acta physica slovaca vol. 47 No. 2, 127 – 156 April 1997

ON THE ANISOTROPIES OF COSMIC MICROWAVE BACKGROUND RADIATION

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Received 20 November 1996, accepted 21 January 1997

The work gives a brief overview of the topic of cosmic microwave background radiation anisotropies. Then it deals with the so-called Rees-Sciama effect; i.e. with the anisotropies arising between the last scattering surface and us due to transparent huge irregularities. Using the formulas of Special Theory of Relativity it is proven that in the neighbourhood of expanding spherical body the Mészáros' calculations (Mészáros 1994) are correct; the inaccuracy is maximally of order 10^{-12}. Then the profile of the blueshift of expansion caused by an expanding sphere is calculated for the case, when the radius of this sphere is much smaller than the relevant Hubble radius. Hence the profiles of the shifts of light periods through a void and through a supercluster are given in the most general cases. These cases contain all the three Friedmannian models and both the synchronous and asynchronous clusters. Then the obtained profiles are explicitly decomposed into the sum of the multipole terms, and it is shown that the observed difference between the measured direction of the maximum of dipole anisotropy of CMBR and the result of Lauer and Postman (1994) is not explainable by the Rees-Sciama effect. This means that no alternative exists to the two possibilities for the explanation of the data of Lauer and Postman; either the either the huge system of Abell clusters is streaming, or the Friedmannian model is querried. The third possibility is, of course, that the data of observations of Lauer and Postman are incorrect. However, any of these three possibilities seem to be strange enough; hence, the problems coming from data of Laurer and Postman further holds. This is the key result of paper. As a further technical result it is also shown that in principle there is no upper limit of Rees-Sciama effect.

1. Introduction

This work deals with the anisotropies of cosmic microwave background radiation (hereafter CMBR). This problem is exciting, because CMBR is the remnant radiation from the early stages of Universe, and hence informations concerning the history of Universe may be obtainable from these studies.

0323-0465/97

The paper has a theoretical character. It is mainly based on the measurement of nisotropy of CMBR done by COBE satellite (Smoot et al. 1992), and the measurement f Lauer and Postman (1994). In fact, it tries to give unusual explanations of these bservational data.

The article consists from two main parts. Section 2 describes the theoretical and he observational background of calculations, and it contains no new results. Section 3 iscusses a new possible explanation of given observational facts; it contains new results. ome of these results were already published (Mészáros and Molnár 1996); nevertheless, or the sake of completeness and also for the readers convenience, some parts of this rticle are again repeated.

In the whole work the signature of metrics $(+,-,-,-)$ is used. The most used uantities are: c is the velocity of light, G is the gravitational constant, H is the Iubble-parameter (which depends on time), $h = H/(100km\ sec^{-1}Mpc^{-1})$, z is the edshift, $\Omega = \rho/\rho_c$ is the ratio of density ρ to the critical density ρ_c, χ is the comoving istance (dimensionless), η is the conformal time (also dimensionless), k is - if it is not therwise stated - the scalar curvature of space ($k = 0, -1, +1$), $a(\eta)$ is the expansion unction, which is given by the solution of Friedmann equations in cosmology, $a_0 =$ $4/(3c^2))\pi G\rho a^3$ (ρ is the density) is a constant having the dimension of length.

2. Survey of facts

2.1. The history of CMBR

The way of the discovery of CMBR is exciting and instructive, and therefore it is riefly recapitulated here.

The first ones, who were thinking about the existence of an observable remnant rom the early stages of expansion of the Universe were Lemaitre (1931) and Tolman 1934). The search for the origin of the chemical elements caused people to consider the ossibility that matter passed through a phase dense and hot enough to have promoted uclear reactions that could have built up the elements. These reactions are very natural n the stars but they were existed through the early dense epochs of the Universe, oo. Chandrasekhar and Henrich (1942) gave an explanation. They had a conclusion hat if matter had relaxed into the thermal equilibrium at a density $\sim 10^7 gcm^{-3}$ and emperature $10^{10}K$, and if the abundances had been frozen in at that point because of he rapid expansion and cooling of the Universe, then the relative abundances of the ighter elements would agree reasonably well with cosmic abundances. But this theory id not work for the abundances of the heavier elements.

Gamow (1942, 1946) had an idea that the thermal equilibrium model is not so good, ecause the high mass density in the early Universe causes a rapid rate of expansion. Ie argued that an analysis of the element abundances that would have been left over om the early Universe, really involves a dynamic rather than equilibrium calculation, aking account of reaction rates in rapidly expanding and cooling material. Alpher, Bethe and Gamow (1948) and Alpher and Herman (1948) corrected some Gamow's naccuracy and they have shown that the present temperature of the Universe have to

be around 5 degrees over the absolute zero. This remnant energy, which left over from the Big Bang, now looks like a weak background blackbody radiation which is coming to us almost with the same intensity from all directions; this radiation should be isotropic. Unfortunately, these ideas were forgotten for ten years. Even earlier, Mc Kellar (1941) studied the excitation of the diatomic molecule CN in the diffuse molecular clouds, and obtained the conclusion that if the rotational excitation of the CN molecules were in statistical equilibrium with the background radiation field at the resonance for the transition between ground and first rotationally excited states, then the parameter T (from Boltzmann equation) would be the effective background radiation temperature. Mc Kellar found the temperature $T = 2.3K$.

After World War II the level of the radioastronomy reached the same level like the optical astronomy. In the early 1960s many of them had an idea that hydrogen formed before the star formation and perhaps originated in the hot Big Bang (Osterbock and Rogerson 1961; O'Dell, Peimbert and Kinman 1964). Already in 1946 Dicke constructed a radiotelescope and measured a microwave radiation from galaxies and obtained the conclusion that the temperature of this radiation have to be lower than $20K$ (Dicke et al. 1946). In sixties, when Dicke (1968) restored the idea of cosmic background radiation, Roll and Wilkinson (1966) began to built a modern Dicke radiometer for the identification of this radiation. In that time Peebles wrote a report about that thing which the experimentators need to find (Peebles 1965). The CMBR - if really left over from the Big Bang - have to be a blackbody radiation. Its temperature depends only on the whole energy of Big Bang and Peebles hoped that this radiation is enough strong now, too, and observable in the microwave part of spectrum.

The discovery of CMBR was done by Penzias and Wilson (1965) in Bell Laboratories in Holmdel (state New Jersey, USA).

2.2. The theory of Sachs-Wolfe effect

Shortly after the discovery of CMBR it was noted that the measurements of anisotropies of CMBR would have an essential importance for the theories of galaxy formation (Sachs and Wolfe 1967). After the publication of this paper a great effort was done both to develope the theory of these anisotropies and to measure these anisotropy terms.

In this part we shortly repeat the standard theory of the anisotropies of CMBR based on ideas of Sachs and Wolfe (1967).

Sachs and Wolfe (1967) mean that this effect consists from four parts. They are: A. Two Dopplerian parts; B. Impact on the present temperature of CMBR caused by the primordial density fluctuations at $z = 1000$; C. Additional shifts of the present temperature of CMBR due to the gravitational potential caused by the fluctuations existing between $z = 1000$ and $z = 0$.

A. The two Dopplerian parts.

The first two kinds of anisotropies are connected with the Doppler effect and the Lorentz transformation. In the Friedmann-Robertson-Walker (hereafter FRW) model of Universe CMBR can appear to be isotropic only in one frame called "preferred frame"

Peebles 1993). If an observer is moving relatively to this frame, he measures hotter radiation in the direction of motion because of Dopplerian shift. This means that CMBR acts as an aether, giving a local definition for preferred frame. It is consistent with the relativity, because the motion of an observer was defined relatively to the homogeneous sea of radiation. The observers motion should be caused by the departures from FRW metric around the observer.

From the Special Theory of Relativity we know that an observer moving with the velocity v relatively to the preferred frame - in which CMBR is isotropic - sees that the thermodynamic temperature T of the radiation is a function of direction. To the first order of (v/c) there is a dipole anisotropy, i.e. the temperature T is given by

$$T(\Theta) = T_0\left(1 + \frac{v}{c}\cos\Theta\right), \qquad (2.2.1)$$

where Θ is the angle between the line of sight and the direction of motion; T_0 is the temperature of blackbody radiation in the preferred frame.

Hence, for the eventually observed dipole anisotropy of CMBR given by equ.(2.2.1) there exists a standard interpretation. It assumes that this is the result of peculiar motion of Sun caused by the gravitational field of the irregularities in the mass distribution in the neighbourhood of Sun. If this is the case then the direction $\Theta = 0$ must be identical to the direction of the peculiar motion of Sun in Galaxy, Local Group and in higher structures of galaxies determined independently on CMBR by the detailed observational studies of the spatial distribution of galaxies.

The second Dopplerian shift is similar to this one, and reflects the motion of matter at $z = 1000$. In Sachs and Wolfe (1967) it is shown that this term is unimportant, and also here it will not be considered later.

Note still that these two terms are usually not considered to be the Sachs-Wolfe effect, although in the paper of Sachs and Wolfe (1967) these terms are clearly mentioned.

B. Relationship between present temperature of CMBR and the primordial density fluctuations at $z = 1000$.

When cosmologists are speaking about the Sachs-Wolfe effect, many times they consider only this part as the "Sachs-Wolfe effect". We briefly explain this phenomena.

Be given an overdensity at $z = 1000$. The density ρ is the Friedmannian one at $z = 1000$. Be given two light beams with blackbody spectrum coming to the Earth ($z = 0$) from the overdensity (beam (1)) and from an arbitrary other point at distance ($z = 1000$) (beam (2)), respectively (see Fig.1.). The relevant temperatures of light beams at the instant, when they arrive to the Earth, will be T and $(T+\delta T)$, respectively. The relationship between the density fluctuation $\delta\rho$ of the matter and the temperature fluctuation δT of CMBR is given by (see, e.g., Weinberg 1972; Molnár 1991; Börner 1993)

$$\frac{\delta\rho}{\rho} = 3\frac{\delta T}{T}. \qquad (2.2.2)$$

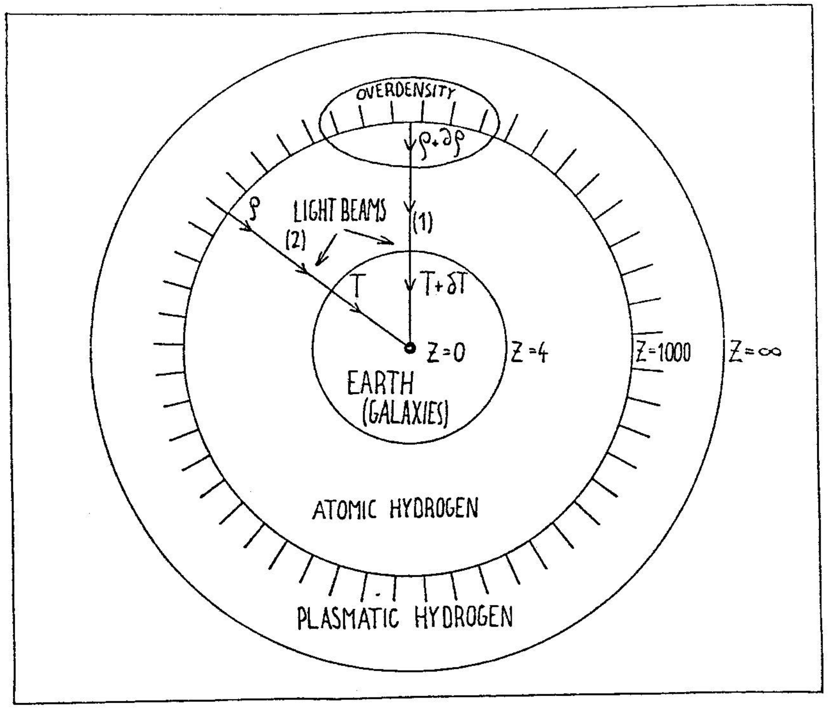


Fig.1. Illustration of the Sachs-Wolfe effect. The density ρ is Friedmannian at $z = 1000$, except for the overdense region with a density $(\rho + \delta\rho)$. Two blackbody radiations coming to the Earth ($z = 0$) from the overdensity (beam(1)) and from an arbitrary another point at redshift $z = 1000$ (beam(2)), respectively, will have different temperatures. The relevant temperatures of light beams at the instant, when they arrive to the Earth, will be T and $(T + \delta T)$, respectively.

The terms of higher order than linear were neglected. Equ.(2.2.2) is the sought relationship between the density fluctuations and the temperature of CMBR. Many times only this effect is called as Sachs-Wolfe one, and in what follows in fact we will do this, too.

C. Additional shifts due to the non-linearities between $z = 1000$ and $z = 0$

Although this effect was also considered by Sachs and Wolfe (1967), it is usual to call it as the Rees-Sciama effect. We also will do this, and in what follows the whole paper deals with this phenomena. Therefore, this question needs a more detailed discussion which is done in the following subsection.

2.3. The theory of Rees-Sciama effect

Rees and Sciama (1968) analysed in detail the changes of the period of light going hrough the large-scale structures existing between $z = 1000$ and $z = 0$. The calcula-ions were quite different than the earlier ones done by Sachs and Wolfe (1967). This ffect is one of the basic pillar of this work.

The key idea is the following. Be given two points A and B in FRW metric, and he photons move from A to B. This means that between B and A there will be a tandard FRW redshift. Then the basic idea is based on the fact that the Friedmannian edshift between A and B can be generalized as the product of the following three shifts:)opplerian (hereafter Dopp) shift, gravitational (or Schwarzschildian - hereafter Sch) hift and the third term is some special blueshift (hereafter blue), which is called "the lueshift of expansion". The formal mathematical expression of this fact is

$$\frac{T_B}{T_A}|_{FRW} = \frac{T_B}{T_A}|_{Dopp} \times \frac{T_B}{T_A}|_{Sch} \times \frac{T_B}{T_A}|_{blue} . \qquad (2.3.1)$$

Further details will be given in Section 3. Here we note only that this decomposition f the FRW shift into three components allows simple estimations of the additional shifts f photons of CMBR caused by, e.g., spherically symmetric clusters and voids between $= 0$ and $z = 1000$. (For example, if there is a void, then the "blueshift of expansion" imply does not exist; Mészáros 1994.) The literature dealing with this effect since the aper of Rees and Sciama (1968) is quite rare, and will continuously be discussed during he article. Here we mention only some of them.

Dyer (1976) confirmed the conclusion of Rees and Sciama (1968), and concluded that he additional blueshift of CMBR due to an expanding spherical supercluster increased ubically by the size. Kaiser (1982); Nottale (1984) and Dyer and Ip (1988) developed he calculations of Dyer (1976) concerning the case of spherical supercluster. At the last ears - trying to explain the COBE data by this effect (see Section 2.4) - several papers ccurred in the topic (Fang and Wu 1993; Arnau et al. 1993; Sáez, Arnau and Fullana 993; Wu and Fang 1994; Mészáros 1994; Tuluie and Laguna 1995; Nakao et al. 1995). n essence, the first four papers again discussed the model of spherical supercluster. Contrary them, the paper of Mészáros (1994) deals with an empty spherical void. The ubical increasing is again obtained by a new analytical calculation. The calculations re also confirmed by Nakao et al. (1995). Tuluie and Laguna (1995) did not consider he exact spherical symmetry, but in fact they again obtained the same size of effect nown previously for the case of spherical symmetry. This suggests - together with 'anek (1992) and Martínez-Gonzáles, Sanz and Silk (1994) - that the restriction to the pherical symmetry is not a loss of generality.

In this paper, in essence, we continue the discussion of Rees-Sciama effect. We evelop the considerations of Mészáros (1994) for the spherical model of a void and upercluster, and estimate the expected dipole and quadrupole anisotropies caused by uch a void and supercluster. These calculations will be applied to be interpretation of ne data of Lauer and Postman (1994) (see Section 2.4). In addition, the theoretical pper limit of the Rees-Sciama effect will also be discussed.

2.4. Observations

The dipole anisotropy was first convincingly seen by Conklin (1969) and Henry (1971). The present result is an average over many elegant experiments as summarized by Smoot et al. (1991, 1992). One has:

$$T = T_0 + \delta T \cos\omega, \quad T_0 = (2.735 \pm 0.060)K, \quad \delta T = (3.36 \pm 0.1)mK,$$

where ω is the angle between the direction defined by $l = (264.7 \pm 0.8)$, $b = (48.2 \pm 0.5)$, (l, b are the galactic coordinates), and the measured direction, and T is the measured temperature of the CMBR having the spectrum of blackbody radiation.

The most important measurements concerning the further anisotropy terms of CMBR give the upper limits $\delta T/T \simeq (1.5 - 4.0) \times 10^{-5}$ on angular scales $\sim (0.05 - 10)$ degrees (Weiss 1980; Uson and Wilkinson 1984; Readhead et al. 1987; Davies et al. 1987) or direct values $\delta T/T \simeq 1.1 \times 10^{-5}$ on angular scales ~ 10 degrees (Smoot et al. 1992; "COBE data"). Similar direct values on smaller scales were also measured at the last years (Ganga et al. 1993; Bennett et al. 1992; Bennett et al. 1994; Cheng et al. 1994; de Bernardis et al. 1994; Devlin et al. 1994; Dragovan et al. 1994; Gundersen et al. 1995; Ruhl et al. 1995; Lineweaver et al. 1995; Netterfield et al. 1995). Hence, one may claim that there are two well measured anisotropy terms of CMBR: the dipole anisotropy of order $\sim 10^{-3}$, and the high spherical harmonics of order $\sim 10^{-5}$ on typical angular scales $\sim (0.05 - 10)$ degrees.

The standard interpretation of the first term assumes that this is a Dopplerian shift caused by the peculiar motion of Local Group (Peebles 1993). In this case the direction of this motion must be identical to the direction of maximum of dipole anisotropy of CMBR. If this is correct (see Section 2.2), then the motion of the Solar System relatively to the preferred frame following from this dipole anisotropy of CMBR is given by the velocity $v_0 = (370 \pm 10) kms^{-1}$ toward the direction $\alpha = 11.2^h$, $\delta = -7^o$ or $l = (264.7 \pm 0.8)^o$, $b = (48.2 \pm 0.5)^o$, where α, δ are the ecliptic coordinates.

The second terms are caused either by the Sachs-Wolfe effect (Wright et al. 1992; White, Scott and Silk 1994; de Oliveira-Costa and Smoot 1995; White and Bunn 1995); or by the Rees-Sciama effect (Fang and Wu 1993; Arnau et al. 1993; Sáez et al. 1993; Mészáros 1994; Wu and Fang 1994; Tuluie and Laguna 1995).

The first comparison of the motion of Sun defined by the dipole anisotropy of CMBR relatively with the peculiar motion of Sun defined by the galaxies were done by Sciama (1967), who used de Vaucouleurs' and Peters (1968) earlier analysis of the motion of Sun in Galaxy, the motion of Galaxy in Local Group, and motion of Local Group within the Supercluster. A good general summarization of the history of these data are given by Börner (1993) and by Peebles (1993).

The peculiar motion of Sun is clearly caused by the cosmologically near objects. For these and for the farther objects there is a wide literature dealing with the observational data concerning the existence of large-scale structures. We briefly mention some of them. A clear evidence of large-scale structures (mainly the existence of superclusters) are given by Gregory et al. (1980); Davis et al. (1982); Huchra et al. (1983); Batuski and Burns (1985); de Lapparent et al. (1986); Tully (1986). Evidence of the voids are presented, e.g., by Joeveer et al. (1978); Kirschner et al (1981). Summing up these observations one may claim that both superclusters and voids of sizes $\sim (10 -$

00)$h^{-1}Mpc$ are doubtlessly existing in the realm of galaxies at, say, $z < 1$.

At the realm of quasars the situation should be similar. For example, this is sup-orted by the investigations of X-ray background radiation arising mainly at $z \sim (1-5)$ nd caused dominantly by quasars (Mészáros and Mészáros 1988; Bagoly, Mészáros and lészáros 1988; Bi, Mészáros and Mészáros 1991). It is even possible that some objects nay exist at distances $z \sim (5-20)$, because the newest studies of gamma-ray bursters uggest that a fraction of these objects may arise in these extreme redshifts (Mészáros nd Mészáros 1995; Horváth, Mészáros and Mészáros 1996; Mészáros and Mészáros 996). Hence, it is natural to expect that the structures observed for galaxies may be xtrapolated for larger redshifts, too, where the Rees-Sciama may also occur.

Surveying the observational data one necessarily must mention that recently Lauer nd Postman (1994) seriously queried the fulfilment of the standard Friedmannian mo-el of Universe. They announced that the direction of the maximum of the dipole nisotropy of CMBR and the direction of motion of Local Group with respect to the bell clusters are different. They concluded that either the huge system of Abell clus-rs moves toward the direction $l \simeq 220^o$ and $b \simeq 52^o$ with velocity $\simeq 689$ km/s, or ere is an intrinsic dipole anisotropy of CMBR of size $\simeq 2 \times 10^{-3}$ with the maximum this direction. At the first case the Friedmannian model is saved, but in the second ise, because the intrinsic anisotropy is assumed to be caused by a global inhomo-neity across the whole Hubble radius, the Friedmannian model of Universe should obably be rejected. Both possibilities are strange enough, because the Friedmannian se needs a never observed stream of the huge system of Abell clusters, and the second se - advocated at the last decade by several authors (Mészáros 1986; Mészáros and anýsek 1988; Gunn 1988; Paczyński and Piran 1990; Turner 1991) - is recently in oubt (Jaroszyński and Paczyński 1995). To be as complete as possible, two further ssibilities may occur, too. First, it is quite possible that the conclusion of Lauer and ostman (1994) is incorrect; see, for example, Riess, Press and Kirshner (1995) sup-orting this point of view (but see also Graham (1996) again confirming the conclusion Lauer and Postman (1994)). Second, the conclusion of Lauer and Postman (1994) is rrect, but neither of the two explanations are acceptable, and one has to search for her interpretation. The study od this last possibility is the main aim of this work. oncerning the Rees-Sciama effect it is essential to note that its value increases cubi-lly by the region causing this phenomenon; hence, for large structures this effect may en be bigger than $\sim 10^{-5}$. For example, at the first paper of this topic (Rees and iama 1968) it is especially noted that a hypothetical mass concentration of size $\simeq 750$ pc may cause a $\sim 10^{-3}$ departure from the Friedmannian value. This estimation is actly confirmed by the analytical calculation of Dyer (1976) and Mészáros (1994). nce, at least in principle, it seems that even the dipole anisotropy of CMBR may caused by this effect, because - from the observational point of view - structures of ler $\sim (500-1000)h^{-1}$ Mpc cannot be excluded.

Therefore, hypothetically, it is possible that the Friedmannian model is correct, but ere is no bulk flow of the system of Abell clusters, and, simultaneously, there is an rinsic dipole anisotropy of CMBR of size $\simeq 2 \times 10^{-3}$ with the maximum at direction 220^o and $b \simeq 52^o$. This anisotropy should be caused by a mass concentration of scale

$\simeq (500-1000)h^{-1}$ Mpc via the Rees-Sciama effect; the mass concentration should be at direction $l \simeq 220^o$ and $b \simeq 52^o$. This hypothetical case is highly similar to the standard Friedmannian interpretation of the Lauer and Postman's data. Nevertheless, there are three essential differences. First of all, no flow of the Abell clusters is needed. Second, there is a well defined size of mass concentration. Third, this mass concentration need not be near, because it may be in essence at any distance between us and the last scattering surface. Of course, no observations suggest that mass concentrations of sizes $\simeq (500-1000)h^{-1}$ Mpc exist. But, on the other hand, such structures cannot be rejected from the observational point of view. Hence, it is fully requested to discuss also the hypothetical possibility that the intrinsic $\simeq 10^{-3}$ dipole anisotropy of CMBR is caused by the Rees-Sciama effect. The investigation of this possibility is the key effort of this paper.

3. On the Rees-Sciama effect

3.1. Blueshift of expansion

In this Section the Chapter 2. of paper Mészáros (1994) is in essence recapitulated, into which some new author's considerations and further technical calculations are added.

Consider the situation that is illustrated on Fig.2.

Be given an expanding sphere with diameter AB and center O. Inside of this sphere there is a homogeneous non-relativistic matter expanding in accordance with the matter dominated spatially flat FRW model. The comoving radius is χ $(0 < \chi \ll 1)$. In the exterior there is a Schwarzschildian vacuum, where the points U and V are at constant distance $r_U = r_V$ from O. Points A and B move away from O, but U and V are at fixed distance from O.

Be sent a light from U across A O, B to V. The period of light will be T_U, T_A, T_O, T_B, T_V, respectively, at points U, A, O, B, V. It is known (Rees and Sciama 1968) that there will occur a blueshift (i.e. $(T_V/T_U) < 1$). Rees and Sciama (1968) explained - by physical arguments - the origin of this blueshift. In this paper the concrete value will be calculated up to the order $\sim \chi^3$. The terms $\chi^n; n \geq 4$; will always be omitted. If some calculations will be exact, this will be said.

Consider, first, the path of light from U to A. This path is in a Schwarzschildian vacuum and the point A moves to point U (sphere is expanding). Therefore, the final shift of light period will be

$$\frac{T_A}{T_U} = \frac{T_A}{T_U}|_{Dopp} \times \frac{T_A}{T_U}|_{Sch} \,. \tag{3.1.1}$$

The Dopplerian shift will be a blueshift (because point A moves toward point O with velocity v_A) and the gravitational shift will be a blueshift, too (because the light moves toward the central body). Hence it follows

$$\frac{T_A}{T_U} = \frac{(1-\frac{v_A}{c})^{1/2}}{(1+\frac{v_A}{c})^{1/2}} \times \frac{(1-\frac{r_g}{r_A})^{1/2}}{(1-\frac{r_g}{r_U})^{1/2}}. \tag{3.1.2}$$

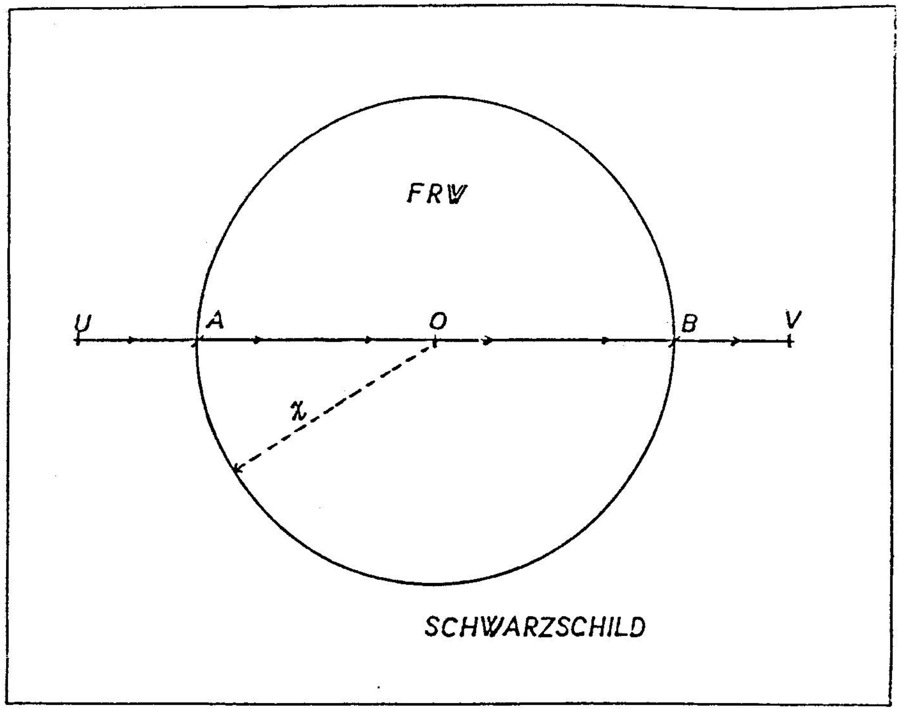


Fig.2. Illustration of the path of light that crosses an expanding Friedmannian sphere surrounded by a vacuum. The light moves from U to V. "FRW" denotes the fact that in the sphere with comoving radius χ there is an FRW metric; "Schwarzschild" denotes the fact that in the exterior there is a Schwarzschildian vacuum.

Here $r_g = (2GM/c^2) = 2a_0\chi^3$ is the gravitational radius (for the flat model this is exact), where M is the mass of sphere; $a_0 = (4\pi G\rho a^3(\eta))/(3c^2)$ is a constant length; $r_A = a(\eta)\chi$ is the distance of point A from O at the time instant $t(\eta)$, when the light misses A (η is the conformal time for this instant); $a(\eta)$ is the expansion function; $v_A = (da/dt)|_\eta \chi$ is the velocity of point A at time $t(\eta)$. For the flat model η is a free parameter and we can choose it arbitrarily. Without loss of generality we choose $\eta = 1$, and in this case $a_0 = 2c/H$. (For the sake of precision, it is necessary to note that H is the value of Hubble parameter at the time instant, when the photon misses the point A.) The right-hand-side of equ.(3.1.2) is a product of a Dopplerian blueshift (A moves toward to U with velocity v_A); and of a gravitational blueshift (light moves toward the central body; $r_U > r_A$).

Consider, second, the path from A to B. Here one has a standard Friedmannian redshift. Because the light misses B at the conformal time $(1+2\chi)$, one obtains

$$\frac{T_B}{T_A}|_{FRW} = \frac{a(1+2\chi)}{a(1)} = 1 + 4\chi + 4\chi^2, \qquad (3.1.3)$$

because $a(\eta) = a_0\eta^2$.

For the shift of light between B and V we receive analogically

$$\frac{T_V}{T_B} = \frac{T_V}{T_B}|_{Dopp} \times \frac{T_V}{T_B}|_{Sch}, \qquad (3.1.4)$$

and hence

$$\frac{T_V}{T_B} = \frac{(1-\frac{v_B}{c})^{1/2}}{(1+\frac{v_B}{c})^{1/2}} \times \frac{(1-\frac{r_g}{r_U})^{1/2}}{(1-\frac{r_g}{r_B})^{1/2}}. \qquad (3.1.5)$$

One has $r_B = a(1+2\chi)\chi$ and $v_B = ((da/dt)|_{(1+2\chi)})\chi$. The Dopplerian shift is again a blueshift, because point B also approaches to V, but the gravitational shift will be red, because the light is going up in the gravitational field of sphere.

Finally, we express the whole shift as the product of the partial shifts

$$\frac{T_V}{T_U}|_{blue} = \frac{T_A}{T_U}|_{Dopp} \times \frac{T_A}{T_U}|_{Sch} \times \frac{T_B}{T_A}|_{FRW} \times \frac{T_V}{T_B}|_{Dopp} \times \frac{T_V}{T_B}|_{Sch}, \qquad (3.1.6)$$

or

$$\frac{T_V}{T_U}|_{blue} = \left(\frac{T_B}{T_A}|_{FRW}\right) \times \left(\frac{T_A}{T_U}|_{Dopp} \times \frac{T_V}{T_B}|_{Dopp}\right) \times \left(\frac{T_A}{T_U}|_{Sch} \times \frac{T_V}{T_B}|_{Sch}\right). \qquad (3.1.7)$$

Now we calculate these partial shifts in the brackets.

A. The Friedmannian shift is given by equ.(3.1.3), where we used Friedmannian solution for flat model (case $k = 0$).

B. Next we calculate the term in the second bracket of equ.(3.1.7), which is given by

$$\frac{T_A}{T_U}|_{Dopp} \times \frac{T_V}{T_B}|_{Dopp} = \frac{(1-\frac{v_A}{c})^{1/2}}{(1+\frac{v_A}{c})^{1/2}} \times \frac{(1-\frac{v_B}{c})^{1/2}}{(1+\frac{v_B}{c})^{1/2}}. \qquad (3.1.8)$$

We have

$$r_{A,B} = a(\eta_{A,B})\chi, \qquad (3.1.9)$$

and

$$v_{A,B} = dr_{A,B}/dt = (da(\eta_{A,B})/dt)\chi = c\frac{a'(\eta_{A,B})}{a(\eta_{A,B})}\chi, \qquad (3.1.10)$$

where $a'(\eta) = da(\eta)/d\eta$. Now, using equ.(3.1.10), after a short calculation we obtain

$$\beta_A = \frac{v_A}{c} = 2\chi, \quad \beta_B = \frac{v_B}{c} = \frac{2\chi}{1+2\chi}. \qquad (3.1.11)$$

We substitute these velocities into the equ.(3.1.8), and after some calculation we obtain

$$\frac{T_A}{T_V}|_{Dopp} \times \frac{T_V}{T_B}|_{Dopp} = 1 - 4\chi + 12\chi^2 - 40\chi^3 + O(\chi^4), \qquad (3.1.12)$$

where $O(\chi^4)$ means the sum of terms $\chi^n; n \geq 4$. Equ.(3.1.1.12) defines a blueshift.

C. Third, we calculate the gravitational part in equ.(3.1.7) (terms in third bracket). Analogically to the Dopplerian case, we obtain

$$\frac{T_A}{T_U}|_{Sch} \times \frac{T_V}{T_B}|_{Sch} = \frac{(1-\frac{r_g}{r_A})^{1/2}}{(1-\frac{r_g}{r_U})^{1/2}} \times \frac{(1-\frac{r_g}{r_V})^{1/2}}{(1-\frac{r_g}{r_B})^{1/2}}, \tag{3.1.13}$$

where U, V are fixed points and $r_U = r_V$. Therefore the form of equ.(3.1.13) will be

$$\frac{T_A}{T_U}|_{Sch} \times \frac{T_V}{T_B}|_{Sch} = \frac{(1-\frac{r_g}{r_A})^{1/2}}{(1-\frac{r_g}{r_B})^{1/2}}, \tag{3.1.14}$$

where

$$r_A = a(1)\chi = \frac{a_0}{2}\chi, \quad r_B = a(1+2\chi)\chi = \frac{a_0}{2}(1+2\chi)^2\chi. \tag{3.1.15}$$

With these r_A, r_B, r_g we calculate (3.1.14) and obtain

$$\frac{T_A}{T_U}|_{Sch} \times \frac{T_V}{T_B}|_{Sch} = 1 - 8\chi^3 + O(\chi^4). \tag{3.1.16}$$

This is again a blueshift.

Finally, we can write the total shift of light between the points U and V as the product of these three (Friedmannian, Dopplerian and Schwarzschildian) shifts according to equ.(3.1.7). We obtain the blueshift

$$\frac{T_V}{T_U}|_{blue} = 1 - 16\chi^3 + O(\chi^4). \tag{3.1.17}$$

This is the blueshift of expansion. The physical meaning is explained by Rees and Sciama (1968).

We transform the result (3.1.17) into physical variables using the physical radius y instead of comoving radius χ.

Be given the Friedmannian solution for the flat model. From these equations it follows that $da/dt = 2c/\eta$. But we also know that $(da/dt)/a = H$ and therefore we obtain the relationship $a = (2c/H)\eta^2$. Because $y = a\chi$, and one may take $q = 1$, we obtain the identity

$$\chi = \frac{H}{2c}y. \tag{3.1.18}$$

Substituting equ.(3.1.18) into equ.(3.1.17) get

$$\frac{T_V}{T_U}|_{blue} = 1 - 16\chi^3 = 1 - \frac{2H^3}{c^3}y^3. \tag{3.1.19}$$

We get the effect of third order, and one may say that the blueshift of expansion increases by the cube of size.

Repeating these calculation to the case of the hyperbolic model the whole calculation will be analogical to the case of flat model. Only some relationships should be changed. For example, the expansion function has form (cf. Weinberg 1972)

$$a(\eta) = a_0(\cosh\eta - 1), \quad \cosh\eta = \frac{2}{\Omega} - 1, \tag{3.1.20}$$

where again $a_0 = (4/(3c^2))\pi G\rho$, and $\Omega = \rho/\rho_c$ (ρ is the density and ρ_c is the critical density). Then the calculation of the three different shifts (Friedmannian, Dopplerian and gravitational) of the light period for the case of hyperbolic model is practically identical to the case of flat model; hence no details are needed here. The total shift, which is the product of the partial shifts (Friedmannian, Dopplerian and Schwarzschildian shift), is given by

$$\frac{T_V}{T_U}|_{blue} = 1 - \frac{2}{3}\frac{1+2\Omega}{(1-\Omega)^{3/2}}\chi^3 + O(\chi^4). \tag{3.1.21}$$

This is the final result in the case of hyperbolic model with $\Omega < 1$.

To transform the result (3.1.21) into physical variables one has to use the physical radius y instead of comoving radius χ). One has: $da/dt = c(\sinh\eta)(\cosh\eta - 1)^{-1} = c(1-\Omega)^{-1/2}$. Because $(da/dt)/a = H$, we obtain the relationship $a = cH^{-1}(1-\Omega)^{-1/2}$. Because $y = a\chi$, we finally obtain the identity

$$\chi = \frac{H}{c}(1-\Omega)^{1/2}y. \tag{3.1.22}$$

Now we substitute equ.(3.1.22) into equ.(3.1.21) and we get

$$\frac{T_V}{T_U}|_{blue} = 1 - \frac{2}{3}\frac{1+2\Omega}{(1-\Omega)^{3/2}}\chi^3 + O(\chi^4) = 1 - \frac{2H^3}{3c^3}(1+2\Omega)y^3 + O(y^4). \tag{3.1.23}$$

We can see that this result for limit $\Omega = 1$ gives equ.(3.1.19).

In case of the elliptic model the whole calculation is analogical to the case of flat and hyperbolic model. As the result one obtains that equ.(3.1.23) holds for any $\Omega > 0$.

3.2. Correctness of calculation

In this Section we show that the use of specially relativistic Doppler formula in the neighbourhood of a big mass sphere is correct. This is still a not published new result and is in fact an addendum to Mészáros' calculations (Mészáros 1994).

Consider the situation illustrated on Fig.3. This is a space-time diagram that describes the behaviour of two null world-lines in the Schwarzschildian field.

Be given two fixed points U and A' in the Schwarzschildian vacuum. The point A is on the surface of expanding sphere, and moves toward the point A'. Be sent a light beam from point U toward the point A, and when it reaches the point A, let the distance between points A and A' is s. The light moves on a null world-line. After a time interval T_U be sent a second light beam from point U toward A. It moves also on a null world-line. Let it be arriving into A exactly in that moment, when this point misses the point A'. One has $T_{A'} = T_U\sqrt{1 - r_g/r'_A}$, where $T_{A'}$ is the period seen by observer at A'. We can also write $s = v_A T_{A'}$, where v_A is the velocity of the point A toward A'. (This distance is measured by observer at A'.) We can write

$$s = v_A T_{A'} = v_a T_U\sqrt{1 - r_g/r_{A'}} < cT_U = \lambda, \tag{3.2.1}$$

because $r_g/r'_A < 1$, $v_A < c$, and $\sqrt{1 - r_g/r'_A} < 1$, where λ is the wavelength. We obtain the result $s < \lambda_0$. Therefore, for us it is enough to test the change of the

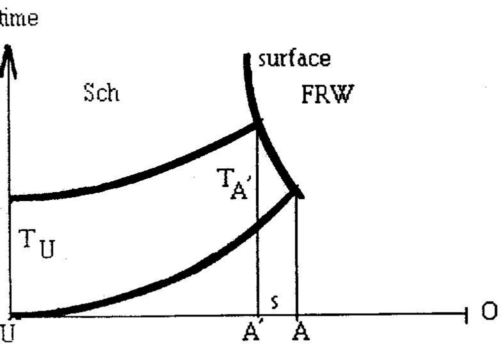


Fig.3. Space-time diagram illustrating the correctness of the formulas of Special Theory of Relativity; for more explanations see the text of Section 3.2.

chwarzschildian metric for distance not bigger than λ on the surface of the mass phere.

In this distance range near the surface of the mass sphere the metric is Minkowskian ith a very high accuracy. This precision may be estimated as follows.

Our initial formula is the expression for Schwarzschildian metric in isotropic Caresian coordinates. This is the following (see, e.g., Kuchař 1968, Section 3.2, page 16)

$$ds^2 = \left(\frac{1-\frac{r_g}{4r}}{1+\frac{r_g}{4r}}\right)^2 c^2dt^2 - (1+\frac{r_g}{4r})^4(dx^2+dy^2+dz^2), \qquad (3.2.2)$$

here $r = \sqrt{x^2+y^2+z^2}$. Here unusually we use the Cartesian coordinates, because n these ones we can see better the deviations from Minkowskian metric. Now we have o find a coordinate transformation, which transforms metric (3.2.2) into Minkowskian etric around the point $(x_0, 0, 0)$, where $x_0 \gg r_g > 0$. The basic criterion for the Minkowskiness" of a given metric is that at the given point the coefficients in the etric should be exactly Minkowskian ones and in the neighbourhood of this points the etric should be nearly Minkowskian.

One of the possible transformations, which fulfils this expectation, is the following:

$$x' = q(x-x_0), \;\; y' = qy, \;\; z' = qz, \;\; t' = Qt, \qquad (3.2.3)$$

here

$$q = \left(1+\frac{r_g}{4x_0}\right)^2, \;\; Q = \frac{1-\frac{r_g}{4x_0}}{1+\frac{r_g}{4x_0}}. \qquad (3.2.4)$$

hen we obtain

$$ds^2 = B_1(x',y',z')c^2dt'^2 - B_2(x',y'z')(dx'^2+dy'^2+dz'^2), \qquad (3.2.5)$$

here

$$B_1(x',y',z') = Q^{-2}\left(1-\frac{r_g}{4\sqrt{(x'/q+x_0)^2+(y'^2+z'^2)/q^2}}\right)^2 \times$$

$$\left(1+\frac{r_g}{4\sqrt{(x'/q+x_0)^2+(y'^2+z'^2)/q^2}}\right)^{-2} \qquad (3.2.6)$$

and

$$B_2 = q^{-2}\left(1+\frac{r_g}{4\sqrt{(x'/q+x_0)^2+(y'^2+z'^2)/q^2}}\right)^4. \qquad (3.2.7)$$

One has: $B_1(x_0,0,0) = B_2(x_0,0,0) = 1$, i.e. at point $(x_0,0,0)$ (in new coordinates at point $x' = y' = z' = 0$) the metric is exactly Minkowskian. Without loss of generality we restrict ourselves to $y' = z' = 0$, and $|x'| \leq \lambda$. For these values one may write

$$B_1 = 1 + \frac{r_g}{x_0}\frac{x'}{x_0} + O((x'/x_0)^2), \qquad (3.2.8)$$

$$B_2 = 1 - \frac{r_g}{x_0}\frac{x'}{x_0} + O((x'/x_0)^2), \qquad (3.2.9)$$

where x_0 is big comparing with r_g, and where $O((x'/x_0)^2)$ means terms of order $(x'/x_0)^2$.

For illustration we choose the case $x_0 = 10r_g$, and therefore $r_g/x_0 = 0.1$. Obviously $x' \simeq \lambda$ is not bigger than, say, 10^6 cm, and the minimal characteristic cosmological distance is $x_0 \sim 1$ Mpc $\sim 10^{17}$ cm. This means that $x'/x_0 \leq 10^{-11}$. Hence, equs.(3.2.8) and (3.2.9) show that the departure of B_1 and B_2 from one is maximally of order $\sim 10^{-12}$. This is the maximal departure from the Minkowskian metric around the point $(x_0,0,0)$.

We see that the use of the Doppler formula in our case is - up to the precision $\sim 10^{-12}$ - correct.

3.3. The profile of the blueshift of expansion

In Mészáros (1994) there is given the blueshift of expansion for lights crossing the center. In Sections 3.3-3.7. the key ideas of this paper are recapitulated and further calculations are added.

Consider the situation illustrated on Fig.4. Similarly to Mészáros (1994), be given an expanding sphere with diameter AB and center O. Inside of this sphere there is a homogeneous nonrelativistic matter expanding in accordance with the matter-dominated Friedmann-Robertson-Walker (FRW) model determined unambiguously by the Hubble-parameter H and by the ratio of density to the critical one Ω. The comoving radius is χ $(0 < \chi \ll 1)$. In the exterior there is a Schwarzschildian vacuum, where the points U, V and H are at constant distances from the center ($r_U = r_V < r_H$). On the other hand, points A, B, C, D and E move away from O.

Let a photon be sent from U to V and further to H. The periods of photon at these points will be T_U, T_V, T_H, respectively. The purpose of this Section is to calculate $[T_V/T_U]$ and $[T_H/T_U]$, respectively, for any $0 \leq \alpha \leq \frac{\pi}{2}$. In Mészáros (1994) and also in Section 3.1 only the special case $\alpha = 0$ was considered.

Trivially, one has

$$\frac{T_V}{T_U} = \frac{T_C}{T_U} \times \frac{T_D}{T_C} \times \frac{T_V}{T_D}, \qquad (3.3.1)$$

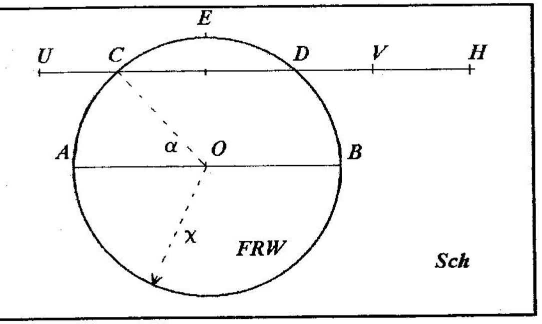


Fig.4. Illustration of the blueshift of expansion due to an expanding sphere with comoving radius χ. The photon moves from U to H across C, D and V. The line OE is perpendicular to CD and AB.

vhere T_C and T_D are the periods of photon, when it crosses the points C and D, espectively.

We assume that photon crosses the line OE at the conformal time $(\eta_A + \chi)$. Then it nisses the point C at the conformal time $\eta_C = (\eta_A + \chi(1 - \cos\alpha))$. At this instant the istance of point C from O is $r_C = a(\eta_C)\chi$, and the velocity of C in direction OC is $v_C =$ $^{-1}(\eta_C)(da(\eta)/d\eta)|_{\eta_C} c\chi$, where c is the velocity of light, and where $a(\eta) = a_0[dQ(\eta)/d\eta]$ s the expansion function; a_0 is a non-zero constant having the dimension of length, and ither $Q(\eta) = (\sinh\eta - \eta)$ for the hyperbolic FRW model, or $Q(\eta) = (\eta^3/6)$ for the arabolic model, or $Q(\eta) = (\eta - \sin\eta)$ for the elliptic model.

$[T_C/T_U]$ will be defined as a product of a general Dopplerian shift, and of a Schwarzchildian blueshift $(r_C < r_U)$. It follow

$$\frac{T_C}{T_U} = \frac{T_C}{T_U}|_{Dopp} \times \frac{T_C}{T_U}|_{Sch} = \frac{T_C}{T_U}|_{Dopp} \times \frac{(1 - \frac{r_g}{r_C})^{1/2}}{(1 - \frac{r_g}{r_U})^{1/2}}, \qquad (3.3.2)$$

vhere $r_g = 2a_0\chi^3$ is the gravitational radius (see Mészáros (1994)). The concrete value f Dopplerian shift need not be specified (see equ.(3.3.6)).

Between points C and D there is a Friedmannian redshift; at point D the light will e at the conformal time $\eta_D = (\eta_C + 2\chi\cos\alpha) = (\eta_A + \chi(1 + \cos\alpha))$. Hence, one obtains

$$\frac{T_D}{T_C} = \frac{a(\eta_D)}{a(\eta_C)} = \frac{T_D}{T_C}|_{blue} \times \frac{T_D}{T_C}|_{Dopp} \times \frac{T_D}{T_C}|_{Sch}, \qquad (3.3.3)$$

vhere

$$\frac{T_D}{T_C}|_{Sch} = \frac{(1 - \frac{r'_g}{r'_D})^{1/2}}{(1 - \frac{r'_g}{r'_C})^{1/2}}, \qquad (3.3.4)$$

nd $r'_g = r_g\cos^3\alpha$, $r'_{C,D} = r_{C,D}\cos\alpha$. This means that the Friedmannian redshift is lecomposed into its three components (Rees and Sciama 1968; Mészáros 1994); applying he results of Mészáros (1994) one uses the fact that between the points C and D the

comoving length is $2\chi\cos\alpha$. Then, obviously, the "blue" term in equ.(3.3.3) is $\cos^3\alpha$ times smaller than the relevant blueshift of expansion given for $\alpha = 0$ by Mészáros (1994). The Dopplerian part again need not be specified (see equ.(3.3.6)).

Obviously, $[T_V/T_D]$ will be defined by the product of a Dopplerian shift, and of a Schwarzschildian redshift $(r_D < r_V)$. Hence, it follows

$$\frac{T_V}{T_D} = \frac{T_V}{T_D}|_{Dopp} \times \frac{T_V}{T_D}|_{Sch} = \frac{T_V}{T_D}|_{Dopp} \times \frac{(1 - \frac{r_g}{r_V})^{1/2}}{(1 - \frac{r_g}{r_D})^{1/2}}. \qquad (3.3.5)$$

One must have

$$\frac{T_C}{T_U}|_{Dopp} \times \frac{T_D}{T_C}|_{Dopp} \times \frac{T_V}{T_D}|_{Dopp} = 1. \qquad (3.3.6)$$

This relation must be fulfilled, because between the points V and U there should be no Dopplerian shift (V does not move relatively to U).

Under this condition one obtains from equs.(3.3.1 - 3.3.6)

$$\frac{T_V}{T_U} = \frac{T_V}{T_U}|_{blue} = \frac{T_C}{T_U}|_{Sch} \times \frac{T_D}{T_C}|_{Sch} \times \frac{T_D}{T_C}|_{blue} \times \frac{T_V}{T_D}|_{Sch}. \qquad (3.3.7)$$

Calculating the concrete value it is essential to note that

$$\frac{T_C}{T_U}|_{Sch} \times \frac{T_D}{T_C}|_{Sch} \times \frac{T_V}{T_D}|_{Sch} = \frac{(1 - \frac{r_g}{r_C})^{1/2}}{(1 - \frac{r_g}{r_D})^{1/2}} \frac{(1 - \frac{r'_g}{r'_D})^{1/2}}{(1 - \frac{r'_g}{r'_C})^{1/2}} \neq 1 \qquad (3.3.8)$$

in the general case. After a straightforward calculation one obtains

$$\frac{T_V}{T_U}|_{blue} = 1 - \frac{2c^3y^3}{H^3}\cos\alpha\Big(\frac{\Omega}{2}\sin^2\alpha + \frac{(1 + 2\Omega)}{3}\cos^2\alpha\Big), \qquad (3.3.9)$$

where $y = a(\eta_A)\chi$.

This relation reproduces - for the special case $\alpha = 0$ - the result of Mészáros (1994). On the other hand, for the special case $\alpha = \pi/2$ it gives no blueshift; from the physical point of view a quite correct result. Equ.(3.3.9) defines the profile of the blueshift of expansion (i.e. the dependence of the blueshift on α).

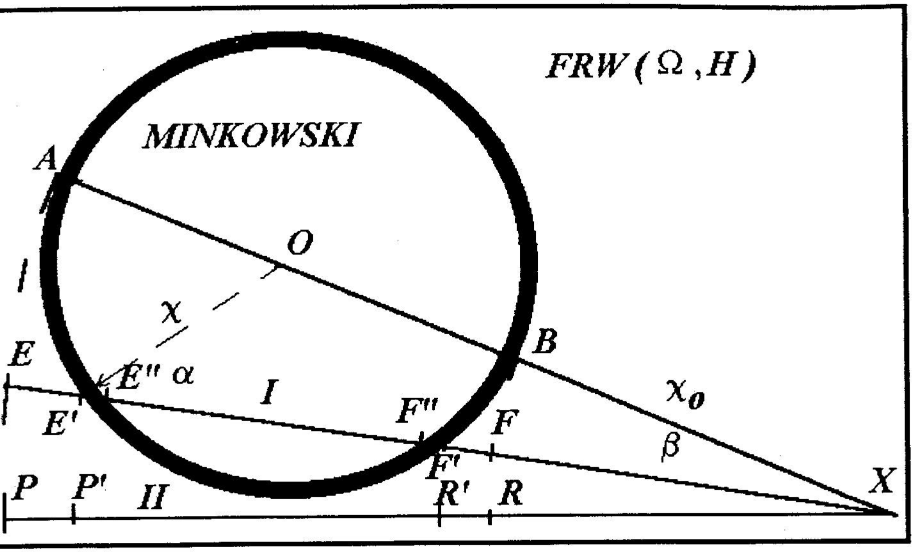
Between the points H and V there is only a further Schwarzschildian redshift. Hence it follows

$$\frac{T_H}{T_U} = \frac{T_V}{T_U}|_{blue} \times \frac{T_H}{T_V}|_{Sch} = \frac{T_V}{T_U}|_{blue} \times \frac{(1 - \frac{r_g}{r_H})^{1/2}}{(1 - \frac{r_g}{r_U})^{1/2}}. \qquad (3.3.10)$$

3.4. The profile of additional redshift due to a void

Similarly to Mészáros (1994) consider again the model of spherical void illustrated on Fig.5. Its actual and comoving radii are y and χ, respectively.

Outside of void there is a matter expanding in accordance with FRW model; in it H is the Hubble parameter and Ω is the ratio of density to the critical density. These two



ig.5. Illustration of the non-Friedmannian shift due to a void. The interior is a Minkowskian acuum; on the edge of void there is matter with a negligible thickness. Outside of the void here is Friedmannian exterior. Two photons (I and II) are sent from E and P, respectively, o X. The significance of other points and angles is explained in Section 3.4.

arameters define unambiguously the Friedmannian exterior; χ defines unambiguously he size of void. On the edge of void there is a transparent layer of matter with a egligible small thickness; if this matter were distributed homogeneously in the sphere vith diameter AB, then the density would be identical to the density of exterior. Inside f void there is a Minkowskian vacuum.

Consider two photons arriving into X at the same time. Point X is at the comoving istance $\chi_0 \geq 0$ from the edge of void. (Note that it may also be $\chi_0 = 0$.) Photon I rosses the void; photon II does not cross the void. Photon II will have a Friedmannian hift, and is taken for comparison. Photon I will have a greater redshift than II, and ts value is known (Mészáros 1994) for $\alpha = \beta = 0$. Here we find its value for arbitrary $\leq \alpha \leq (\pi/2)$. The period of photon I arriving into X is T_{XI}, the period of photon II t X will be T_{XII}. One also has $T_E = T_P$.

Immediately it follows

$$\left(\frac{T_{XI}}{T_E} \Big/ \frac{T_{XII}}{T_P}\right) = \left(\frac{T_{F'}}{T_{E'}} \Big/ \frac{T_{R'}}{T_{P'}}\right) = \left(\frac{T_{F'}}{T_{E'}}|_{blue}\right)^{-1}. \tag{3.4.1}$$

The first step in this relation is obvious; between points E' and E (X and F; F and ') there is the same Friedmannian shift as between P' and P (X and R; R and R'). The second step follows from the following consideration. Between E" and E' (F' and ") there is a pure Schwarzschildian shift, because between them there is a constant

distance. On the other hand, between E" and F" there is a pure Dopplerian shift, because these two points move in fact in a Minkowskian vacuum. Hence, between F' and E' the shift is a product of Dopplerian and Schwarzschildian ones. Thus, one obtains (see equ.(3.3.9))

$$\left(\frac{T_F}{T_{\bar{E}}}|_{blue}\right)^{-1} = 1 + \frac{2c^3y^3}{H^3}\cos\alpha\left(\frac{\Omega}{2}\sin^2\alpha + \frac{(1+2\Omega)}{3}\cos^2\alpha\right) = 1 + \frac{2c^3y^3}{H^3} f(\alpha,\Omega), \tag{3.4.2}$$

where

$$\sin\beta = \frac{\chi}{\chi + \chi_0}\sin\alpha = k\sin\alpha, \tag{3.4.3}$$

and where y_0 is the actual distance between X and B. Of course, in function $f(\alpha, \Omega)$ the dependence on α may be changed into a dependence on β and on $k = y/(y_0 + y)$. Then the function $f(\beta, k, \Omega)$ define the profile of the non-Friedmannian shift due to a spherical empty void for $0 \leq \beta \leq \tilde{\beta}$, where $\sin\tilde{\beta} = k \leq 1$. For $\tilde{\beta} \leq \beta \leq \pi$ the shift is clearly Friedmannian.

3.5. The profile due to a cluster

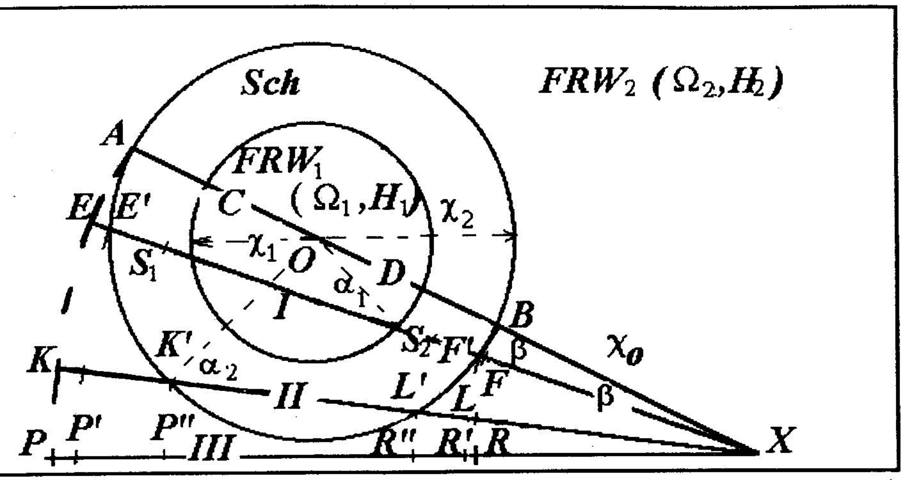
Consider the model of spherical cluster illustrated on Fig.6. There is an inner expanding Friedmannian region (FRW$_1$; "interior") surrounded by a Schwarzschildian vacuum. All this is immersed into a second Friedmannian region (FRW$_2$; "exterior"). Then the Hubble parameters and the ratios of densities to the critical ones are H_1, Ω_1 and H_2, Ω_2, respectively. The comoving radii of interior and exterior are χ_1 and χ_2, respectively.

These six quantities define unambiguously the model of cluster. They are not independent. One of these parameters (say χ_1) is defined by the remaining five ones, because the total mass of interior must be identical to the mass of a sphere with radius OA in the FRW$_2$ metric (Einstein and Strauss 1945). This means that one must have

$$a_1^3\chi_1^3\Omega_1 H_1^2 = a_2^3\chi_2^3\Omega_2 H_2^2, \tag{3.5.1}$$

where a_1 (a_2) is the expansion function defined unambiguously by H_1 and Ω_1 (by H_2 and Ω_2) by the standard relations of Friedmannian cosmological models. (E.g., if $\Omega_2 < 1$, then $a_2 = c/(H_2\sqrt{1-\Omega_2})$.)

The cluster has arisen at a given time at past. This question is discussed in detail by Dyer and Ip (1988), and therefore only a short recapitulation of some ideas of this paper is presented here. There are in fact two possibilities: the cluster is either "synchronous" or "asynchronous". The pairs H_1, Ω_1 and H_2, Ω_2, respectively, define unambiguously the age of interior t_1 and exterior t_2, respectively, by the standard relations of Friedmannian cosmology. (For example, if $\Omega_1 = 1$, then $t_1 = 2/(3H_1)$. Or, for example, if $\Omega_2 < 1$, then $t_2 = H_2^{-1}[(1-\Omega_2)^{-1} - \Omega_2(2(1-\Omega_2)^{3/2})^{-1}\text{arccosh}((2/\Omega_2)-1)]$.) For the synchronous case one has $t_1 = t_2$. This means that here the four quantities $H_1, H_2, \Omega_1, \Omega_2$ are not independent, and one of them (say H_1) is defined by the remaining three ones unambiguously. On the other hand, for the asynchronous model one should have $t_1 - t_2 = t_{lag} > 0$; formally the birth of interior is before the birth of exterior.



;.6. Illustration of the non-Friedmannian redshift due to a spherical supercluster. The erior of expanding sphere with a Friedmannian metric is immersed into a Schwarzschildian cuum. These two parts together give the region of supercluster. Beyond this region there is Friedmannian exterior. Three photons (I, II, III) are sent from E, K and P, respectively, to The significance of other points and angles is explained in Section 3.5.

fact, the physical birth of cluster is after the initial singularity. (This is quite possible, deed; see, Bonnor and Chamorro (1990) and Mészáros (1991).) Nevertheless, once the ncrete value of t_{lag} is introduced by physical arguments, the equation $t_1 - t_2 = t_{lag}$ again solvable unambiguously similarly to the synchronous case. Hence, in any case, e four quantities $H_1, H_2, \Omega_1, \Omega_2$ are not independent, and one of them is defined by e remaining three ones unambiguously. The concrete formula of this dependence can written down in the general case. (For example, let it be $\Omega_2 < 1 < \Omega_1$, and $t_{lag} > 0$. hen one will have: $H_2^{-1}[(1-\Omega_2)^{-1} - \Omega_2(2(1-\Omega_2)^{3/2})^{-1}\text{arccosh}((2/\Omega_2)-1)] + t_{lag} =$ $_1^{-1}[(\Omega_1-1)^{-1} - \Omega_1(2(\Omega_1-1)^{3/2})^{-1}\arccos((2/\Omega_1)-1)]$.) As is discussed by Dyer and (1988), several combinations are possible; both Ω_1 and Ω_2 may be either smaller, equal or bigger than unity; the interior may also be collapsing for $\Omega_1 > 1$; any case ould be considered twice, because it can be either synchronous or asynchronous. In dition, some combinations are excluded. (For example, if $\Omega_1 = \Omega_2 = 1$, and $t_{lag} = 0$, en one should have $H_1 = H_2$, and thus no cluster is existing.) In order to keep the nerality and, simultaneously, to avoid the listing and detailed discussion of several mbinations, the best procedure is simply to calculate the profile of Rees-Sciama effect any four $H_1, H_2, \Omega_1, \Omega_2$ quantities. In what follows this is done.

Let three photons (denoted as photons I, II and III) be sent to X from points E, K d P. The periods are the same at the instant of emission, and they are also sent at the me time instant (defined by the conformal time η_A). This means $T_E = T_K = T_P$. The oton I (II, III) - arriving into the point X - will have the period T_{XI} (T_{XII}, T_{XIII}).

For photon II one has

$$\left(\frac{T_{XII}}{T_K} \Big/ \frac{T_{XIII}}{T_P}\right) = \left(\frac{T_{L'}}{T_{K'}} \Big/ \frac{T_{R''}}{T_{P''}}\right)$$

$$= 1 + \frac{2c^3 y_2^3}{H_2^3}\cos\alpha_2\left(\frac{\Omega_2}{2}\sin^2\alpha_2 + \frac{(1+2\Omega_2)}{3}\cos^2\alpha_2\right), \tag{3.5.2}$$

where y_2 is the actual radius AO at conformal time η_A. Between the points F'and E' the shift is identical to the shift of photon I in the case of void. This is given by the fact that the motion of these points does not depend on the distribution of matter inside the sphere; we again have a shift given by the product of Schwarzschildian and Dopplerian ones. Between the angles the relation is

$$\sin\beta = \frac{\chi_2}{\chi_2 + \chi_0}\sin\alpha_2 = \frac{y_2}{y_2 + y_0}\sin\alpha_2. \tag{3.5.3}$$

Here the non-Friedmannian shift is identical to the case of void; it is clear from equs.(3.4.2 - 3.4.3).

For photon I one may proceed as follows. Consider two auxiliary points S_1 and S_2 being in Schwarzschildian vacuum at constant distances r_{S_1} and r_{S_2} from O. Then one has

$$\frac{T_{F'}}{T_{E'}} = \frac{T_{S_1}}{T_{E'}} \times \frac{T_{S_2}}{T_{S_1}} \times \frac{T_{F'}}{T_{S_2}} = \frac{T_{S_1}}{T_{E'}}\Big|_{Dopp} \times \frac{(1-\frac{r_g}{r_{S_1}})^{1/2}}{(1-\frac{r_g}{r_{E'}})^{1/2}}$$

$$\times \frac{T_{S_2}}{T_{S_1}} \times \frac{T_{F'}}{T_{S_2}}\Big|_{Dopp} \times \frac{(1-\frac{r_g}{r_{F'}})^{1/2}}{(1-\frac{r_g}{r_{S_2}})^{1/2}}. \tag{3.5.4}$$

The first and last term were immediately obtainable as products of Schwarzschildian and Dopplerian shift. T_{S_2}/T_{S_1} is given by equ.(3.3.10). Using this and equs.(3.5.2 - 3.5.4), one obtains

$$\left(\frac{T_{XI}}{T_E} \Big/ \frac{T_{XIII}}{T_P}\right) = \left(\frac{T_{F'}}{T_{E'}} \Big/ \frac{T_{R'}}{T_{P'}}\right) =$$

$$1 + \frac{2c^3 y_2^3}{H_2^3}\cos\alpha_2\left(\frac{\Omega_2}{2}\sin^2\alpha_2 + \frac{(1+2\Omega_2)}{3}\cos^2\alpha_2\right)$$

$$-\frac{2c^3 y_1^3}{H_1^3}\cos\alpha_1\left(\frac{\Omega_1}{2}\sin^2\alpha_1 + \frac{(1+2\Omega_1)}{3}\cos^2\alpha_1\right)$$

$$= 1 + \frac{2c^3 y_2^3}{H_2^3} f(\alpha_2, \Omega_2) - \frac{2c^3 y_1^3}{H_1^3} f(\alpha_1, \Omega_1), \tag{3.5.5}$$

where y_1 is the actual distance between O and D, and where

$$\sin\beta = \frac{y_1 \sin\alpha_1}{y_2 + y_0}. \tag{3.5.6}$$

Thus, the profile of non-Friedmannian shift is given by equs.(3.5.5 - 3.5.6) for $0 \le \beta \le \beta_1$, where $\sin\beta_1 = y_1/(y_0 + y_2)$, and by equs.(3.5.2 - 3.5.3) for $\beta_1 \le \beta \le \beta_2$, where $\sin\beta_2 = y_2/(y_0 + y_2)$. For $\beta_2 \le \beta \le \pi$ there is, of course, a Friedmannian shift.

3.6. The decomposition

For the case of void it is a mathematical exercise to decompose the function $f(\beta, k, \Omega)$ lefined by equ. (3.4.2) - into trigonometric Fourier-series. Function is defined for $\leq \beta \leq \tilde{\beta}$. It may also be defined for $[\tilde{\beta}, \pi]$; in this region it is identically zero. It may o be generalized for $-\pi \leq \beta \leq 0$ by relation $f(\beta, k, \Omega) = f(-\beta, k, \Omega)$.

One obtains

$$f(\beta, k, \Omega) = \frac{f_0(k,\Omega)}{2} + \sum_{n=1}^{\infty} f_n(k,\Omega)\cos n\beta,$$

$$f_n(k,\Omega) = \frac{1}{\pi}\int_{-\pi}^{\pi} f(\beta,k,\Omega)\cos n\beta\, d\beta = \frac{2}{\pi}\int_0^{\tilde{\beta}} f(\beta,k,\Omega)\cos n\beta\, d\beta$$

$$= \frac{2k}{\pi}\int_0^{\pi/2} f(\alpha,\Omega)\frac{\cos\alpha\cos n\beta}{\sqrt{1-k^2\sin^2\alpha}}d\alpha, \qquad (3.6.1)$$

nere $\cos n\beta$ should be explained by the powers of $\cos\alpha$. The coefficients $f_n(k,\Omega)$ are alytically calculable; for $n = 0, 2, 4, ...$ via the complete elliptic integrals, and for $= 1, 3, 5$ even exactly.

The relevant relations for $n = 0, 1$ and 2 are the following:

$$f_0(k,\Omega) = \frac{\Omega}{\pi k}\Big[E\Big(\frac{\pi}{2},k\Big) - (1-k^2)F\Big(\frac{\pi}{2},k\Big)\Big]$$

$$+\frac{2+\Omega}{9\pi k^3}\Big[(4k^2-2)E\Big(\frac{\pi}{2},k\Big) + (3k^2-5k^2+2)F\Big(\frac{\pi}{2},k\Big)\Big].$$

$$f_1(k,\Omega) = k\Big[\frac{5\Omega}{16}+\frac{1}{8}\Big],$$

$$f_2(k,\Omega) = -f_0(k,\Omega) + \frac{2\Omega}{3\pi k}\Big[(1+k^2)E\Big(\frac{\pi}{2},k\Big) - (1-k^2)F\Big(\frac{\pi}{2},k\Big)\Big]$$

$$+\frac{2(2+\Omega)}{45\pi k^3}\Big[2(1-k^2)(1-3k^2)E\Big(\frac{\pi}{2},k\Big) + (3k^4+7k^2-2)F\Big(\frac{\pi}{2},k\Big)\Big] \qquad (3.6.2)$$

here (see, e.g., Gradshteyn and Ryzhik, 1980)

$$\int_0^{\pi/2}\sqrt{1-k^2\sin^2\gamma}\; d\gamma = E\Big(\frac{\pi}{2},k\Big),$$

$$\int_0^{\pi/2}\frac{d\gamma}{\sqrt{1-k^2\sin^2\gamma}} = F\Big(\frac{\pi}{2},k\Big), \quad 0<k<1. \qquad (3.6.3)$$

Here $f_0(k,\Omega), f_1(k,\Omega), f_2(k,\Omega)$ define the monopole, dipole and quadrupole anitropies, respectively, seen by the observer being at the point X. For rough order timation, if $k \sim 1$, one may take $|f_0| \sim |f_1| \sim |f_2| \sim 1$.

For cluster model no further details are necessary. The only difference is that e above calculations must be done two times; first, for $f(\alpha_2, \Omega_2)$, and, second, for $f(\alpha_1, \Omega_1)$. In the first case the above results are immediately usable, if one substitutes k by $k_2 = y_2/(y_0+y_2)$, $\tilde{\beta}$ by β_2 and Ω by Ω_2; similarly, in the second case the above results are immediately usable, if one substitutes k by $k_1 = y_1/(y_0+y_2)$, $\tilde{\beta}$ by β_1 and Ω by Ω_1. Hence, again, for rough order estimation, if $k_1 \sim k_2 \sim 1$, one may take $|f_0| \sim |f_1| \sim |f_2| \sim 1$.

3.7. Discussion of Sections (3.3 - 3.6)

Under the assumptions used in Sections 3.3 - 3.6 (exact spherical symmetry, Minkowskian vacuum in the void, etc...) the obtained relations seem to be exact. Nevertheless, there are possible three modifications even under these conditions, which will shortly be discussed here.

First, the considered lines are not straightlines in general case. For example, the line between U and H on Fig.4 is not exactly a straightline. Clearly, when the photon misses the point C, the distance of this point from the line AB is smaller than the distance of point D from line AB, when the photon misses point D. The difference is $(r_D(\eta_D) - r_C(\eta_C))\sin\alpha = (da(\eta)/d\eta)|_{\eta=\eta_A}\chi^2\sin 2\alpha$. In addition, even the lines UC and DH are not straightlines, because they are geodetics in Schwarzschildian metric. The departure may be estimated as follows. Specially, for $\alpha = (\pi/2)$ on Fig.4, between U and V the photon moves on a hyperbola in a Schwarzschildian metric, where the departure from the straightline (in radians) is $(2r_g/r_E) = (8\chi^2/\eta_E^2)$. This is the inaccuracy in the direction for $\alpha = (\pi/2)$. For $\alpha < (\pi/2)$ the deviation from the straightline between U and C, and between H and D should be smaller. Hence, these deviations from the straighlines are $\sim \chi^2$ corrections (in radians), which are clearly negligible. This means that line I in the void model and lines I and II in cluster model are also roughly straightlines; lines II in the void model, and line III in the cluster model are exact straightlines.

The second problem concerns the whole calculations of Sections 3.4 and 3.5. In Section 3.4 (in Section 3.5) one assumes that photons I and II (photons I, II and III) are sent at the same time instant, and they also arrive at the same time instant into X. But, in the general case, once the photons arrive at the same time into X, it is not sure that they also were sent at the same time instant; the time needed for photon I to cross, e.g., distance between points F" and E" (Fig.4) need not be identical to the time needed for photon II to cross the distance between R' and P'. This so-called "time delay" effect is discussed in detail by Dyer (1976). He shows that at the lowest order this effect is negligible; i.e. only for terms χ^n; $n \geq 4$; is this effect essential. Hence, fortunately, the time-delay effect is also negligible in our calculations.

One has to remark that the assumption of spherical symmetry is surely a simplification allowing analytical calculations. Nevertheless, as it is discussed in detail by Rees and Sciama (1968); Panek (1992) and also by Tuluie and Laguna (1995), the departures from the spherical symmetry should not play an essential role. Thus, one may expect that without the spherical symmetry the conclusion of this paper will not be changed.

Thus, there is a good hope that the profiles - obtained here for the most general cases of spherical void and cluster models in analytical forms - are correct.

Discussing the physical significance of calculations of Sections (3.3 - 3.6) one may the following.

The decomposition into the usual dipole and quadrupole terms of the departure from Friedmannian shift - caused either by a void or by a cluster - shows that the dipole the quadrupole terms are of the same order. This means that vain is the dipole sotropy of order $\sim 10^{-3}$ due to a suitable large void or supercluster (this happens, ., for a void with $yH/c \sim 10^{-1}$, i.e. for a void with radius $\sim 300h^{-1}Mpc$), in this e the quadrupole term should also be of order $\sim 10^{-3}$. This theoretical prediction is an obvious contradiction with the observational data; any quadrupole terms, if exist, uld be of order $\sim 10^{-5}$ or smaller (Benett et al. 1994) (see also Section 2.3). Hence, seems to be doubtless that no intrinsic dipole anisotropy of CMBR of order $\sim 10^{-3}$ n exist due to the Rees-Sciama effect. Hence one may also categorically conclude that e alternative explanation of the data of Lauer and Postman (1994) - namely that the rinsic dipole anisotropy is caused by the Rees-Sciama effect - falls.

Note still that the significance of these Sections is not given only by the fact that rejects the possibility of an intrinsic dipole anisotropy of CMBR of order $\sim 10^{-3}$ e to the Rees-Sciama effect. The calculations may be useful at different cases, too. e ring-like profiles of the non-Friedmannian shift of CMBR caused by spherically mmetric superclusters and voids are characteristic and in principle measurable also r objects with sizes $\sim (100 - 300)h^{-1}Mpc$; in this case one expects anisotropies of der $\sim 10^{-5}$. The measurements of such a behaviour would empirically confirm the ees-Sciama effect.

3.8. The upper limit

In this section we calculate the upper limit of the Rees-Sciama effect. The ideas of is Section were not published yet.

From several papers it may seem that the Rees-Sciama effect is maximally of order $T/T_0) \sim (10^{-5} - 10^{-3})$, where T_0 is the temperature of cosmic microwave background, d δT is the departure from this value (Rees and Sciama 1968; Dyer 1976; Kaiser 1982; ottale 1984; Dyer and Ip 1988; Panek 1992; Fang and Wu 1993; Arnau et al. 1993; áez, Arnau and Fullana 1993; Wu and Fang 1994; Mészáros 1994; Martínez-Gonzáles, anz and Silk 1994; Tuluie and Laguna 1995; Nakao et al. 1995; Mészáros and Molnár 996; Mészáros and Vanýsek 1996). Nevertheless, the effect is growing cubically (Dyer 976; Mészáros 1994) by the scale of causing objects. So it is not so sure that the effect ill always remain so small. Hence, it is interesting to ask for the theoretical maximum f this effect. This is the topic of this Section.

Be given a void (see Fig.7.), which comoving distance from us is $\chi_0 \geq 0$. The omoving radius of the void is χ. Inside of the void there is vacuum, and the matter is n the surface of void. The light coming from the last scattering surface being in redshift $= 1000$ (z is the redshift) toward to observer being at point O enters into the void at e point A, and leaves it at point B. Outside of the void there is a Friedmannian metric, which a second light moves across X and Y to O. This light is taken for comparison nd has a Friedmannian redshift. We consider only the case for $\Omega = 1$, where Ω is the atio of the density to the critical one. The cases with $\Omega \neq 1$ are different only in the

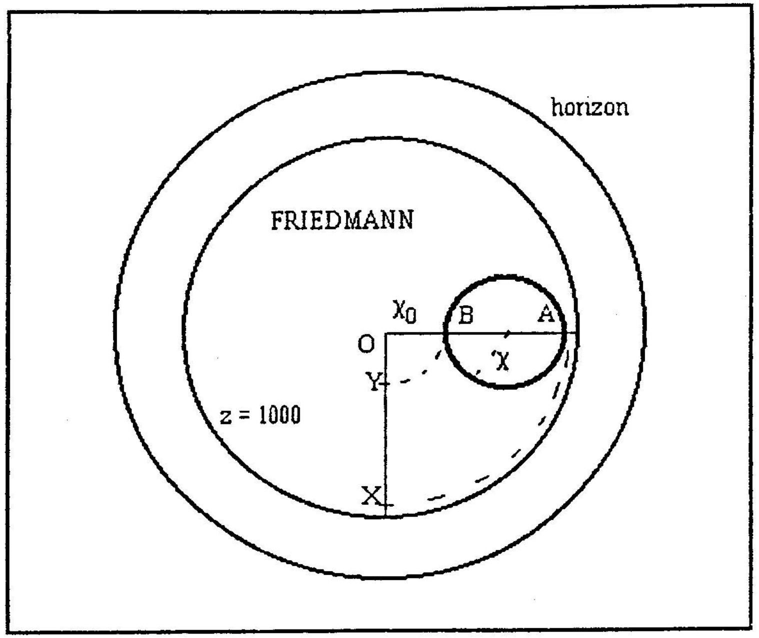


Fig.7. Illustration of the maximal size of void. In the expanding sphere with diameter AB (comoving diameter is 2χ) there is a Minkowskian vacuum. Outside of sphere there is a Friedmannian metric. A light is sent from A across B to O.

factor 3 (Mészáros 1994), and therefore give the same order of results, which is enough for the purpose of this Section.

In principle the allowed maximum of χ may be estimated as follows.

It is sure that at $z = 1000$ did not exist any objects yet. The value of the anisotropy of microwave background radiation caused by the Sachs-Wolfe effect shows that the perturbations of matter existing at $z = 1000$ were not larger than $\sim 10^{-5}$ (Sachs and Wolfe 1967; Mészáros 1986; Mészáros and Vanýsek 1988; Paczyński and Piran 1990; Turner 1991; Mészáros and Molnár 1991; Mészáros 1991; Wright et al. 1992; Börner 1993, Chapt.11.2.2; Peebles 1993; Mészáros 1993a; Mészáros 1993b; Lauer and Postman 1994; White, Scott and Silk 1994; Mészáros 1995; Jaroszyński and Paczyński 1995; White and Bunn 1995; Mészáros 1996; Krasiński 1996). The present value of the conformal time for the flat Friedmannian model is arbitrary, and therefore we choose $\eta_0 = 1$. The conformal time at $z = 1000$ is η_{1000}. One has $a(\eta_z)/a(\eta_0) = \eta_z^2 = 1/(1+z)$, and hence $\eta_{1000} = 10^{-3/2}$. (The expansion function is given by $a(\eta) = (a_0/2)\eta^2$, where a_0 is a constant length.) We can define the condition (see Fig.8.)

$$(\chi_0 + 2\chi) \leq \eta_0 - \eta_{1000} = 1 - 10^{-3/2} = 0.97, \qquad (3.8.1)$$

where χ_0 and χ are free parameters.

We can write the relative departure of redshift caused by the Rees-Sciama effect om the Friedmannian one as (Rees and Sciama 1968; Mészáros 1994)

$$\frac{\frac{T_B}{T_A}}{\frac{T_Y}{T_X}} = \sqrt{\frac{1+w/c}{1-w/c}} \times \sqrt{\frac{1-r_g/r_B}{1-r_g/r_A}} \times \frac{a(\eta_X)}{a(\eta_Y)}, \qquad (3.8.2)$$

here w is the relative velocity between the points A and B; $r_g = 2a_0\chi^3$ is the gravita- onal radius of the matter that defines the void; r_A and r_B are the radii of the void at $_A$ and η_B, respectively; T_A, T_B,... are the periods of light, when it crosses the points. ne has: $r_A = a(\eta_A)\chi$ and $r_B = a(\eta_B)\chi$, where $\eta_B = (\eta_A + 2\chi) = (1-\chi_0)$.

The third term in equ.(3.8.2) is surely finite, because both $a(\eta_A)$ and $a(\eta_B)$ are nite.

In the first term it can be $w = c$, and then the relative redshift can be infinite. ne has $w = (v_A + v_B)/(1 + v_A v_B/c^2)$, where $v_A = 2c\chi/\eta_A$, $v_B = 2c\chi/\eta_B$. Because $_B < v_A$, $w = c$ occurs for $v_A = c$. This means that already for $2\chi = (1 - \chi_0 - 2\chi)$ this the case. Hence, the maximal allowed comoving radius of void is given by

$$\chi = \frac{1-\chi_0}{4}. \qquad (3.8.3)$$

In the denominator of the second term it can be $(1 - r_g/r_A) = 0$ and the relative edshift can again be infinite. When we express r_A, r_g as the functions of χ, then we btain the maximal possible value of χ. One has:

$$\frac{r_g}{r_A} = \frac{4\chi^2}{(1-\chi_0-2\chi)^2} = 1. \qquad (3.8.4)$$

his gives the same result as equ.(3.8.3).

For $\chi_0 = 0$ we obtain the maximal possible comoving diameter $2\chi = 1/2$; i.e. exactly he half of the Hubble-radius having the size 6000 h^{-1} Mpc.

All this means that if we suppose the existence of a void with size around $\sim$ $000h^{-1}Mpc$, then the Rees-Sciama effect becomes infinite. This means that such reat objects should not exist in nature, because the observations suggest (Section 2.4) hat the Rees-Sciama effect should be of order $\sim 10^{-5}$.

Note that in this Section the Rees-Sciama effect due to the void was considered. he case of the supercluster does not lead to the essentially different results; except for he sign of effect (Einstein and Strauss, 1945; Rees and Sciama 1968; Dyer 1976; see lso Sections 3.4-3.5). The observed objects are of order $\sim 100h^{-1}Mpc$, and this is the ause that the anisotropies of microwave background are small. On the other hand, the bserved high spherical harmonics of order $\sim 10^{-5}$ on typical angular scales $\sim (0.1-10)$ egrees (Smoot et al. 1992; Ganga et al. 1993; Bennett et al. 1992; Bennett et al. 1994; heng et al. 1994; de Bernardis et al. 1994; Devlin et al. 1994; Dragovan et al. 1994; undersen et al. 1995; Ruhl et al. 1995; Lineweaver et al. 1995; Netterfield et al. 1995) nay be caused by the Rees-Sciama effect.

4. Summary

The main results of this article may be listed as follows:

1. The work gives an useful - of course, never complete - overview of the literature about the topic (Section 2).

2. Using the formulas of Special Theory of Relativity it is proven that in the neighbourhood of expanding spherical body the Mészáros' calculations (Mészáros 1994) are correct. The inaccuracy is maximally of order 10^{-12} (Section 3.1.- 3.2).

3. The profile of the blueshift of expansion caused by an expanding sphere is presented for the case, when the radius of this sphere is much smaller than the relevant Hubble radius (Section 3.3).

4. The profiles of the shifts of light periods through a void and through a supercluster in the most general cases are given. These cases contain all the three Friedmannian models and both the synchronous and asynchronous clusters (Sections 3.4 - 3.5).

5. The mentioned profiles are explicitly decomposed into the sum of the multipole terms (Section 3.6).

6. It is shown that the observed difference between the measured direction of the maximum of dipole anisotropy of CMBR and the result of Lauer and Postman (1994) is not explainable by the Rees-Sciama effect (Section 3.7). This means that no alternative exists to the two possibilities for the explanation of the data of Lauer and Postman; either the either the huge system of Abell clusters is streaming, or the Friedmannian model is querried. The third possibility is, of course, that the data of observations of Lauer and Postman is incorrect. However, any of the three possibilities seem to be strange enough. This is the key result of this paper.

7. It is shown that - from the theoretical point of view - there is no upper limit of Rees-Sciama effect (Section 3.8).

Acknowledgement The author would like to thank G. Börner, A. Mészáros, and Z. Stuchlík for the valuable remarks and interesting discussions.

References

[1] R.A. Alpher, H.A. Bethe, G. Gamow: *Phys. Rev.* **73** (1948) 803
[2] R.A. Alpher, R. Herman: *Nature* **162** (1948) 774
[3] J.V. Arnau et al.: *Astrophysical Journal* **402** (1993) 359
[4] Z. Bagoly, A. Mészáros, P. Mészáros: *Astrophysical Journal* **333** (1988) 54
[5] D.J. Batuski, J.O. Burns: *Astrophysical Journal* **299** (1985) 5
[6] C.L. Bennett et al.: *Astrophysical Journal* **396** (1992) L7
[7] C.L. Bennett et al.: *Astrophysical Journal* **436** (1994) 423
[8] H.G Bi, A. Mészáros, P. Mészáros: *Astronomy and Astrophysics* **243** (1991) 16
[9] W.B. Bonnor, A. Chamorro: *Astrophysical Journal* **361** (1990) 21
[10] G. Börner: *The Early Universe*, Springer, Berlin (1993)
[11] S. Chandrasekhar, L.R. Heinrich: *Astrophysical Journal* **95** (1942) 288
[12] E.S. Cheng et al.: *Astrophysical Journal* **422** (1994) L37

3] E.K. Conklin: *Nature* **222** (1969) 971
4] R.D. Davies et al.: *Nature* **326** (1987) 462
5] M. Davis et al.: *Astrophysical Journal* **253** (1982) 423
6] P. de Bernardis et al.: *Astrophysical Journal* **422** (1994) L33
7] V. de Lapparent, M.J. Geller, J.P. Huchra: *Astrophysical Journal* **302** (1986) L1
8] A. de Oliviera-Costa, G.F. Smoot : *Astrophysical Journal* **448** (1995) 477
9] G. de Vaucouleurs, W.L. Peters: *Nature* **220** (1968) 868
0] M.J. Devlin et al.: *Astrophysical Journal* **430** (1994) L1
1] R.H. Dicke et al.: *Phys. Rev.* **70** (1946) 340
2] R.H. Dicke: *Astrophysical Journal* **152** (1968) 1
3] M. Dragovan et al.: *Astrophysical Journal* **427** (1994) L67
4] C.C. Dyer: *Monthly notices of Royal Astronomical Society* **175** (1976) 429
5] C.C. Dyer, P.S.S. Ip: *Monthly notices of Royal Astronomical Society* **235** (1988) 895
6] A. Einstein, E.G. Strauss: *Rev. Mod. Phys.* **18** (1945) 148
7] L.Z. Fang, X.P. Wu: *Astrophysical Journal* **408** (1993) 25
8] G. Gamow: *J. Wash. Acad. Sci.* **32** (1942) 353
9] G. Gamow: *Phys. Rev.* **70** (1946) 572
0] K. Ganga et al.: *Astrophysical Journal* **410** (1993) L57
1] I.S. Gradshteyn, I.M. Ryzhik: *Table of Integrals, Series, and Products* Academic, New York (1980)
2] A.W. Graham: *Astrophysical Journal* **459** (1996) 27
3] S.A. Gregory, L.A. Thompson, W.A. Tifft: *Astrophysical Journal* **243** (1980) 411
4] J.O. Gundersen et al.: *Astrophysical Journal* **443** (1995) L57
5] J.E. Gunn, J.E.: *in* ASP Conf.Ser.4, The Extragalactic Distance Scale, *eds. S. van den Bergh, Pritchet*, San Francisco, (1988) p. 344
6] P.S. Henry: *Nature* **231** (1971) 516
7] I. Horváth, P. Mészáros, A. Mészáros: *Astrophysical Journal* **470** (1996) 56
8] J.P. Huchra et al.: *Astrophysical Journal Suppl.* **53** (1983) 89
9] M. Jaroszyński, B. Paczyński: *Astrophysical Journal* **448** (1995) 488
0] M. Joeveer, J. Einasto, E.Tago: *Monthly notices of Royal Astronomical Society* **185** (1978) 357
] N. Kaiser: *Monthly notices of Royal Astronomical Society* **198** (1982) 1033
] R.P. Kirschner et al.: *Astrophysical Journal* **248** (1981) L57
] A. Krasiński: *The Inhomogeneous Universe: Models and their Properties*, Cambridge University Press, Cambridge (1996)
] K. Kuchař: *Základy Obecné Teorie Relativity* Academia, Prague (1968, in Czech)
] L.D. Landau, E.M. Lifshitz: *Teoriya polya*, Nauka, Moscow (1973, in Russian)
] T.R. Lauer, M. Postman: *Astrophysical Journal* **425** (1994) 418
] G. Lemaitre: *La Revue des Questions Scientifiques*, 4e série, **20** (1931) 391
] C.H. Lineweaver et al.: *Astrophysical Journal* **448** (1995) 482
] E. Martínez-Gonzáles, J.L. Sanz, J. Silk: *Astrophysical Journal* **436** (1994) L1
] A. Mc Kellar: *Publ. Dominion Astrophys. Obs.* **7** (1941) 251

[51] A. Mészáros: *Acta Phys. Hung.* **60** (1986) 75
[52] A. Mészáros: *Monthly notices of Royal Astronomical Society* **253** (1991) 619
[53] A. Mészáros: *Astronomy and Astrophysics* **278** (1993a) 1
[54] A. Mészáros, A.: *Astrophys. Space Sci.* **207** (1993b) 5
[55] A. Mészáros: *Astrophysical Journal* **423** (1994) 19
[56] A. Mészáros: *Acta Phys. Hung. New Series* **1** (1995) 107
[57] A. Mészáros: *Astronomy and Astrophysics* submitted (1996)
[58] A. Mészáros, P. Mészáros: *Astrophysical Journal* **327** (1988) 25
[59] A. Mészáros, P. Mészáros:*Astrophysical Journal* **466** (1996) 29
[60] A. Mészáros, Z. Molnár: *Astrophys. Space Sci.* **181** (1991) 323
[61] A. Mészáros, Z. Molnár: *Astrophysical Journal* **470** (1996) 49
[62] A. Mészáros, V. Vanýsek: *Bull. Astron. Inst. Czech.* **39** (1988) 185
[63] A. Mészáros, V. Vanýsek: *Astronomy and Astrophysics* in press (1997)
[64] P. Mészáros, A. Mészáros: *Astrophysical Journal* **449** (1995) 9
[65] Z. Molnár: Thesis, Charles University, Prague (1991)
[66] K. Nakao et al.: *Astrophysical Journal* **453** (1995) 541
[67] C.B. Netterfield et al.: *Astrophysical Journal* **445** (1995) L69
[68] L. Nottale: *Monthly notices of Royal Astronomical Society* **206** (1984) 713
[69] C.R. O'Dell, M. Peimbert, T.D. Kinman: *Astrophysical Journal* **140** (1964) 119
[70] D.E. Osterbrock, J.B. Rogerson: *PASP* **73** (1961) 129
[71] B. Paczyński, T. Piran: *Astrophysical Journal* **364** (1990) 341
[72] M. Panek: *Astrophysical Journal* **388** (1992) 225
[73] P.J.E. Peebles: *Astrophysical Journal* **142** (1965) 1317
[74] P.J.E. Peebles: *Physical Cosmology*, Princeton University Press, Princeton (1970)
[75] P.J.E. Peebles: *The Large-Scale Structure of the Universe*, Princeton University Press, Princeton (1980)
[76] P.J.E. Peebles: *Principles of Physical Cosmology*, Princeton University Press, Princeton (1993)
[77] A.A. Penzias, R.W. Wilson: *Astrophysical Journal* **142** (1965) 419
[78] A.C.S. Readhead et al.: *in* Proc. IAU Symp. on "Large Scale Structure", Balaton (1987)
[79] M. Rees, D.W. Sciama: *Nature* **217** (1968) 511
[80] K. Rektorys et al.: Přehled užité matematiky, Státní nakladatelství technické literatury, Praha (1963, in Czech)
[81] A.G. Riess, W.H. Press, R.P. Kirshner: *Astrophysical Journal* **445**, (1995) L91
[82] J.E. Ruhl et al.: *Astrophysical Journal* **453** (1995) L1
[83] P.G. Roll, D.T. Wilkinson: *Phys. Rev. Lett.* **16** (1966) 405
[84] R.K. Sachs, A.M. Wolfe: *Astrophysical Journal* **147** (1967) 73
[85] D. Sáez, J.V. Arnau, M.J. Fullana: *Monthly notices of Royal Astronomical Society* **263** (1993) 681
[86] D.W. Sciama: *Phys. Rev. Lett.* **18** (1967) 1065
[87] G.F. Smoot et al.: *Astrophysical Journal* **371** (1991) L1
[88] G.F. Smoot et al.: *Astrophysical Journal* **396** (1992) L1

9] R.C. Tolman: *Relativity Thermodynamics and Cosmology*, Clarendon Press, Oxford (1934)

0] R.B. Tully: *Astrophysical Journal* **303** (1986) 25

1] R.B. Tully et al.: *Astrophysical Journal* **388** (1992) 9

2] R. Tuluie, P. Laguna: *Astrophysical Journal* **445** (1995) L73

3] M.S. Turner: *Phys. Rev.* **D44** (1991) 3737

4] J.M. Uson, D.T. Wilkinson: *Astrophysical Journal* **277** (1984) L1

5] S. Weinberg: *Gravitation and Cosmology* J.Wiley, New York (1972)

6] R. Weiss: *Annual Review of Astronomy and Astrophysics* **18** (1980) 489

7] M. White, E.F. Bunn: *Astrophysical Journal* **450** (1995) 477

8] M. White, D. Scott, J. Silk: *Annual Review of Astronomy and Astrophysics* **32** (1994) 319

9] E. L. Wright et al.: *Astrophysical Journal* **396** (1992) L13

0] X.P. Wu, L.Z. Fang: *Astrophysical Journal* **424** (1994) 53