acta physica slovaca vol. 47 No. 3/4, 259 – 262 June 1997

WIGNER FUNCTION DESCRIPTION OF "ATOMIC SCHRÖDINGER CATS"[1]

M. G. Benedict[2], A. Czirják[3], Cs. Benedek[4]
Department of Theoretical Physics, Attila József University,
H-6720 Szeged, Tisza L. krt. 84, Hungary

Received 30 April 1997, accepted 12 May 1997

We consider a class of states in the Dicke model, which can be regarded as atomic analogues of the states usually called Schrödinger cat states in quantum optics. The properties of these states are investigated by the spherical Wigner function which is especially convenient for visualizing quantum interference effects.

1. Introduction

In quantum optics one usually speaks of a Schrödinger cat (SC) state if one has a superposition of two different coherent states of an oscillator field mode. For the properties and the possible generation of these states see [1]. Another type of SC like state can be created [2] in a collection of two-level atoms. The individual atoms can be regarded as the "cells" of the cat, and the cat is definitely alive, if all of its cells are alive, they are in the $|+\rangle$ state, and it is definitely dead, if all the cells are in the ill, $|-\rangle$ state. In the case of N atoms a prototype of a SC like state is then:

$$|\Psi_{SC}\rangle = \frac{1}{\sqrt{2}}(|+,+,...+\rangle + |-,-,...-\rangle), \tag{1}$$

where each of the terms contain N pluses and N minuses. This state is in the totally symmetric subspace of the whole Hilbert space [3, 4], and if such states are manipulated by a light mode with dipole interaction, then the atomic system will remain in this $N+1$ dimensional subspace. The N atom dipole interaction with light is equivalent to the dynamics of a spin of $j = N/2$, and the phase space for this atomic subsystem is the surface of a sphere of radius $\sqrt{j(j+1)}$, $(\hbar = 1)$, which is essentially the Bloch sphere. This phase space and a quasiprobability distribution function on it was first introduced

[1]Presented at the **Fifth Central-European Workshop on Quantum Optics**, Prague, Czech Republic, April 25 - 28, 1997
[2]E-mail address: benedict@physx.u-szeged.hu
[3]E-mail address: czirjak@sol.cc.u-szeged.hu
[4]E-mail address: bence@physx.u-szeged.hu

by Stratonovich [5], and have been considered later by several authors: [6, 7]. We use the original definition [5], and we call it the Wigner distribution:

$$W(\theta,\phi)=\sqrt{\frac{2j+1}{4\pi}}\sum_{K=0}^{2j}\sum_{Q=-K}^{K}\rho_{KQ}Y_{KQ}(\theta,\phi), \tag{2}$$

where $\rho_{KQ}=\mathrm{Tr}(\rho T^{\dagger}_{KQ})$ is the characteristic function (actually a matrix), corresponding to the density operator ρ, and the T_{KQ}-s are the spherical multipole operators.

In the work of Dowling, Agarwal and Schleich [8] graphical representations of the Wigner functions of the number, coherent and squeezed atomic states were presented.

2. The Wigner function of the SC states

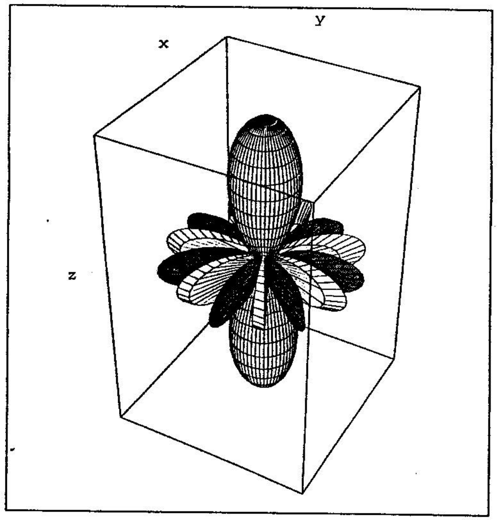

Fig. 1. Wigner function for a SC state, Eq. (3) for the case of $N=7$ atoms. The absolute value of the function is measured on the radius in the direction θ,ϕ, and the surface is shown in light where the function is positive and in dark where it takes on negative values.

We give here first the result of a calculation of the Wigner function of the SC state of Eq. (1). From Eqs. (2) and (1) one can derive the following result:

$$W_{SC}(\theta,\phi)=\frac{1}{2}\sqrt{\frac{N+1}{4\pi}}\left\{\sum_{l=0}^{N}\frac{\sqrt{2l+1}N!}{\sqrt{(N-l)!(N+l+1)!}}[Y_{l0}(\theta)+Y_{l0}(\pi-\theta)]\right.$$
$$\left.+(-1)^N\sqrt{\frac{(2N+1)!}{4\pi}}\frac{(\sin\theta)^N\cos(N\phi)}{2^N N!}\right\} \tag{3}$$

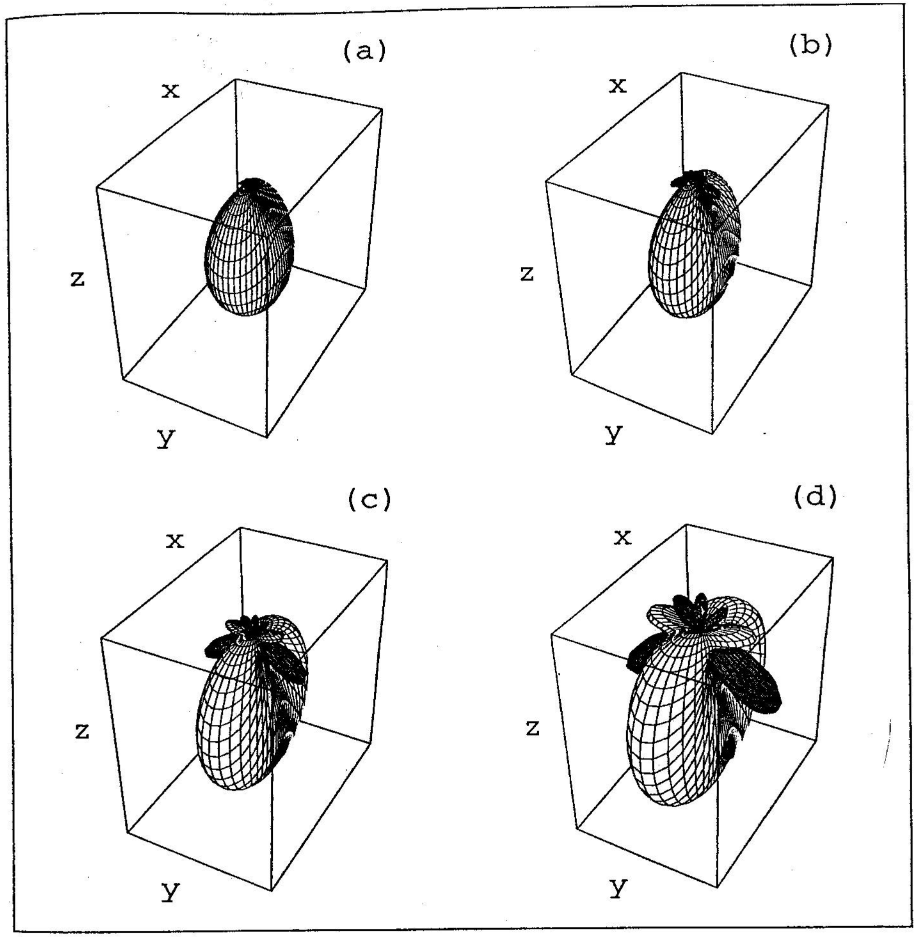

Fig 2. Wigner functions for the state $|\Psi_{12}\rangle$ for $N=4$ atoms, and for several different values of β: (a) $\beta=30°$, (b) $\beta=40°$, (c) $\beta=52°$, (d) $\beta=60°$. For smaller values of β the state goes over into a single coherent state, and then it has essentially only one positive lobe. This graphical presentation shows qualitatively that the y component of the dipole moment is squeezed, the maximal value of the squeezing in the present case ($N=4$) comes about $\beta=40°$.

The first term, containing the weighted sum of two spherical harmonics corresponds to the individual states $|+,+...+\rangle$, and $|-,-...-\rangle$, while the last term arises from the interference term. Fig. 1 shows the polar diagram of this Wigner function for 7 atoms. The two bumps to the "north" and "south" correspond to the quasiclassical coherent nstituents, while the ripples along the equator, where the function takes periodically ositive and negative values are the result of interference between the two kets of Eq. (1). ccording to the factor $\cos(N\phi)$ in (3) the number of both the negative and the positive vings" along the equator are equal to the number of the atoms.

3. More general SC states

One can also construct more general SC states by taking the superposition of two omic coherent states [6]:

$$|\Psi_{12}\rangle = \frac{|\tau_1\rangle + |\tau_2\rangle}{\sqrt{2(1 + Re\langle\tau_1|\tau_2\rangle)}}, \tag{4}$$

nd the corresponding Wigner function still can be calculated. The explicit formula ill not be given here, instead we present graphical plots for this state in Fig. 2, with atoms. Here we have chosen $\tau_1 = \tan\beta$, $\tau_2 = -\tan\beta$, where β is the polar angle of e classical Bloch vector corresponding to the atomic coherent state (it is measured om the south pole). This means that the x component of the expectation value of the pole moment in these states is proportional to $\pm(N/2)\sin\beta$, respectively, and the y mponent is zero.

For small β values, the interference is weak, and the maximum of the Wigner function around $\theta = 0$. For larger β-s the function has two maxima around $\theta = \pm\beta$, and the terference gets more pronounced along the equator perpendicular to the x axis. When $= \pi/2$, the two maxima corresponding to the individual coherent states point in the and $-x$ directions, respectively. In this case we get back the Wigner function of the C state of Eq. (1), rotated around the y axis by $\pi/2$.

Except for this latter extremal case, the J_y quadrature is squeezed in the $|\Psi_{12}\rangle$ state, nd for the maximal value of the squeezing one obtains 27% for large values of N. The etails of these calculations will be given elsewhere.

cknowledgements The authors thank J. Janszky and W. Schleich for enlighteng discussions on the subject. This work was supported by the National Research oundation of Hungary (OTKA) under contracts No. T 22281 and No. F 23336.

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