=0}^{\infty} \frac{(-1)^{m_2+n_2}}{\sqrt{m_2!n_2!}} \left|\frac{R}{T}\right|^{m_2+n_2} \hat{a}_1^{m_2} |T|^{\hat{a}_1^\dagger \hat{a}_1} \hat{\varrho}_{\rm in1} |T|^{\hat{a}_1^\dagger \hat{a}_1} (\hat{a}_1^\dagger)^{n_2} \otimes |m_2\rangle\langle n_2|, \tag{2}$$

where $|T| = \cos\theta$ and $|R| = \sin\theta$, and $|n_k\rangle$ are the eigenstates of the photon-number operators $\hat{a}_k^\dagger \hat{a}_k$ of the two input modes ($k = 1, 2$). From Eq. (2) we see that the output modes are, in general, highly correlated. When the photon number of the mode in the second output channel is measured and m_2 photons are detected, then the mode in the first output channel is prepared in a quantum state whose density operator $\hat{\varrho}_{\rm out1}$ reads

$$\hat{\varrho}_{\rm out1}(m_2) = \frac{\langle m_2|\hat{\varrho}_{\rm out}|m_2\rangle}{{\rm Tr}_1(\langle m_2|\hat{\varrho}_{\rm out}|m_2\rangle)}. \tag{3}$$

The probability of such an event is given by

$$P(m_2) = {\rm Tr}_1(\langle m_2|\hat{\varrho}_{\rm out}|m_2\rangle) = \sum_{n_1=m_2}^{\infty} \binom{n_1}{m_2} \left(1-|T|^2\right)^{m_2} |T|^{2(n_1-m_2)} \langle n_1|\hat{\varrho}_{\rm in1}|n_1\rangle. \tag{4}$$

In particular, if the first input mode is prepared in a squeezed vacuum state, we may write $\hat{\varrho}_{\rm in1} = \hat{S}(\xi)|{\rm vac}_1\rangle\langle{\rm vac}_1|\hat{S}^\dagger(\xi)$, where $\hat{S}(\xi) = \exp\{-\frac{1}{2}[(\xi\hat{a}_1^\dagger)^2 - {\rm h.c.}]\}$. In this case, combining Eqs. (2) and (3) and using Eq. (1), we obtain

$$\hat{\varrho}_{\rm out1}(m_2) = |\Psi_{m_2}\rangle\langle\Psi_{m_2}|, \tag{5}$$

where

$$|\Psi_{m_2}\rangle = |\Psi_{m_2}(\alpha)\rangle = \frac{1}{\sqrt{\mathcal{N}_{m_2}}} \sum_{n_1=0}^{\infty} c_{m_2,n_1}(\alpha)\,|n_1\rangle, \tag{6}$$

$$c_{m_2,n_1}(\alpha) = \frac{(n_1+m_2)!}{\Gamma\left[\frac{1}{2}(n_1+m_2)+1\right]\sqrt{n_1!}} \frac{1}{2}\left[1+(-1)^{n_1+m_2}\right]\left(\tfrac{1}{2}\alpha\right)^{\frac{1}{2}(n_1+m_2)}, \tag{7}$$

with $|\alpha| = |T|^2\kappa$, $\kappa = \tanh|\xi|$, and $\mathcal{N}_{m_2} = \sum_{n_1=0}^{\infty} |c_{m_2,n_1}(\alpha)|^2$. The probability $P(m_2)$ is derived to be

$$P(m_2) = \sqrt{\frac{1-\kappa^2}{1-\alpha^2}} \left[\frac{\alpha^2(1-|T|^2)}{|T|^2(1-\alpha^2)}\right]^{m_2} \sum_{k=0}^{[\frac{1}{2}m_2]} \frac{m_2!}{(m_2-2k)!(k!)^2(2\alpha)^{2k}} \tag{8}$$

($[x]$, integral part of x). From Eqs. (6) and (7) we easily see that when the detected number of photons in one output channel, m_2, is even (odd), then the mode in the other output channel is prepared in a quantum state $|\Psi_{m_2}\rangle$ that contains only contributions from photon-number states with even (odd) numbers of photons. In particular, when the number of detected photons, m_2, is zero, then the output state is again a squeezed vacuum, but with the parameter α in place of κ.

(a) (b)

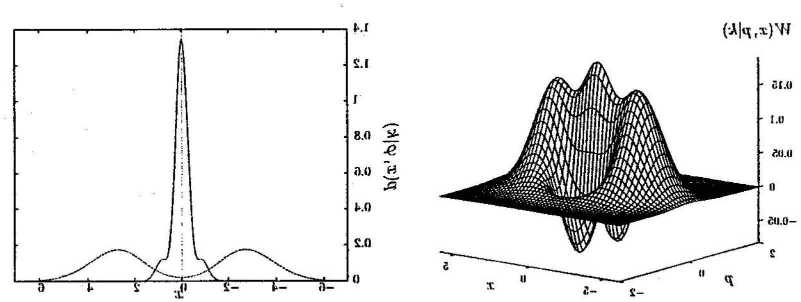

Fig. 1. Quadrature distribution (a) [for the phase parameters $\varphi = 0$ (full line) and $\varphi = \pi/2$ (broken line)] and the Wigner function (b) of the conditional state with realistic photodetection ($N = 10$, $\eta = 0.8$) for $k = 2$ coincident events, and $\kappa = 0.75$, $|T|^2 = 0.8$ ($\alpha = 0.6$).

Let us briefly discuss the properties of the conditional states (for details, see [6]). The photon-number distribution can easily be obtained from Eqs. (6) and (7),

$$P(n|m) = |\langle n|\Psi_m\rangle|^2 = \mathcal{N}_m^{-1}\,|c_{m,n}(\alpha)|^2 \tag{9}$$

(for notational convenience, here and in the following we omit the subscripts 1 and 2 introduced above to distinguish between the two output channels). In particular, the mean photon number is given by

$$\langle \hat{n} \rangle = \alpha \frac{\partial}{\partial \alpha} \log\left[\frac{\mathcal{N}_m}{\alpha^m}\right] \frac{\alpha^2}{1-\alpha^2} + m\frac{1+\alpha^2}{1-\alpha^2} - 2\sum_{k=0}^{[\frac{1}{2}m]} k\, a_{k,m} \left[\sum_{k=0}^{[\frac{1}{2}m]} a_{k,m}\right]^{-1}, \tag{10}$$

where $a_{k,m} = (2\alpha)^{-2k}/[(m-2k)!\,(k!)^2]$. Note that when no photons are detected, $m=0$, then $\langle \hat{n} \rangle$ reduces to the mean number of photons of a squeezed vacuum, $\langle \hat{n} \rangle = \alpha^2/(1-\alpha^2)$. The quadrature-component distribution reads as

$$p(x,\varphi|m) = \frac{|\alpha|^m}{\mathcal{N}_m \sqrt{\pi \Delta^{m+1}}\, 2^m} \exp\left(-\frac{1-\alpha^2}{\Delta} x^2\right) \left|\mathrm{H}_m\left[\sqrt{(\alpha e^{i2\varphi} - \alpha^2)/\Delta}\, x\right]\right|^2, \tag{11}$$

where the abbreviation $\Delta = 1+\alpha^2 - 2\alpha\cos(2\varphi)$ is used. For φ near $\pi/2$ the distribution $p(x,\varphi|m)$ exhibits two separated peaks, whereas for φ close to 0 or π an interference pattern is observed (cf. Fig. 1), which is a typical signature of a Schrödinger cat-like state. Finally, the Wigner function $W(x,p|m)$ takes the form

$$W(x,p|m) = \frac{\exp\left(-\lambda x^2 - \frac{p^2}{\lambda}\right)}{\pi \mathcal{N}_{1m}} \sum_{k=0}^{m} \frac{(-2|\alpha|)^k}{k![(m-k)!]^2} \left|\mathrm{H}_{m-k}\left[i\sqrt{\alpha\lambda}\left(x + i\frac{p}{\lambda}\right)\right]\right|^2, \tag{12}$$

where

$$\lambda = \frac{1-\alpha}{1+\alpha}, \quad \mathcal{N}_{1m} = \sum_{k=0}^{[\frac{1}{2}m]} \frac{(2|\alpha|)^{m-2k}}{(m-2k)!\,(k!)^2}, \tag{13}$$

and the Husimi function $Q(x,p|m)$ is given by

$$Q(x,p|m) = \frac{|\alpha|^m}{\pi \mathcal{N}_m 2^{m+1}} \left|\mathrm{H}_m\left[\tfrac{1}{2} i\sqrt{\alpha}(x+ip)\right]\right|^2 \exp\left\{-\tfrac{1}{2}\left[(1-\alpha)x^2 + (1+\alpha)p^2\right]\right\}. \tag{14}$$

From Eqs. (6) and (7) it is seen that $|\Psi_m\rangle$ can be regarded as a superposition of states $|\Psi_m^{(+)}\rangle$ and $|\Psi_m^{(-)}\rangle$ as $|\Psi_m\rangle = A\left(|\Psi_m^{(+)}\rangle + |\Psi_m^{(-)}\rangle\right)$, where

$$|\Psi_m^{(\pm)}\rangle = \frac{1}{\sqrt{\mathcal{N}_m^{(\pm)}}} \sum_{n=0}^{\infty} c_{m,n}^{(\pm)}(\alpha)|n\rangle, \tag{15}$$

$$c_{m,n}^{(\pm)}(\alpha) = \frac{(n+m)!}{\Gamma[(n+m)/2+1]\sqrt{n!}} \left(\pm\sqrt{\tfrac{1}{2}\alpha}\right)^{n+m}, \tag{16}$$

and $\mathcal{N}_m^{(\pm)} = \sum_{n=0}^{\infty} |c_{m,n}^{(\pm)}(\alpha)|^2$. A more detailed analysis [6] shows that the states $|\Psi_m^{(\pm)}\rangle$ are very close to squeezed coherent states.

3. Realistic photon counting

In order to produce the conditional states $|\Psi_m\rangle$, highly efficient and precise photocounting is needed which requires photodetectors that have been not available at present. The difficulties may be overcome using photon chopping [9]. For such a multichannel $2N$ port detection device the probability of recording k coincident events is given by

$$\tilde{P}_{N,\eta}(k|m) = \sum_l \tilde{P}_N(k|l)\, M_{l,m}(\eta) \tag{17}$$

(η, detection efficiency), where $\tilde{P}_N(k|m) = M_{l,m}(\eta) = 0$ for $k,l > m$ and

$$\tilde{P}_N(k|m) = \frac{1}{N^m}\binom{N}{k} \sum_{l=0}^{k} (-1)^l \binom{k}{l} (k-l)^m \quad \text{for} \quad k \le m, \tag{18}$$

$$M_{l,m}(\eta) = \binom{m}{l} \eta^l (1-\eta)^{m-l} \quad \text{for} \quad l \le m. \tag{19}$$

Since detection of k coincident events can result from various numbers m of photons, the conditional state is in general a statistical mixture. Therefore in place of Eq. (5) we now have

$$\hat{\varrho}_{\mathrm{out}}(k) = \sum_m P_{N,\eta}(m|k)\, |\Psi_m\rangle\langle\Psi_m|, \tag{20}$$

where the conditional probability $P_{N,\eta}(m|k)$ can be obtained using the Bayes rule as

$$P_{N,\eta}(m|k) = \frac{1}{\tilde{P}_{N,\eta}(k)} \tilde{P}_{N,\eta}(k|m) P(m). \tag{21}$$

Here $P(m)$ is the prior probability (8) of m photons being present, and

$$\tilde{P}_{N,\eta}(k) = \sum_m \tilde{P}_{N,\eta}(k|m) P(m) \tag{22}$$

is the prior probability of recording k coincident events.

4. Conclusions

The feasibility of generating Schrödinger cat-like states via conditional measurements on a beam splitter has been studied. We have shown that when a squeezed vacuum and an ordinary vacuum are mixed by a beam splitter and in one of the output channels the number of photons is measured, then the conditional quantum state in the other output channel reveals all the properties of a Schrödinger cat-like state. Analytical results for the photon-number and quadrature-component distributions and the Wigner and Husimi functions of the conditional states have been given. We have expressed the conditional states as a superposition of two quasiclassical component states that are very close to squeezed coherent states and approach coherent states for sufficiently large numbers of detected photons. Assuming multichannel detection using highly efficient avalanche photodiodes, we have also discussed the problem of generating the Schrödinger cat-like states under the conditions of realistic photocounting. As expected, the interference structure is somewhat smeared. Nevertheless, the interference structure can still be found even for a realistic detection scheme (cf. Fig. 1).

Acknowledgement This work was supported by the Deutsche Forschungsgemeinschaft.

References

[1] E. Schrödinger: *Naturwissenschaften* **23** (1935) 807

[2] R.L. de Matos Filho, W. Vogel: *Phys. Rev. Lett.* **76** (1996) 608; C. Monroe, D.M. Meekhof, B.E. King, D.J. Wineland: *Science* **272** (1996) 1131; M.M. Nieto: *Phys. Lett. A* **219** (1996) 180

[3] V. Bužek, P.L. Knight: in *Progress in Optics XXXIV*, ed. by E. Wolf (North-Holland, Amsterdam, 1995)

[4] La Porta, R.E. Slusher, B. Yurke: *Phys. Rev.Lett.* **62** (1989) 28; S. Song, C.M. Caves, B. Yurke: *Phys. Rev. A* **41** (1990) 5261; B. Yurke, W. Schleich, D.F. Walls: *Phys. Rev. A* **42** (1990) 1703

[5] P. Tombesi, D. Vitali: *Phys. Rev. Lett.* **77** (1996) 411

[6] M. Dakna, T. Anhut, T. Opatrný, L. Knöll, D.-G. Welsch: *Phys. Rev. A* **55** (1997) 3184

[7] R.A. Campos, B.E.A. Saleh, M.C. Teich: *Phys. Rev. A* **40** (1989) 1371

[8] M. Ban: *Phys. Rev. A* **49** (1994) 5078

[9] H. Paul, P. Törmä, T. Kiss, I. Jex: *Phys. Rev. Lett.* **77** (1996) 2446