acta physica slovaca vol. 47 No. 3/4, 185 – 192 June 1997

QUANTUM OPTICS AS A CONCEPTUAL TESTING GROUND[1]

J. A. Bergou[2]
Department of Physics and Astronomy,
Hunter College of the City University of New York,
695 Park Avenue, New York, NY 10021, USA

Received 6 May 1997, accepted 14 May 1997

Entangled states provide the necessary tools for conceptual tests of quantum mechanics and other alternative theories. Here our focus is on a test of the time symmetric, pre- and postselective quantum mechanics and its relation to the consistent histories interpretation. First, we show how to produce a nonlocal entangled state, necessary for the test, where there is precisely one photon hiding in three cavities. This state can be produced by sending appropriately prepared atoms through the cavities. Then, we briefly review the proposal for an experimental test of pre- and postselective quantum mechanics using the three-cavity state. Finally, we show that the outcome of such an experiment can be discussed from the viewpoint of the consistent histories interpretation of quantum mechanics and therefore provides an opportunity to subject quantum cosmological ideas to laboratory tests.

1. Introduction

Two quantum systems are entangled if their state cannot be expressed as a product of states of the individual systems. This implies that the two systems are correlated. It also implies that, even though the entire system may be in a pure state, neither of its subsystems has a wavefunction. In fact, if the degree of entanglement is large enough, Bell's inequality can be violated [1]. Consequently, entangled states feature prominently in investigations of the foundations of quantum mechanics. The concept of entanglement can easily be generalized to more than two systems. For example, Greenberger, Horne and Zeilinger proposed a strong test of local hidden variables theories which involves the use of a highly entangled state (GHZ state) of three systems [2]. Tests of quantum mechanics itself also require that highly entangled states be used [3 – 5].

Experimental realizations of these tests require that methods of producing entangled states be found. Previous works have concentrated primarily on producing entangled

[1]Presented at the **Fifth Central-European Workshop on Quantum Optics**, Prague, Czech Republic, April 25 - 28, 1997

[2]E-mail address: jbergou@shiva.hunter.cuny.edu

states of atoms. Cirac and Zoller showed how to produce a maximally entangled state of two two-level atoms [6]. Their method could also generate a GHZ state of three atoms if the cavity through which the atoms pass is prepared in a superposition of a three photon state and the vacuum. A method of producing entangled pairs of atoms using two micromaser cavities was discussed by Bogar and Bergou [7]. Tests of the nonlocality of quantum mechanics employing two cavities and, subsequently, two atoms were proposed by Freyberger [8] and Gerry [9]. Sleator and Weinfurter, as a byproduct of their work on teleportation and quantum logic gates, found how to create an entangled state of one cavity and an arbitrary number of two-level atoms [10]. A method of generating particular entangled states of two cavities occured as an intermediate step in the quantum optical adaptation of teleportation proposed by Davidovich, Zagury, Brune, Raimond, and Haroche [11].

In the next section we show how a particular entangled state of a photon hiding in one of three spatially separated cavities can be produced. The method is based on the techniques of cavity quantum electrodynamics [12]. Further details on the techniques as well as the generation of more general entangled states can be found in [5] and in [13]. Then we review a proposal to test time symmetric quantum mechanics, originally suggested by Aharonov, Bergmann, and Lebowitz [14] and its relation to the consistent histories interpretation of quantum mechanics [15] which has widely been considered as the only vital alternative to the standard Copenhagen interpretation.

2. Generation of a three-cavity entangled state

In this section we begin by reviewing the available techniques for the manipulation of single-mode cavity fields and two-level atoms. They include preparation and further manipulation of an atom via a resonant classical field before and after the cavities, quite frequently in the Ramsey configuration, and both resonant (Jaynes-Cummings) and nonresonant (QND type) dispersive interactions of an atom with the quantized cavity fields. Then, we show that an appropriate combination of these techniques can be used to generate the desired three-cavity entangled states.

We shall consider two-level atoms with excited state $|e\rangle$, ground state $|g\rangle$, and energy separation E_0. First, we describe their interaction with the resonant classical field. If the atom and the field interact for a time t the evolution of the atomic states is given by

$$\begin{aligned} |e\rangle &\rightarrow \cos\theta_1(t)|e\rangle + \sin\theta_1(t)|g\rangle, \\ |g\rangle &\rightarrow -\sin\theta_1(t)|e\rangle + \cos\theta_1(t)|g\rangle, \end{aligned} \tag{1}$$

where $\theta_1(t) = \frac{1}{2}\Omega t$ and Ω is the Rabi frequency. Later, we shall be interested in specific values of $\theta_1(t)$. Notice that $\theta_1(t)$ is half the classical Rabi rotation angle.

When the field is quantized, we have to give both the state of the atom and the number of photons to completely specify the state of the system. That is, our states are of the form $|e,n\rangle$ or $|g,n\rangle$ where n is the photon number. The field is assumed to exist inside a cavity which the atom traverses. For our purposes, we are only interested in the cases $n=0$ and $n=1$. For $n=0$ we have

$$\begin{aligned} |e,0\rangle &\rightarrow \cos\theta_2(t)|e,0\rangle + \sin\theta_2(t)|g,1\rangle, \\ |g,0\rangle &\rightarrow |g,0\rangle, \end{aligned} \tag{2}$$

where $\theta_2(t) = gt$ (with g being the coupling constant between the atom and the cavity mode). For $n = 1$ we are only interested in what happens to an atom injected in its ground state

$$|g,1\rangle \rightarrow -\sin\theta_2(t)|e,0\rangle + \cos\theta_2|g,1\rangle. \tag{3}$$

Here $\theta_2(t)$ is half the vacuum Rabi flipping angle.

Finally, when a two-level atom interacts with a far off-resonant quantized field mode, it acquires a phase shift as follows [12]. When the photon number is zero the states of the atom are unaffected. If it is one, we have

$$|e\rangle \rightarrow e^{-i\theta_3(t)}|e\rangle, \text{ and } |g\rangle \rightarrow |g\rangle, \tag{4}$$

where $\theta_3(t) = (g^2/\delta)t$ is the relative phase shift between the two atomic levels (δ is the detuning on the intermediate transition), induced by the dispersive interaction. That is, the $|g\rangle$ state is unchanged and the $|e\rangle$ state is multiplied by a phase factor.

These are the basic ingredients we need to create the desired entangled three-cavity state. It is now only necessary to arrange them in the proper sequence and to choose the proper values of θ_1, θ_2, and θ_3. Our first objective is to establish the appropriate initial condition for the three-cavity system. We begin with the three cavities in their vacuum states. An atom in the state $|e\rangle$ is sent through a region before the cavities where it interacts with a classical field (Ramsey zone) with $\cos(\theta_1) = \frac{1}{\sqrt{3}}$, putting the atom in the state

$$|in\rangle = \frac{1}{\sqrt{3}}(|e\rangle + \sqrt{2}|g\rangle). \tag{5}$$

Then, this atom is sent through the first cavity where it undergoes resonant interaction. The interaction time has been adjusted so that $\theta_2 = \pi/2$, yielding the atom-cavity state

$$\frac{1}{\sqrt{3}}(|1\rangle_1 + \sqrt{2}|0\rangle_1)|0\rangle_2|0\rangle_3|g\rangle. \tag{6}$$

The states $|0\rangle_j$ and $|1\rangle_j$ are, respectively, the zero and one photon states of cavity j ($j = 1,2,3$). This first step is necessary only to establish the appropriate initial condition, Eq. (6), for the three-cavity system. Alternatively, we could omit this step from our considerations and take this initial condition as given.

In any case, the state of the first atom factorizes and the atom can be discarded after establishing the proper intial condition. A second atom in its excited state is sent into the system. The atom is first sent through the Ramsey zone with $\theta_1 = \pi/4$ which prepares it in the state $|+\rangle$ [see Eq. (8) below]. Then, it passes through the first cavity and a second Ramsey zone afterwards. It interacts off-resonantly in the first cavity and the interaction time has been chosen so that $\theta_3 = \pi$. After this interaction the state of the full atom-cavity system is

$$\frac{1}{\sqrt{3}}(|1\rangle_1|-\rangle + \sqrt{2}|0\rangle_1|+\rangle)|0\rangle_2|0\rangle_3, \tag{7}$$

where

$$|\pm\rangle = \frac{1}{\sqrt{2}}(|g\rangle \pm |e\rangle). \tag{8}$$

The atom now passes through a second Ramsey zone with $\theta_1 = -\pi/4$ which has the effect

$$|+\rangle \to |e\rangle, \text{ and } |-\rangle \to |g\rangle, \tag{9}$$

so that the total atom-cavity state is now

$$\frac{1}{\sqrt{3}}(|1\rangle_1|g\rangle + \sqrt{2}|0\rangle_1|e\rangle)|0\rangle_2|0\rangle_3. \tag{10}$$

The atom then passes through the second cavity where it interacts resonantly with $\theta_2 = \pi/4$. The resulting state of the system is

$$\frac{1}{\sqrt{2}}(|1\rangle_1|0\rangle_2|g\rangle + |0\rangle_1|1\rangle_2|g\rangle + |0\rangle_1|0\rangle_2|e\rangle)|0\rangle_3. \tag{11}$$

Finally the atom passes through the third cavity where it interacts resonantly with $\theta_2 = \pi/2$. This results in the following final state for the system

$$\frac{1}{\sqrt{3}}(|1\rangle_1|0\rangle_2|0\rangle_3 + |0\rangle_1|1\rangle_2|0\rangle_3 + |0\rangle_1|0\rangle_2|1\rangle_3)|g\rangle. \tag{12}$$

The atom can now be discarded and we are left with the final highly entangled three-cavity field state

$$|\Psi_3\rangle = \frac{1}{\sqrt{3}}(|1\rangle_1|0\rangle_2|0\rangle_3 + |0\rangle_1|1\rangle_2|0\rangle_3 + |0\rangle_1|0\rangle_2|1\rangle_3). \tag{13}$$

This is just the desired three-cavity entangled state that we were set out to generate.

3. A test of pre- and postselective quantum mechanics

Next we describe how a three-cavity entangled state such as the state $|\Psi_3\rangle$ of Eq. (13) can be used to test the propositions of pre- and post selected quantum mechanics originally suggested by Aharonov, Bergmann, and Lebowitz (ABL rule) [14]. A more detailed account is about to appear elsewhere [16]. In order to understand how the ABL rule is different from conventional quantum mechanics, we first briefly review what is the quantum mechanical prediction for sequential measurements. At time t_1 the system is prepared in state $|a\rangle$ by measuring an operator A and finding the eigenvalue a (we use a for both the eigenvalue and to label the corresponding quantum state). Suppose, at time $t > t_1$, we measure the nondegenerate operator C on this system. The probability amplitude of finding the eigenvalue c_n is $\langle c_n|a\rangle$. Then, at time $t_2 > t$, the measurement of an operator B is carried out. The amplitude of finding the eigenvalue b after finding c_n in this sequential measurement is $\langle b|c_n\rangle\langle c_n|b\rangle$. The probability is simply the absolute square of this amplitude,

$$p(b, c_n|a) = |\langle b|c_n\rangle\langle c_n|a\rangle|^2, \tag{14}$$

where the notation reflects that this, in fact, is a conditional probability which is conditioned on the initial state. In order to facilitate later comparison with the consistent histories interpretation, we note that this expression can be written as

$$p(b, c_n|a) = \mathrm{Tr}[P_b P_n P_a P_n], \tag{15}$$

where P is the projection operator onto the eigenstate specified by its label.

This expression is not symmetric under time reversal, as was first observed by ABL. It makes predictions starting from a fixed initial state (the preselected state in their terminology) in a definite direction in time such that $t_1 < t < t_2$. According to their reasoning, this apparent lack of time reversal symmetry is due to the measurement itself. The actual measurement of A has been carried out and all information from times before t_1 is lost. ABL therefore defined a pre- and postselected system where the final measurement of B at t_2 is carried out and the actual result b is found. A collection of systems all satisfying the same pre- and postselection condition is the "pre- and postselected ensemble". It is natural then to ask what is the probability that a measurement of C, at time t on this ensemble, will yield c_n. In general, the answer will be different from Eq. (14). ABL suggested that the probability is given by the following expression,

$$p(c_n|a, b) = \frac{p(b, c_n|a)}{\sum_i p(b, c_i|a)}. \tag{16}$$

This simply means that the probability is given by the weight of the particular outcome relative to the weight of all possible outcomes, a very natural looking proposition, indeed. At this point it should be mentioned that this is the simplest form of the ABL rule, based on the assumption of a discrete and nondegenerate spectrum. Various generalizations are available but this is sufficient for our purposes. The notation shows explicitly that this is a conditional probability which is now conditioned on both the initial and final state. Furthermore, it is explicitly time reversal symmetric: from the point of view of the initial condition, this is a prediction and from the point of view of the final condition, this is a retrodiction. For uniqueness, it is sometimes termed a proposition.

A relatively simple experimental test of Eq. (16) can be carried out in the following way. Let us assume that the initial state $|a\rangle$ is the one given by $|\Psi_3\rangle$ in Eq. (13) and the final state $|b\rangle$ differs only in the sign of the last term in the r.h.s. in Eq. (13),

$$|b\rangle = \frac{1}{\sqrt{3}}(|1\rangle_1|0\rangle_2|0\rangle_3 + |0\rangle_1|1\rangle_2|0\rangle_3 - |0\rangle_1|0\rangle_2|1\rangle_3). \tag{17}$$

Since $|a\rangle$ and $|b\rangle$ are not orthogonal, our pre- and postselected ensemble is not empty. Suppose now that, at time t, we open cavity i, where $i = 1$ or 2, to see if the photon is there. If we introduce the notation $|1\rangle = |1\rangle_1|0\rangle_2|0\rangle_3$ and $|2\rangle = |0\rangle_1|1\rangle_2|0\rangle_3$ then such a measurement obviously corresponds to a projection on $|i\rangle$ (again $i = 1$ or 2).

The other possibility, photon not in cavity i, corresponds to the projection onto the orthogonal subspace, $1-|i\rangle\langle i|$. Upon substituting into Eq. (16), it is a simple matter of algebra to show that the ABL rule yields unit probability for this kind of measurement. In other words, we will always find the photon in the cavity that we care to open. Furthermore, it assigns unit probability to mutually exclusive propositions which is a highly counterintuitive suggestion. This is, however, not the end of it. If, at time t, we decide to measure the operator A on this pre- and postselected ensemble, i.e., we choose $C = A$ then Eq. (16) tells us that the value a is found with unit probability again. Finally, if at time t we decide to measure operator B on this ensemble, i.e., we make the choice $C = B$ then the ABL rule predicts that the outcome b is found, again with certainty. Let me summarize the situation that we have encountered so far. At time t, we can perform at least four different measurements on our pre- and postselected system, the outcome of each of which can be retrodicted with unit probability. In addition, the first two of these are mutually exclusive propositions.

Is this a satisfactory situation? Hardly, if we are to assign any element of reality to the outcome of these intermediate measurements and the states the system is found in. It appears that quantum mechanics is contextual and "reality" which happens with certainty depends on the question we ask from the system. A natural assumption would be that this is so beacuse of the way quantum mechanics constructs probabilities. To show that this is not quite the case, let us have a look at Eq. (16) from a broader perspective, viz. from the point of view of the consistent histories interpretation of quantum mechanics [15]. It was first shown by Griffiths [17] that quantum mechanical probabilities conform to the rules of classical probability theory if the following condition is satisfied,

$$\mathrm{Re}[\mathrm{Tr}(P_B C_i P_A C_j)] = 0 \text{ for } i \neq j. \tag{18}$$

Here Re[...] denotes the real part of [...]. This is the off-diagonal generalization of Eq. (15). It is easy to check that in our case, when we use i and j as defined after Eq. (17), this condition is met. One consequence of the above equation is that the denominator in Eq. (16) can be written in the following way,

$$\sum_i p(b, c_i|a) \equiv \sum_i |\langle b|c_i\rangle\langle c_i|a\rangle|^2 = \left|\sum_i \langle b|c_i\rangle\langle c_i|a\rangle\right|^2 = |\langle b|a\rangle|^2. \tag{19}$$

Here, in the first step, we made use of Eq. (18) and, in the second step, of the closure relation. This means that the denominator in Eq. (16) becomes independent of the particular decomposition of the unity operator and the probability can be written as

$$p(c_n|a,b) = \frac{p(b, c_n|a)}{p(b|a)}. \tag{20}$$

The expression in the denominator is now simply the transition probability for the $|a\rangle \to |b\rangle$ transition and our notation accounts for the fact that it is just the conditional probability for this transition since the state $|a\rangle$ is normalized to unity. Thus, under these conditions Eq. (16) reduces to Eq. (20) which has the form of a classical conditional probability where the outcome c_n is conditioned on two conditions.

The consistent histories formalism was introduced to deal with a closed quantum system (possibly the universe in quantum cosmologies). One particular question addressed by this formalism is whether it is possible to retrodict the past history of a closed system, including, in particular, its initial state, knowing, e.g., its present quantum state. Based on the retrodictions by Eq. (16) or Eq. (20) this is highly doubtful since even less cannot be achieved. Knowing both the initial and final state of the system mutually exclusive histories can be retrodicted, each with unit probability. A test of this kind, therefore, would be extremely interesting because, if it confirms the proposition of Eq. (16), it just adds to the mysteries of quantum mechanics. If it contradicts Eq. (16) then, clearly, the consistent histories interpretation (or its stronger version, the decoherent histories [18]) need some further rules to select possible scenarios from among the permitted ones. At this stage, it appears this second path is more likely to be followed and further constraints are to be found in the future.

4. Conclusion

Highly entangled states are useful in testing local hidden variables theories, quantum mechanics itself, and the pre- and postselective quantum mechanics of Aharonov, Bergmann, and Lebowitz. They also feature prominently in certain schemes to transmit quantum information, such as teleportation [19] and quantum cryptography [20]. One, therefore, wants to have a method of producing them.

As we have shown cavity QED gives us the necessary tools to do this. It is possible to produce maximally entangled states of multiple cavities. It should be possible to use these states to process and transmit quantum information.

With regard to experimental feasibility it should be noted that, in the experiments of Brune *et al* [21, 22] using circular Rydberg states, both large Rabi rotation angles and, in the off-resonant case, large phase shifts have been achieved. Since the lifetime of the circular Rydberg atoms is much longer (30 ms) than the transit time through the apparatus, it does not pose a serious limitation on the suggested scheme.

Acknowledgements This research was supported by the Office of Naval Research and by a grant from PSC-CUNY. The author is grateful to Dr. Igor Jex for organizing the 5th Central European Workshop on Quantum Optics in Prague.

References

[1] J.S. Bell: *Physics* **1** (1964) 195

[2] D. Greenberger, M. Horne, A. Zeilinger: in *Bell's Theorem, Quantum Theory, and Conceptions of the Universe*, ed. by M. Kafatos (Kluwer, Dordrecht, 1989) p. 62

[3] B.S. Tsirelson: *Lett. Math. Phys.* **4** (1980) 93

[4] M. Hillery, B. Yurke: *Quantum Opt.* **7** (1995) 215

[5] J.A. Bergou: *Acta Physica Slovaca* **46** (1996) 241

[6] J. Cirac, P. Zoller: *Phys. Rev. A* **50** (1994) R2799

[7] P. Bogar, J. Bergou: *Phys. Rev. A* **53** (1996) 49

[8] M. Freyberger: *Phys. Rev. A* **51** (1995) 3347
[9] C.C. Gerry: *Phys. Rev. A* **53** (1996) 4583
[10] T. Sleator, H. Weinfurter: in *Fundamental Problems in Quantum Theory*, Annals of th New York Academy of Sciences **755** (1995) 715
[11] L. Davidovich, N. Zagury, M. Brune, J.M. Raimond, S. Haroche: *Phys. Rev. A* **5** (1994) R895
[12] M. Brune, S. Haroche, J.M. Raimond, L. Davidovich, N. Zagury: *Phys. Rev. A* **4** (1992) 5193
[13] J. Bergou, M. Hillery: *Phys. Rev. A*, to appear
[14] Y. Aharonov, P.G. Bergmann, J.L. Lebowitz: *Phys. Rev.* **134** (1964) B1410
[15] R. Omnès: *Rev. Mod. Phys.* **64** (1992) 339
[16] J.A. Bergou: *J. Mod. Opt.*, to appear
[17] R.B. Griffiths: *J. Stat. Phys.* **36** (1984) 219
[18] M. Gell-Mann, J.B. Hartle: in *Complexity, Entropy and the Physics of Information*, e by W. H. Zurek (Addison-Wesley, Redwood City, California, 1991)
[19] C.H. Bennett, G. Brassard, C. Crepeau, R. Jozsa, A. Peres, W. Wooters: *Phys. Re Lett.* **70** (1993) 1895
[20] A. Ekert: *Phys. Rev. Lett.* **67** (1991) 661
[21] M. Brune, F. Schmidt-Kaler, A. Maali, J. Dreyer, E. Hagley, J.M. Raimond, S. Haroch *Phys. Rev. Lett.* **76** (1996) 1800
[22] M. Brune, E. Hagley, J. Dreyer, X. Maître, A. Maali, C. Wunderlich, J.M. Raimond, S Haroche: *Phys. Rev. Lett.* **77** (1996) 4887