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SECOND ORDER FLOW OF PIONS IN MIDRAPIDITY[1]

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In this work we study consequences of the rescattering process on azimuthal momentum distribution of pions. For this purpose we have performed computer simulation of the expanding pion gas created in non-central Pb-Pb 158GeV/n collisions. Second order asymmetry in azimuthal distribution of pions is found and studied. The asymmetry is explained as a consequence of the geometry of non-central collisions and the rescattering process including formation time parameter.

1. Introduction

Azimuthal asymmetries in transverse momentum distributions of particles measured in relativistic heavy ion collisions (HIC) were observed for the first time by Plastic Ball detector in Berkeley [1]. Since this pioneering experiment asymmetries in relativistic HIC were measured also at higher energies [2, 3]. Typical types of asymmetries (bounce-off and squeeze-out) are understood as a consequence of the collective behavior of nuclear matter. This understanding of the origin of azimuthal asymmetries in non-central HIC succesfully explains most of experimental data.

However at AGS experiment with Au+Au 11.4 GeV/n collisions E877 collaboration reported [2] about unclear origin of a new type of squeeze-out effect parallel to impact parameter. It seems that E877 collaboration has found first experimental indications of the effect predicted in the work of J.-Y.Ollitrault [4]. Second order flow parallel to impact parameter can originate also from interactions of secondary produced particles among themselves.

In this work we investigate asymmetry in transverse momentum distribution of pions created in the process of HIC. No collective behaviour of nuclear matter is supposed in the calculation. The asymmetry results from the geometry of non-central HIC and the rescattering process.

Paper is organized as follows: In Section 2 we give an intuitive explanation of the origin of the asymmetry. Short description of our computer simulation is given in

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Section 3. In Section 4 we present main results of the simulation - dependence the asymmetry on impact parameter and also other features of the studied effect. We finish this work with short summary and conclusions.

2. Asymmetry as a consequence of rescattering

Usual explanation of azimuthal asymmetries in momentum distributions of particles in HIC is based on the collective behaviour of nuclear matter. Hydrodynamical models [5, 6] describe the process of HIC using the equation of state for nuclear matter. As we shall see these assumptions are not required for the existence of our type of asymmetry. It can be generated without collective effects of the primary nuclear matter contained in the colliding nuclei.

Number of secondary particles created in HIC at present SPS energies (160GeV/n) is substantially larger than number of nucleons contained in the colliding nuclei. Therefore the situation is different than that at lower energies and the interaction of secondaries among themselves may become significant. Rescattering process among secondary produced pions was already studied in the simulation [7] where central S-Pb 200 GeV/n collisions were investigated. This simulation succesfully reproduced $\vec{p}_t$ dependence of transverse size parameters R_t [8] extracted by HBT technique [9].

We think that rescattering of secondaries can demonstrate itself in experimental data also as a second order flow of pions in non-central collisions. For the existence of this effect no collective behaviour of the primary nuclear matter is necessary. The phenomenon is not dependent on (generated by) the flow of nucleons. It might be observed also in future ultrarelativistic HIC experiments on RHIC and LHC.

In this section we explain the origin of the studied transverse flow of pions as a consequence of the geometry of non-central HIC and the rescattering process. In Fig.1a we show geometry of non-central A+A collision in transversal plane. Secondary particles i.e. also pions are created mainly in the overlapping region where collisions of primary nucleons happen. Shape of the overlapping region depends on impact parameter b and is azimuthally asymmetric in transversal plane. Since we do not consider any collective behaviour of nuclear matter it is natural to assume that the initial distribution of pions in transverse momenta $\Psi(\vec{p}_t)$ is azimuthally symmetrical: $\Psi^S(\vec{p}_t) = \Psi(|\vec{p}_t|)$. For the sake of simplicity we do not write longitudinal components of $\vec{p}$ and $\vec{x}$ in this section however simulation described in the next section is performed in 3+1 dimensions.

Denoting the space distribution of the points of creation of pions by $\Phi(\vec{x}_t)$ we have the following initial condition for the pion gas created in HIC:

$$X(\vec{x}_t, \vec{p}_t) = \Phi^A(\vec{x}_t) \cdot \Psi^S(\vec{p}_t) \quad (1)$$

As a consequence of the rescattering process original $\vec{x}-\vec{p}$ non-correlated distribution (1) becomes $\vec{x}-\vec{p}$ correlated [10] and (as we shall see from results of our simulation) the asymmetry in $\Phi^A(\vec{x}_t)$ *leaks* into asymmetry in the resulting transverse momentum distribution.

$$\Psi^S(\vec{p}_t) \cdot \Phi^A(\vec{x}_t) \to X^A(\vec{p}_t, \vec{x}) \quad (2)$$

where ^{A} denotes asymmetry of distribution $X(\vec{p}_t, \vec{x})$ in momentum.

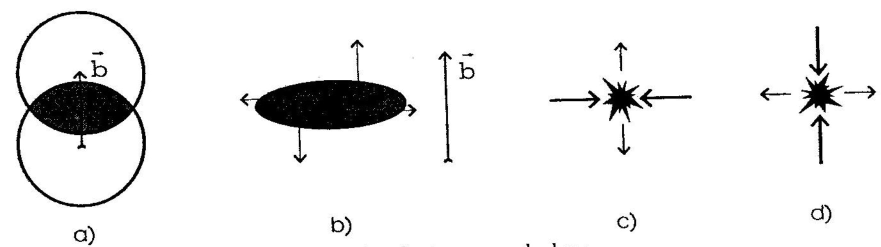

Fig. 1. Geometry of non-central collision in transversal plane

At present we understand this effect in the following way: Let us have two groups of pions: A group of parallel pions with the momentum parallel to $\vec{b}$ and the group of orthogonal pions with the momentum orthogonal to $\vec{b}$. In this simplified situation sketched in Fig.1b) three types of collisions can happen:

1) Collisions of orthogonal pion with parallel pion. This type of collisions cannot change initial ratio of orthogonal/parallel pions because of the conservation of momentum in the process of collisions.

2) Collisions of orthogonal pion with orthogonal pion - Fig.1c) and collisions of parallel pion with parallel pion - Fig.1d). These collisions can influence the ratio of orthogonal/parallel pions and therefore only these collisions can originate asymmetry in the transverse momentum distribution. In the case of symmetrical original cloud of pions (in Fig.1b it is asymmetrical) the probability of collisions Fig.1c) and Fig.1d) would be the same. We think that asymmetrical shape of initial distribution of pions (Fig.1b) together with formation time parameter (see Fig.4 and the text below) make collisions of type Fig.1c) more frequent than collisions of type Fig.1d) and this mechanism leads to the excess in the number of parallel pions. Resulting asymmetry in the momentum distribution of pions freezes in the considered pion gas because of the expansion of the system.

Azimuthal asymmetry in transverse momentum distribution of pions after the rescattering process can be measured by detectors as the excess of pions in the direction parallel to the impact parameter. As we shall see in next sections the predicted asymmetry should demonstrate itself as a second order asymmetry in the Fourier analysis of transverse flow [11].

In Fig. 2 we show the result of a toy simulation of the effect described. Initial distribution of pions in momenta was taken from the central S-Pb 200 GeV/n rescattering simulation [7, 10] and therefore it was azimuthally symmetric. Initial distribution of pions in $\vec{x}_t$ space is artificially asymmetric as it is shown in Fig. 2b). After 20fm/c of time evolution the rescattering process leads to the asymmetry in transverse momentum distribution as it is clearly seen from Fig. 2c). Simulation of the expanding pion gas for "real" Pb-Pb 158 GeV/n non-central collisions is described in the next section.

3. The Simulation

For the simulation of the expanding pion gas created by "real" Pb-Pb 158 GeV/n

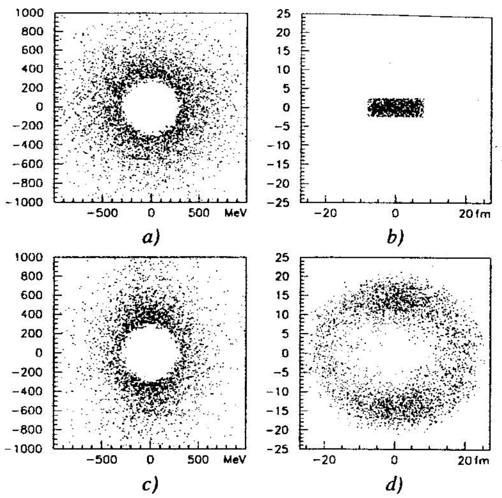

Fig.2. Toy simulation of the effect. *a)* is $\vec{p}_t$ distribution before the rescattering. *c)* is p_t distribution after 20fm/c of rescattering process. Cut in transverse momentum $p_t > 300MeV$ is applied to enhance the visual effect of the asymmetry. *b)* is $\vec{x}_t$ distribution before the rescattering process and *d)* after 20 fm/c of the expansion.

non-central collisions two independent programs) were used:

1) Cascade generator [12] which generates initial momenta and positions of pions as a result of independent nucleon-nucleon collisions.

2) Rescattering program [10] which simulates time evolution of the interacting pion gas created in HIC.

Main interface structure of the simulation is shown in Fig.3. Non-central Pb-Pb 158 GeV/n collisions were simulated by the cascade generator (CG) for random orientations and selected values of impact parameter $\vec{b}$. Information about initial momentum, time and place of creation of pions produced by CG was used as input for rescattering program [10]. Final momenta of the interacting pions were selected from the output of the rescattering program and analyzed for transversal asymmetries. For each event approximate orientation of the impact parameter was determined from the output of the cascade generator. Rotated events with the same orientation of impact parameter were analyzed for the existence of asymmetry in transversal momentum (see Fig.3). Description of internal structure of the program used for the simulation of rescattering process can be found in [10]. This program was build according to the description of the simulation of central S-Pb 200 GeV/n collisions [7]. Here we remind just main features of rescattering program [10].

Pions are treated as point-like objects. Position and momentum of each pion is known during the simulation. Pions move in small time steps $\Delta t = 0.1 fm$ as free particles. Collisions happen if two pions appear to be at a distance smaller than critical

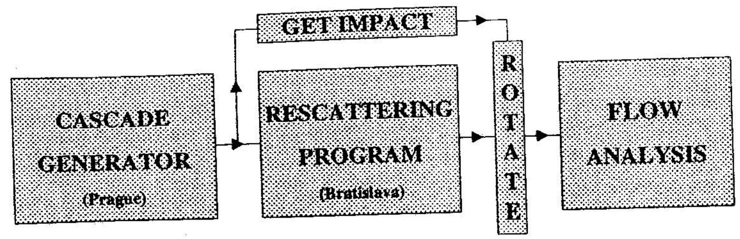

Fig.3 Main interface structure of the simulation

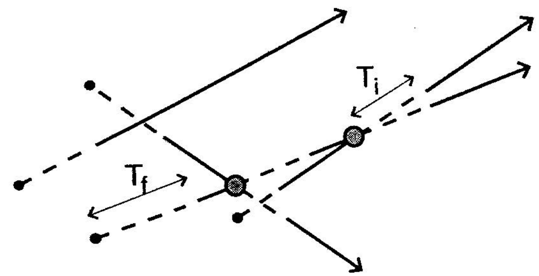

Fig.4. Our scenario of $\pi\pi$ collisions. Collisions can happen only if both pions are allowed to interact (solid lines).

distance D_k which is determined from the isospin averaged total elastic cross-section

$$D_k = \sqrt{\sigma(s)/\pi} \tag{3}$$

Momenta of pions after the collision are determined in CMS of the pair according to the isospin averaged differential cross section

$$d\sigma/d\Omega = a(s) + b(s)\cos^2(\theta) \tag{4}$$

(Numerical values of the functions $a(s), b(s)$ are calculated from data [13, 14].)

Then new momenta are transformed back to the global frame of the simulation. Test for the relative distance is performed for every pair of pions in each time step.

Existing pions are restricted from interactions for time T_f - formation time [15] after their creation and also for time T_i after each collision (see description in [10]). These two phenomenological parameters allow to influence total number of collisions in the simulation and also other features of the expanding of pion gas.

4. Data analysis

Events with exact values of impact parameter $b = 3, 5, 7, 9, 11fm$ were used for the analysis. For each event approximate orientation of impact parameter was determined by a procedure (similar to Danielewicz-Odyniec method [16]) which is described in the preprint [20][3].

Rotated events were used for Fourier type of analysis of transversal flow [11]. For each pion the azimuthal angle of momentum was determined and added to histogram of azimuthal distribution $R(\phi)$ (see Fig.5). Because of the known character of asymmetry and the rotation of events into the direction with the same orientation of impact parameter we have fitted normalized histograms $R^N(\phi)$ to the function

$$R^N(\phi) = 1 + S_2 \cos(2\phi) \tag{6}$$

Numerical results of the analysis are presented and discussed in the next section.

5. Results

First the presence of asymmetry was tested on the set of 24 artificial S-Pb 200 GeV/n asymmetrical events. Strong final asymmetry in azimuthal distribution of pions in transversal momentum is visible directly from Fig.2.

Then "real" non-central Pb-Pb 158 GeV/n collisions were studied using the output of cascade generator [12]. We have run the program for five sets of 10 events with impact parameter $b = 3, 5, 7, 9, 11 fm$ in order to study dependence of the asymmetry on the value of impact parameter and additional 20 events for $b = 7fm$ were run in order to verify whether statistics of 10 events per set was sufficient. Pions in rapidity range $\langle -1, 1\rangle$ were selected for the analysis.

In Fig.5 we show histograms of distribution $R(\phi)$ for the ROT - rotated (to the same orientation of $\vec{b}$) and non-rotated (NoR) events for b=3fm and b=9fm. Normalized distributions $R^N(\phi)$ were used for the fit.

Results of the fit are summarized in Tab.1. Simulation of 20 additional events for b=7fm showed that our statistics of 10 events per the set was sufficient for the qualitative analysis we had performed. Main result of our simulation is visible clearly from Fig.6: Asymmetry increases with impact parameter in range $3 - 9fm$ and it falls down for $b > 9fm$. Turnover point of S_2 lies between 7 and 11 fm. We think that the position of the turnover point depends on numerical values of T_f, T_i parameters used in the simulation. We shall study the dependence in the near future. (Hardware failure of the 2GB disk delayed our progress in this direction for 3 months.)

We have studied also rapidity dependence of the second order asymmetry (represented by parameter S_2) and also existence of the first order asymmetry in our data. First order asymmetry was found to be very weak and the value of S_2 coefficient did not decrease for pions with rapidity in range $\langle 1, 2\rangle$ ($\langle -2, -1\rangle$). Numerical results of this analysis can be found in the preprint [20].

An interesting behaviour of asymmetry coefficient S_2 was found during our data analysis. As it is shown in Tab.1 coefficient S_2 is significantly higher for high p_t pions ($p_t > 300$MeV).

[3] This contribution is shortened and corrected version of the preprint [20].

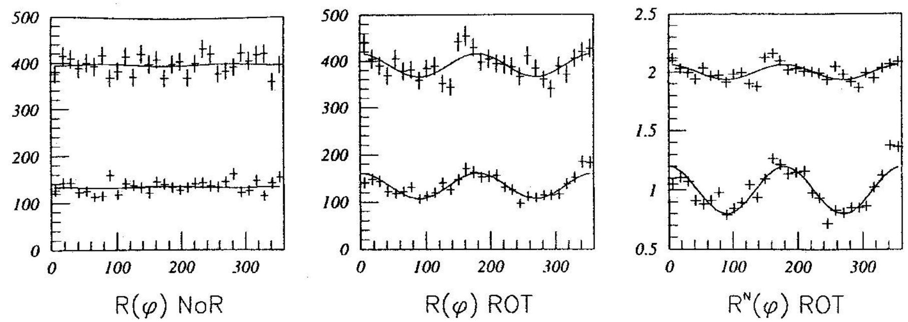

Fig.5. Histograms of the azimuthal distributions of pions for b=3fm and b=9fm. Numbers of particles in the bins of histogram for b=3fm are higher than in the case b=9fm because of the higher total multiplicity of pions in the collisions b=3fm. Normalized histogram $R^N(\phi)$ for b=3fm is artificially shifted up.

b	3 fm	5 fm	7 fm /30	9 fm	11 fm /20
$(P_t > 0)$ S_2	0.06 ± 0.01	0.09 ± 0.01	0.135 ± 0.010	0.20 ± 0.02	0.13 ± 0.02
$(P_t > 300)$ S_2	0.13	0.15	0.21	0.30	0.21
N_{coll}/π	4.5	4.2	3.4	2.6	1.1

Table 1.

Tab.1. Asymmetry coefficients for Pb-Pb 158 GeV/n non-central events. 7 fm/30 means that fit was performed for 30 events. ($T_f = T_i = 0.5fm$)

This seems to be an interesting prediction at present. We think that this behaviour is a consequence of the momentum dependence of formation path of pions which is relativistically dilated in our simulation (see Fig.4).

$$L_f = v_\pi \cdot \frac{T_f}{\sqrt{1 - v_\pi^2/c^2}} \tag{7}$$

This phenomenon also requires further investigation. Preliminary analysis of the simulation performed for different values of T_f parameter showed that S_2 dependence on p_t region of pions is sensitive to the value of T_f parameter (see Appendix).

6. Summary and Conclusions

We have studied azimuthal asymmetry in the transverse momentum distribution of pions in non-central Pb-Pb 158 GeV/n collisions. For the presence of this type of asymmetry no collective behaviour of the primary nuclear matter is necessary. It is a consequence of the geometry of non-central collisions and the rescattering among secondary produced pions.

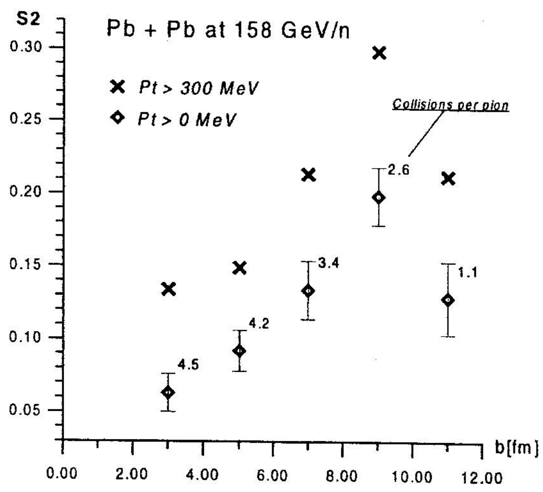

Fig.6. Dependence of S_2 coefficient on impact parameter. For high-p_t pions S_2 coefficient is higher compared to the full p_t range pions.

The simulation showed that the asymmetry increases with the impact parameter in the range up to $9fm$. Turnover point is located between $7-11fm$ in our data. Decrease of asymmetry coefficient S_2 for $b = 11fm$ is most likely a consequence of low number of pions participating in the rescattering process combined with small total size of the overlapping region in comparison to formation path $L_f \simeq c \cdot T_f$ of pions.

We think that impact parameter dependence of S_2 asymmetry parameter studied in this work was already confirmed experimentally by NA49 collaboration [18] using Ring Calorimeter setup [19]. Experimental results [18] for impact parameter dependence of the second order flow in midrapidity are surprisingly similar to our results shown in Fig.6. Quantitative comparison is however not easy. Our simulation studies only interactions of pions among themselves and it does not distinguish between π^+, π^-, π^o. Moreover our results are not corrected for detector effects.

Unexpected p_t dependence of the S_2 asymmetry coefficient seems to be an interesting prediction. Preliminary analysis of our artificially asymmetrical S-Pb 200GeV/n events (Fig.2) showed that p_t dependence of our S_2 coefficient is considerably sensitive to the value of T_f parameter (see Appendix).

At future HIC experiments multiplicities of secondaries will be much higher. It is not excluded that mechanism responsible for transversal flow of the primary nuclear matter at lower (present) energies will play a little role. In this case the studied effect can be substantial for the transversal flow phenomenon on RHIC and LHC heavy ion experiments.

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Appendix

At the time of the revision of this manuscript data about p_t dependence of S_2 asymmetry parameter in artificial S-Pb 200GeV/n events (see Fig.2) were available. In the following table we present our results for different values of T_f, T_i parameters.

T_f ; T_i [fm] p_t:	0-200 MeV	200-400 MeV	400-Inf. MeV	No p_t cut
0.2 ; 0.2 S_2:	0.18	0.43	0.62	0.43
1.0 ; 0.0 S_2:	0.13	0.36	0.36	0.29

Table 2

Tab.2. Dependence of asymmetry parameter S_2 on transversal momentum cut on pions for different values of parameters T_f, T_i.

We would like to attract attention of reader to the preprint SUNY-NTG 96-40 available also as nucl-th/9610026 which is closely related to the subject of this work.

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