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A QUANTUM BEAM SEPARATOR[1]

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We propose a realistic scheme for a quantum beam separator for light. Our model consists of a single atom traversing a resonator. Depending on the internal state of the atom light will be reflected off or transmitted through the resonator. We have investigated the performance of this device under realistic operating conditions using quantum Monte Carlo wavefunction simulations. We have found the time dependence of the coupling and the measurement backaction of the continuous measurement process of the output light to significantly influence the behaviour of the device.

1. Introduction

Quantum mechanics itself imposes no fundamental limit on the size of an object that could be in a superposition state [1]. There is no empirical evidence, however, that macroscopic superpositions occur in our world. The most famous hypothetical example of such a macroscopic superposition is due to Schrödinger who devised a gedanken experiment which leaves a cat in a state of being dead and alive at the same time. This is accomplished by transforming a microscopic superposition state into a superposition of a macroscopic object (now often called a *cat state*) by creating quantum entanglement of both entities. In this way the occurence of superposition states in the macroscopic world seems possible [2, 3]. However, any experimental confirmation would involve the observation of interference between two manifestly macroscopic states of a large object. This necessitates preservation of coherence, i.e., throughout the experiment the system state must not get entangled with the state of the environment (the nature of which depends on the type of system used) and thus poses an increasingly challenging technical problem as the size of the system is increased.

Other theories (beyond standard quantum mechanics) [4] predict fundamental limits for the distances over which coherence can be preserved. The ideal experiment to test

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these theories would make use of a device that can generate a superposition state of large and complex objects, separated by large distances and completely decoupled from the environment. If the objects are subsequently reunited and superimposed they should exhibit interference effects [5]. Recently, an experiment of such type has been carried out on a single ion [6].

Alternative proposals for creating superposition states rely on the use of a large number of photons contained in two separate microwave resonators. By sending a single atom through both resonators a superposition state with all photons being either in the one or the other resonator can be generated [2]. While being in principle within the limits of present day technology, this proposal suffers from the limited size of the achievable spatial separation of the two cavities. Moreover, a direct observation of microwave photons is difficult to achieve.

We will show that with present-day technology it should be feasible to prepare a superposition state of a large number of optical photons in such a way that all of the photons are either in one or the other arm of an optical interferometer (cf. Ref. [7]). As an entrance beam splitter of the interferometer we need a *quantum gate* (quantum mirror) which can be in a superposition state of complete transmission and reflection. In this way the photons (laser pulse) get entangled with the mirror state and will then form a macroscopic superposition of the sought type. In contrast to this a conventional beam splitter puts each individual photon in a superposition state of both paths *independently* of all the other photons, so that no macroscopic superposition is created.

Here we fathom the realisability of a quantum mirror made up of an optical high-Q cavity traversed by an atom in a superposition state of two long-lived atomic levels. Contrary to previous work [7] we consider an *open system*. Using Monte-Carlo wave function simulations (which implement a continuous measurement process of the light leaving the resonator with the concomitant backaction on the system dynamics, cf. e.g. [8]) allows us to get close to experimental reality by including a time dependent atom-cavity coupling, spontaneous emission as well as cavity loss. We found these to significantly affect the performance of the device.

2. The Model

We consider a double ended optical resonator illuminated at one port with off-resonant light. As long as the cavity is empty the light will be reflected back off the driven port. If a switchable active element placed inside the resonator can shift the cavity into resonance so that all light gets transmitted, we have implemented a "classical" gate for light. What now if our active medium is a quantum object that can be in a superposition of its *on* and *off*-states? Is this system equivalent to an ordinary classical random beam splitter or will the quantum nature of the medium leave its fingerprint on the light leaving the resonator?

A single three-level atom acts as our active medium. The relevant two states of the atom are the ground states labelled $|0\rangle_a$ and $|1\rangle_a$. State $|0\rangle_a$ does not couple to the light in the resonator [9], while the coupling between the cavity-field (resonance frequency ω_{cav}) and the atom on the transition from $|1\rangle_a$ to an excited state $|e\rangle_a$ (transition frequency ω_0) can shift the resonator into resonance with the incoming field (frequency ω_p) at port I.

A mathematical description of this system is effected by the following master equation for the density operator ρ of the atom-cavity system:

$$\begin{aligned}\dot{\rho} &= -i[H_p + H_{JC}, \rho] - \sum_{i=1,2} \kappa_i \left(\{a^\dagger a, \rho\}_+ - 2a\rho a^\dagger\right) \\ &- \gamma\left(\{\sigma_{ee}, \rho\}_+ - 2\sigma_{1e}\rho\sigma_{e1}\right), \end{aligned} \tag{1}$$

where the driving of the resonator and the cavity-atom interaction are described by H_p and H_{JC}, respectively:

$$H_p = -i\sqrt{2\kappa_1}\,\alpha_{in}\left(a^\dagger - a\right), \tag{2}$$

$$H_{JC} = \Delta_c a^\dagger a + \Delta_a \sigma_{ee} - ig(t)\left(a^\dagger \sigma_{1e} - \sigma_{e1} a\right), \tag{3}$$

with $\Delta_c = \omega_{cav} - \omega_p$ and $\Delta_a = \omega_0 - \omega_p$. Cavity loss through ports 1 and 2 occurs at rates $2\kappa_j$. Spontaneous decay takes place from level $|e\rangle_a$ to level $|1\rangle_a$ at a rate 2γ. The time-dependence of the coupling strength $g(t) \equiv \tilde{g}(x(t)) = (g_0/\sqrt{2\pi}\,w)\exp([x(t) - x_0]^2/2w^2)$ is due to the assumed ballistic motion of the atom through the Gaussian transverse mode profile of width w.

2.1. Passive dispersive intracavity medium

Let us first consider a resonator filled with a dispersive medium. Using standard input-output formalism [10] we find for the light emanating from the two ports of the resonator

$$\langle a^1_{out}\rangle = r\alpha_{in}, \quad \text{and} \quad \langle a^2_{out}\rangle = t\alpha_{in}, \tag{4}$$

with

$$r = \frac{\kappa_2 - \kappa_1 + i\Delta_g}{\kappa_2 + \kappa_1 + i\Delta_g}, \quad \text{and} \quad t = \frac{-2\sqrt{\kappa_1\kappa_2}}{\kappa_2 + \kappa_1 + i\Delta_g}, \tag{5}$$

where $\Delta_g = \Delta_c + \Delta$ (Δ is the shift induced by the dispersive medium). In the limit of perfect *impedance matching*, i.e., $\kappa_1 \equiv \kappa_2$, a large mistuning $\Delta_g > \kappa_i$ causes all light to be reflected back at port I, while for $\Delta = -\Delta_c$ all light will be transmitted through port II as intended. The cavity thus behaves like an ordinary beam splitter, i.e., $|r|^2 + |t|^2 = 1$, although the finite storage time of energy inside it gives rise to a finite delay in the transmission.

2.2. Coupled cavity-atom system

To get a grasp of the dispersive effect of a single atom [11] inside the resonator we diagonalise H_{JC}, cf. Ref. [12]. The ground state of the coupled atom-cavity system is denoted $|e_0\rangle = |\text{vac}\rangle_c \otimes |1\rangle_a$. Furthermore, there are infinitely many doublets of dressed states (approximately $\hbar\omega_p$ apart) parameterised by an index n, corresponding to the number of cavity photons for the atom in state $|1\rangle_a$. We find

$$H_{JC} = \sum_{n\geq 1} \hbar\left(E_n^{(+)}|e_n^{(+)}\rangle\langle e_n^{(+)}| + E_n^{(-)}|e_n^{(-)}\rangle\langle e_n^{(-)}|\right), \tag{6}$$

with $E_n^{(\pm)}(t) = n\Delta_c + \delta/2 \pm \left[\delta^2/4 + ng^2(t)\right]^{1/2}$ and $\delta = \Delta_a - \Delta_c$. The eigenvectors are given by

$$|e_n^{(\pm)}\rangle = \left(ig(t)\sqrt{n}|n-1\rangle_c \otimes |e\rangle_a + [n\Delta_c - E_n^{(\mp)}]|n\rangle_c \otimes |1\rangle_a\right)/N^{\pm}, \quad (7)$$

where $N^{\pm}$ are suitable normalization factors. In Fig. 1 we schematically depict the three lowest eigenvalues as functions of $g(x(t))$ for $\delta > 0$. We realise that the coupled

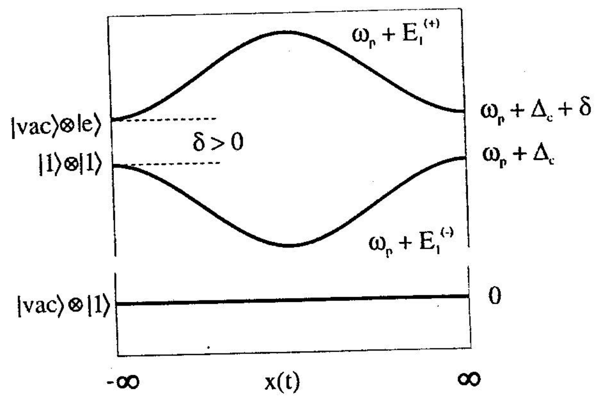

Fig. 1. Adiabatic eigenenergies vs. position of the three energetically lowest eigenstates $|e_0\rangle$, $|e_1^{\pm}\rangle$.

atom-cavity system is shifted into resonance with the coherent pump-field at maximum coupling strength $g_m = \max(g(t))$ if we choose $\Delta_c = -\delta/2 + [\delta^2/4 + g_m^2]^{1/2}$. In this limit we predominantly pump the component $|1\rangle_c \otimes |1\rangle_a$ of $|e_1^{(-)}\rangle$ provided that $\Delta_a \gg g_m$ is satisfied. This minimises spontaneous emission from state $|\mathrm{vac}\rangle_c \otimes |e\rangle_a$. At the same time we have to require that $\Delta_c \gg \kappa_1 + \kappa_2$ is satisfied. This ensures that without an atom present in the proper internal state the resonator will almost completely reflect the driving field. For this we need: $\Delta_a \gg g_m$ and $(\Delta_c = -\delta/2 + [\delta^2/4 + g_m^2]^{1/2}) \gg \kappa_1 + \kappa_2$. This can only hold for sufficiently strong atom-cavity coupling, i.e. $g_m \gg \kappa_1 + \kappa_2, \gamma_j$ [9]. In this limit the resonator switches from almost total reflection to nearly perfect transmission and back as a single atom flies through [11].

3. Gate Dynamics

In two steps we will now show that our device is substantially different from any classical apparatus. First we will demonstrate that we have created an efficient switch. As outlined above, a significant amount of light can only be transmitted if the atom is in its internal state $|1\rangle_a$. If we inject atoms in an equally weighted superposition state

of its ground states $|0\rangle_a$ and $|1\rangle_a$ then there should be a strong correlation between the number of photons having been transmitted through the cavity and the result of a subsequent state measurement on the atom. The detection of a single transmitted photon is thus likely to trigger a whole avalanche of subsequent detections. Likewise, if hardly any photons are transmitted the state measurement should return $|0\rangle_a$. In an ideal setting one would first send the atom through the cavity thereby creating a superposition state of a many photon light pulse in the transmission and reflection path of the mirror which is then subsequently analysed. The long transit time of the atom through the resonator ($\approx 100\mu s$)in this case requires long optical delay lines (≈ 30 km). To demonstrate the basic principle we use a more practical setup and analyse the output light, while the atom is still *inside* the resonator this implementing a continuous measurement [13].

3.1. Direct detection

We numerically simulate such a continuous measurement as depicted in Fig. 2 with the adjustable mirrors in their positions 1. After the atom has flown through the

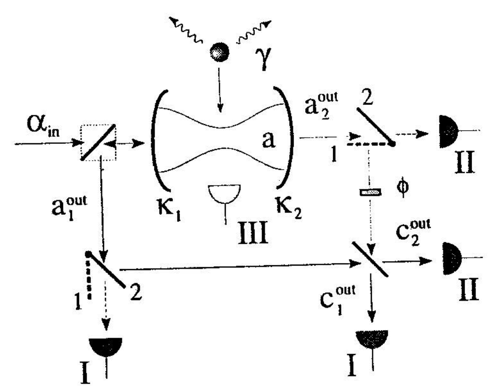

Fig. 2. Diagrammatic representation of the setup. By help of the movable mirrors we may switch between direct detection (1) and interferometric detection (2).

resonator its internal state will be measured by detector III. The stochastic Schrödinger equation (SSE) for the conditional wave function $|\Psi\rangle_c$ corresponding to the density matrix equation (1) can be simulated using a quantum Monte Carlo algorithm. It reads [13,14]

$$d|\Psi\rangle_c = -iH_{\mathrm{eff}}|\Psi\rangle_c\, dt + \sum_j (\lambda_j c_j - 1)\, dN_j |\Psi\rangle_c, \quad (8)$$

with $H_{\mathrm{eff}} = H_c - i\sum_j c_j^\dagger c_j$ and the λ_j arbitrary coefficients. The mean number of counts of type j in $]t, t+dt]$ is given by $\langle dN_j(t)\rangle_c = {}_c\langle\Psi(t)|2c_j^\dagger c_j|\Psi\rangle_c\, dt$. From Fig. 2 and using

$\alpha_{in} = \sqrt{2\kappa_1}\alpha$ we make the identifications: $c_1 = \sqrt{\kappa_1}(\alpha+a)$, $c_2 = \sqrt{\kappa_2}a$ and $c_3 = \sqrt{\gamma}\sigma_{1e}$. In addition we have set $H_c = \frac{1}{2}H_p + H_{JC}$. The number processes $N_j(t)$ denote the number of counts in detector j up to time t, satisfying $dN_j(t)dN_k(t) = \delta_{jk}dN_j(t)$ and $dN_j(t)dt = 0$. In Fig. 3 we have plotted the normalized distribution $p_{II}(n,T)$ of the number n of transmitted photons during the transit time T. Correlating the count

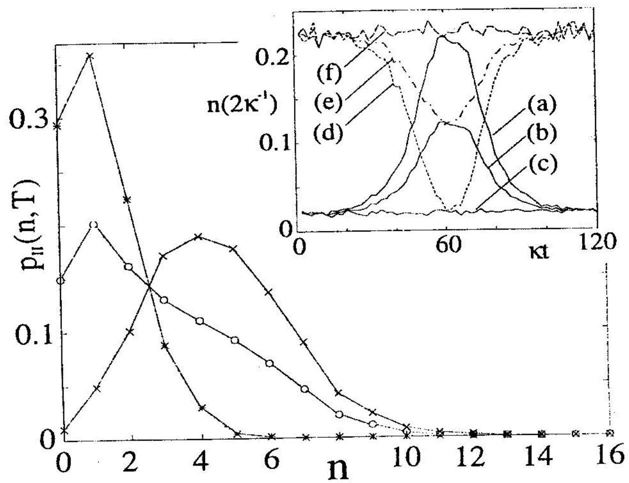

Fig. 3. Distribution (12000 realizations) of the number of photons counted by detector II during the transit time T (o) and the distributions conditioned on a atomic state measurement yielding $|0\rangle$ (*) and $|1\rangle$ (×), respectively. Using $\kappa/2 = \kappa_1 = \kappa_2$ the parameters are $\kappa T = 120$, $g_m/\kappa = 11.5$, $\Delta_a/\kappa = 40$, $\Delta_c/\kappa = 3.32$, $\alpha = 0.35$. The inset depicts the photon flux integrated over a period $2\kappa^{-1}$ for detectors I (e) and II (b). Curves (f/d) and (c/a) represent the same conditioned on a state measurement yielding 0 or 1, respectively.

sequences with the result of an atomic state measurement by detector III reveals the strong projective character of the setup. From such postselection we obtain two partial distributions of distinctively different mean values. In the ideal scenario already the first "click" at detector II would project the atom into state $|1\rangle_a$ and determine all other clicks. In practice due to the finite Q of the cavity a small field builds up even inside the empty resonator leading to a small number of background counts degrading the correlation. The inset of Fig. 3 displays the time dependence of the total and the postselected count rates of the two detectors thereby clearly demonstrating the switching property.

3.2. Interferometric detection

From the evidence presented this far, one cannot conclude that our device is truely quantum, as one would find similar behaviour for a classical random switch, cf. Sec. 2.1.

with Δ_g random. As stated in the introduction, a macroscopic superposition state may only arise from a physical process during which *quantum entanglement* is established between a microscopic and a macroscopic entity. The nonclassical nature of our switch can thus only be ascertained in an experiment sensitive to quantum coherence. The ambiguity between our system and a classical random switch arises from the fact that the setup discussed above provides ***which-path*** information encoded in the atom. This set-up does not distinguish between an atom in an equally weighted mixture of its states $|1\rangle$ and $|0\rangle$ or in a pure superposition state. We thus modify our setup such that clicks at the detectors no longer provide which-path information. Assume that the movable mirrors in Fig. 2 are now in their positions 2. The detectors will then measure both output fields after they have been recombined on a 50−50 beam splitter. The dectectors I and II measure the fields c^j_{out}:

$$c^1_{out} = \sqrt{2}c_1 = \sqrt{\kappa_1}\left(\alpha + [1+Re^{i\phi}]a\right), \tag{9}$$

$$c^2_{out} = \sqrt{2}c_2 = -\sqrt{\kappa_1}\left(\alpha + [1-Re^{i\phi}]a\right), \tag{10}$$

with $R = \sqrt{\kappa_2/\kappa_1}$. The phase ϕ is optional and may be varied by inserting a phase shifter. The photon flux $\Lambda_I = \langle 2c_1^\dagger c_1\rangle$ at detector I at time t will then be given by

$$\begin{aligned}\Lambda_I &= \kappa_1\left(\alpha^2 + [1+R^2]\langle a^\dagger a\rangle + \alpha\langle a + a^\dagger\rangle\right) + \\ &+ \kappa_1 R\cos\phi\left(2\langle a^\dagger a\rangle + \alpha\langle a+a^\dagger\rangle + i\alpha\tan\phi\langle a - a^\dagger\rangle\right).\end{aligned} \tag{11}$$

Choosing $\phi = 0$ detector II will merely see a coherent field, $c^2_{out} = \sqrt{\kappa_1}\alpha$. This means that clicks registered by detector II do not provide any information about the state of the coupled atom-cavity system. Similarily detector I measures a quantity proportional to the total output field which also cannot yield which-path information [3]. Clicks at detector I will only give rise to a rotation of the atomic coherence ($\langle\sigma_{01}\rangle$) in the complex plane. As they occur at random times the net result will be a random phase shift of the atomic coherence vector. This implies that the modulus of the initial atomic coherence will survive the detection sequence and the atom stays in a superposition state.

The quantum nature of our switch becomes apparent by projecting the final atomic state onto a superposition of the two atomic ground states and correlating the result with the count sequence obtained from detector I. In the inset of Fig. 4 we plot the average photocurrent [curve (a)], and the conditioned currents obtained from a projection on $|0\rangle \mp i|1\rangle$ [curves (b/c)] as functions of time. (in practice this is accomplished by applying a $\pi/4$-pulse to the low-frequency transition between the two ground states of the atom before it reaches detector III.) Curves (b) and (c) exhibit a strong variation which cannot be explained classically. The difference count rate $\delta\Lambda$ between detectors I and II is directly related to the correlation between the light in both arms of the interferometer, i.e., $\delta\Lambda(t) = \langle(a^1_{out})^\dagger a^2_{out} + \text{h.c.}\rangle$. Hence, using Eq. (4) this quantity vanishes for a classical random switch [4], as one would intuitively expect. The randomness of the phase-shift incurred by the atomic state somewhat limits the usefulness of our approach.

[3] Provided there is no other significant loss mechanism present in the system.
[4] This follows from Sec. 2.1. for $\kappa_1 = \kappa_2$.

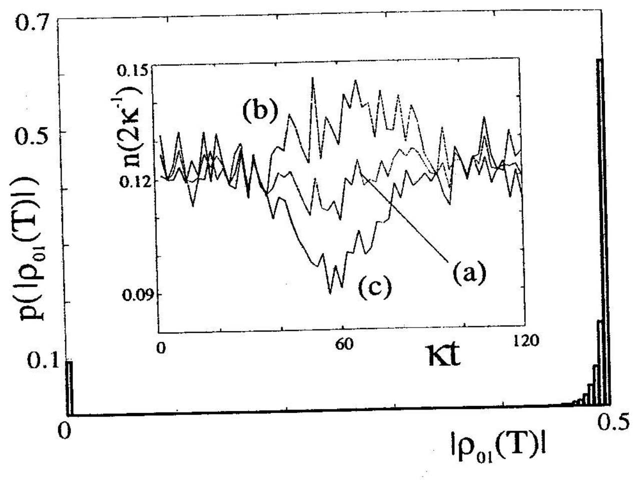

Fig. 4. Distribution of the modulus of the final atomic coherence $|\rho_{01}(T)|$ for the same parameters as in Fig. 3 and $\gamma = \kappa/2$. The inset depicts the photon flux sampled by detector I over periods of $2\kappa^{-1}$ (a). Curves (b/c) are obtained by correlating the click sequences with an atomic state measurement yielding $|0\rangle \mp i|1\rangle$.

Through postselection we check whether a certain total phase shift of the atomic state coincides with a certain pattern in the count sequence. Doing so is of course only meaningful if the spread in the total phase shift is less than 2π. Since the atom interacts with a field whose envelope varies with time, this gives a limit on the maximum size of the pulse area for which this simple postselection technique can be applied.

An alternative method to demonstrate the coherence properties of the reflected and transmitted photon pulses lies with the observation of the atomic coherence. Knowledge of the path of the light pulse immediatly yields information on the atomic state and the atomic coherence between the two ground states collapses. If atomic coherence is preserved during the interferometric measurement this would prove the superposition properties of the light field, i.e., our ignorance of the path the light took. We show this in Fig. 4, where the normalised probability distribution of the modulus of the atomic coherence $\rho_{01}(T) = {}_c\langle\sigma_{10}(T)\rangle_c$ is plotted. The contribution at the origin is due to trajectories involving a spontaneous decay of the atom. The backaction of the interferometric measurement on the atom occurs in the form of random jumps in the phase of the atomic coherence. This is a genuine feature of the chosen setup, where the field measurement is *concurrent* with the atom field interaction. A setup where the measurement of the fields starts after the atom has left the cavity would avoid these complications but require long optical delay. Nevertheless the preservation of the atomic coherence, as demonstrated above suffices to infer the superposition character

of the photon pulses.

4. Conclusions

Based on a numerical experiment we have thus shown that the use of a high-Q optical cavity together with an atom in a long-lived superposition state allows us to prepare a highly delocalized quantum superpostion state of a many photon light pulse. Its coherence and decoherence properties can be analysed by interferomeric techniques. Although a practical realisation seems experimentally challenging, several fundamental tests of quantum theory might be possible with such a source based on currently available technology. In view of the longer storage times and improved detection schemes for atomic coherence, also a setup based on trapped ions seems possible, provided a sufficiently good optical cavity is available. As a final point we wish to remark that the recent success in preparing degenerate quantum states for many atoms (BEC) allows speculations about analogous schemes to create nonlocal superpositions of many particle states.

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