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INTERMITTENCY SIGNALS AND CORRELATIONS[1]

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Experimental and theoretical investigations of intermittency signals are shortly reviewed. The problems related to standard description of correlation effects are discussed in more detail.

1. Introduction

Ten years ago Bialas and Peschanski started a new field of investigations in multiparticle production by a proposal to search for "intermittent" local density fluctuations in momentum space [1]. They have shown that for such fluctuations (imitated by the random cascade "α model" used to describe turbulence) the scaled factorial multiplicity moments F_q defined by

$$F_q(\delta) \equiv \overline{\left(\frac{n!}{(n-q)!}\right)_\delta} \frac{1}{\overline{n_\delta}^q} \qquad (1)$$

grow power-like with the inverse size of the phase-space bin δ: $F_q \sim \delta^{-\varphi_q}$, where φ_q is called "intermittency slope". This was contrasted with Poissonian fluctuations, for which all F_q are equal to one, and with standard correlations, which result in finite values of F_q for the bin size decreasing to zero.

More precisely, the averages in formula (1) should be taken not only over events, but also over the bin position in the phase-space of each event to collect reasonable statistics. In fact, for sufficiently high multiplicities one can get then meaningful results even for single events! Instead of the bin size δ one can obviously use the number of bins M, where $Y = M * \delta$ is some measure of kinematical range in which analysis is performed. We shall come back later to the details of this procedure.

The original motivation of [1] was to look for a possible signal of a transition to the quark-gluon plasma phase. Since at the phase transition point the correlation length becomes infinite, it is natural to expect a power-like increase for the factorial moments (1) (the numerators of which are integrals of the q-particle distributions). The analysis of one of heavy ion collision events from a cosmic ray experiment [2] seemed to support such hopes: F_q were indeed increasing as (small) negative powers of the bin size in

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pseudorapidity, whereas the "smooth fit" (describing correctly other features of the data [3]) did not show any dependence on the bin size. In Fig. 1 this is shown for F_5, where the increase is visible best.

Within a few years, however, it became obvious that the effect was not limited to heavy ion collisions. In hadron collisions and in the e^+e^- annihilation [4] the same pattern was observed: F_q were increasing for small bins in momentum space (see Fig. 2) in clear disagreement with the results from the Monte Carlo generators tuned to describe all other data. Since nobody expected the quark-gluon plasma formation in e^+e^- collisions, this suggested that intermittency signals may result from other effects than phase transition. The QCD parton cascade was proposed as an obvious candidate [5]. The flood of experimental and theoretical investigations followed, resulting in specialized workshops devoted mainly to this subject [6,7,8,9].

However, the very fact of simultaneous observation of the signals in hadron-hadron and heavy ion collisions suggested that the correlations responsible for the increase of moments are of a collective character. Indeed, if the single- and multiparticle distributions in heavy ion collisions are just sums of the contributions from ν separate nucleon-nucleon collisions, all the short range correlations are suppressed by a factor $1/\nu$ when compared with hadron-hadron interactions. Since the numerator in formula (1) is the integral of q-particle distribution over the bin in the phase-space, the bin-size dependent part of the moments in heavy ion collisions should be proportional to $1/\overline{\nu}$. Such a suppression would make the effect inobservably small for heavy nuclei.

Apart from the hypothetical quark-gluon plasma formation, the only obvious source of such collective correlations was the Bose–Einstein interference effect. Thus it was suggested that intermittency signals may result from the space-time structure of the source reflected by the Bose–Einstein enhancement of identical particles [10,11]. However, the data seemed to exhibit no significant difference between the moments for like- and unlike sign hadrons [12]. Only a few years later experimentalists discovered serious contamination of hadronic data by Dalitz- and γ-conversion e^+e^- pairs, which imitated the effect for unlike-sign pairs [13]. Now there seems to be no doubt that the effect in nuclear- and hadronic collisions is dominated by (if not restricted to) the same sign multiplets of hadrons.

Therefore at present it seems unlikely to use intermittency effects as signals of quark-gluon plasma formation. The generally accepted interpretation is that the cascade mechanism in hard processes, and Bose–Einstein interference in all multiparticle production processes are mainly responsible for the observed signals. This does not mean, however, that the effect is explained; these are rather indications in which language one should formulate the proposed explanations. In particular, power-like increase of moments for like-sign particles, if related to the Bose–Einstein interference, suggests rather unusual space-time structure of the source.

We come back later to more detailed discussion of these ideas. In the next section, however, we will present a few of the choices of variables and quantities (other than just factorial moments of formula (1)) used to describe intermittency signals. Then we will shortly present some selected data, and discuss their interpretation.

We will not discuss the general ideas of self-similarity, fractality and multifractality,

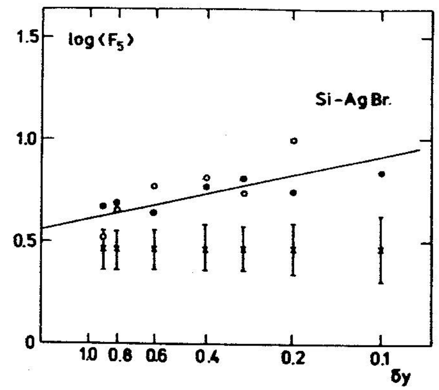

Fig. 1. $\langle F_5 \rangle$ for the cosmic event Si-AgBr [2] vs bin width in pseudorapidity δy in a log-log scale. Black dots are for y in the range (-3.55, 3.65) and circles are for range (-3.65, 3.55). Crosses with error bars are simulated from the fit [3], and the straight line is a power-like fit [1].

which are often associated with intermittency. Rather, we will try to relate standard models and ideas used to describe multiparticle production to the observed effects, and to show what sort of modifications seem to be necessary to describe them. We refer the reader to more complete reviews [14,15] covering other aspects of intermittency.

2. Formalism and variables

As already stated, the numerator of formula (1) is the integral of q-particle density

$$\overline{n_q} \equiv \overline{\frac{n!}{(n-q)!}}_\delta = \int_\delta dx_1 \ldots dx_q \, \rho(x_1 \ldots x_q). \tag{2}$$

Here each of the x_i-s represents a variable in momentum space in which the bin of size δ is defined. Bar denotes averaging over events.

In calculating the average scaled factorial moment $\langle F_q \rangle$ one should take average of (1) over the bin position (which will be denoted by brackets: $\langle X \rangle \equiv \sum_{i=1}^{M} X(\delta_i)/M$). It can be done separately for the numerator and the denominator, as in [1] ("horizontal average")

$$\langle F_q \rangle_H \equiv \langle \overline{n_q} \rangle / \langle \overline{n} \rangle^q, \tag{3}$$

or for the whole right-hand side of (1) ("vertical average")

$$\langle F_q \rangle_V \equiv \langle \overline{n_q} / \overline{n}^q \rangle. \tag{4}$$

The second choice was proposed [16] to avoid the trivial effect of increasing F_q for a non-flat distribution in x. Unfortunately, for small bins it introduces large fluctuations from the ends of the phase-space, where both the numerator and the denominator of

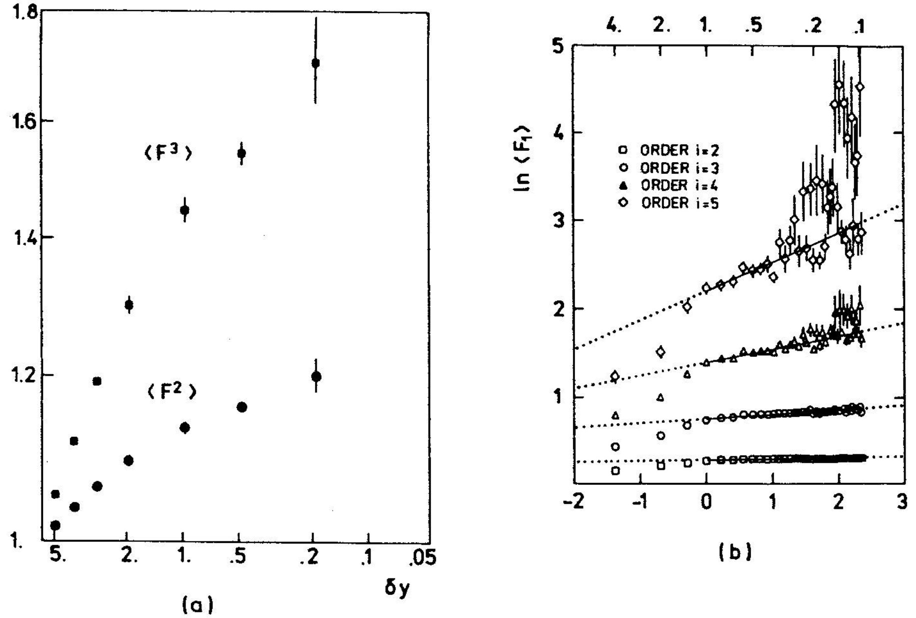

Fig. 2. Scaled factorial moments $\langle F_q \rangle$ a) for e^+e^- collisions, b) for π/Kp collisions vs rapidity bin width in a log-log scale [4].

(1) are very small. The other way to remove the unwanted effect is to divide by the average over bin position of power of the average multiplicity [1,17]

$$\langle F_q \rangle_H^{'} \equiv \langle \overline{n_q} \rangle / \langle \overline{n}^q \rangle \tag{5}$$

However, the simplest solution is to modify the original variable x to $\tilde{x}$, in which every bin, by definition, has the same average multiplicity [18,19].

The power-like increase of $F_q(\delta)$ for decreasing δ can occur only if ρ_q has power-like singularities in differences $x_i - x_k$. This is easiest to see for $q = 2$, where $\rho_2(x_1, x_2)$ should have a term proportional to $|x_1 - x_2|^{-\varphi}$ to get in $F_2(\delta)$ a term proportional to $\delta^{-\varphi}$.

This suggests immediately two modifications of the original analysis. First, ρ_2 is the sum of a product of single particle densities and of the two-particle correlation function. A singularity in $x_1 - x_2$ may obviously appear only in the second term. Thus, unless some strange cancellations occur, it seems to be more natural to expect the dominance of power-like term in the scaled cumulant $K_2 = F_2 - 1$ (which is the integral of two-particle correlation function) rather than in F_2 . Obviously, for δ small enough it should make no difference. However, we cannot really expect a singularity in the mathematical sense, but rather a power-like behaviour in some range of the variables. Plotting the dependence of moments as functions of δ in a double logarithmic scale we may always

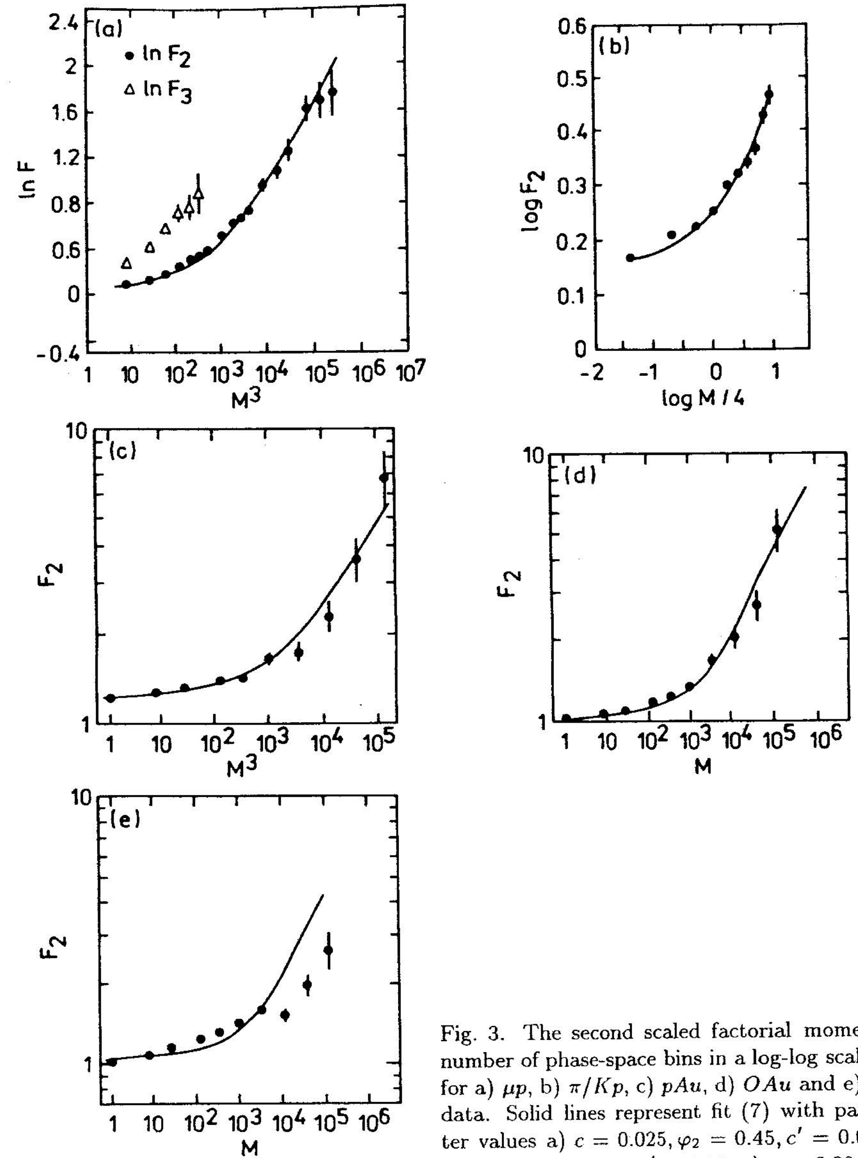

Fig. 3. The second scaled factorial moment vs number of phase-space bins in a log-log scale [23] for a) μp, b) π/Kp, c) pAu, d) OAu and e) SAu data. Solid lines represent fit (7) with parameter values a) $c = 0.025, \varphi_2 = 0.45, c' = 0.02$, b) $c = 0.02, \varphi_2 = 0.45, c' = 0.16$, c) $c = 0.02, \varphi_2 = 0.45, c' = 0$, d) and e) $c = 0.01, \varphi_2 = 0.5, c' = 0$.

select a range, in which a straight line fits well the data. However, to claim serious evidence for intermittent behaviour one should find such a fit for a really broad range of δ. As we shall see, the recent data seem to support the suggestion [20] that this occurs for the scaled cumulants K_q rather than for the moments F_q .

For completeness, let us remind here the relations defining (unscaled) cumulants $\kappa_q = K_q\, \overline{n}^q$ and moments $\overline{n_q}$ by means of the generating function $G(z) \equiv \sum z^n P(n)$, from which all the relations between F_q and K_q can be deduced

$$\overline{n_q} = \frac{d^q}{dz^q} G(z) \,|_{z=1}; \quad \kappa_q = \frac{d^q}{dz^q} [\ln G(z)] \,|_{z=1}. \tag{6}$$

The second modification concerns the choice of the variables. (Pseudo)rapidity bins are convenient to use, but in most of the cases the increase of moments was found to saturate for small bins. In fact, standard exponential form of two-particle correlation function in $\delta y \equiv |y_1 - y_2|$ was found to give similar increase [11]. This is to be expected if there is a power-like singularity of the correlation function in a more natural Lorentz invariant variable, as *e. g.* four-momentum difference squared Q^2 . Since for small Q^2 we may express it as a sum of three terms proportional to δy^2 , $\delta\varphi^2$ and $\delta \ln p_t^2$ (φ is azimuthal angle, and p_t transverse momentum), integrating over φ and p_t smoothes the singularity [21,22]. Thus for real "intermittent signal" one should use three-dimensional bins in y, φ and $\ln p$ rather than a one-dimensional analysis.

Such an analysis suggested indeed a power-like increase of scaled cumulants in 3d bins in a wide range of bin sizes [23]. Moreover, fitting second order moments by a simple formula

$$F_2 \doteq 1 + c(M^3)^{\varphi_2} + c' \tag{7}$$

one observed approximate universality of the value of intermittency slope φ_2 (about 0.5, *cf.* Fig. 3). Unfortunately, since the global number of bins M is the product of the numbers of divisions in each dimension, the empty bins dominate for small bin sizes and the errors of the moments grow very quickly.

The solution of this problem was to replace the integrals over bins in each individual variable by "strip integrals" over areas defined by differences between the variables of two particles [24]. Already for one-dimensional moments in rapidity, binning in rapidity difference reduced drastically fluctuations, as seen in Fig. 4, where the famous "spike event" of NA22 [25] is analyzed by two methods [26]. The scaled factorial moment in strip integral method is defined here as

$$F_q^s(\delta y) \equiv \int_{\Omega_s} \prod dy_i\, \rho_q(y_1 \dots y_q) / \int_{\Omega_s} \prod dy_i \rho_1(y_i) \tag{8}$$

where Ω_s is defined for $q = 2$ by a condition $|y_1 - y_2| < \delta y$, and for higher q by a suitably generalized condition involving differences between rapidities for many pairs of particles.

The gain in statistics is even more striking for 3d moments, if one replaces binning in separate variables by a single condition on differences of four-momenta squared, *e. g.* $\sum Q_{ij}^2 < Q^2$. One defines then F_q^s as integrals analogous to (8) with integration over all the components of momenta and with Ω_s defined by this condition. We will return

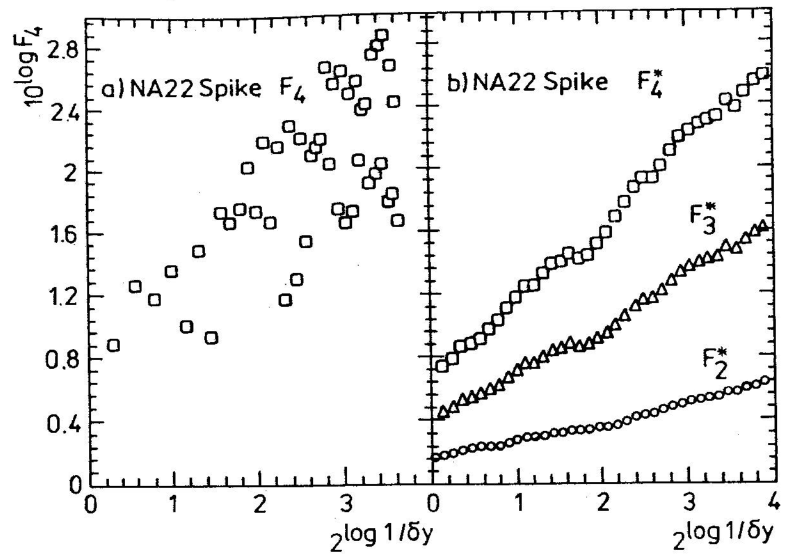

Fig. 4. a) The fourth scaled factorial moment of the NA22 "spike event" [25] vs rapidity bin width (lower order moments, which fluctuate less, are not shown to keep the picture clear), b) the "strip integrals" (8) of the order 2 – 4 vs "strip width", both in a log-log scale [26].

to the data analyzed in this way in the next section. Let us only notice here that in this case the loss of statistics when going from the integral measure (F_q^s) to the ratio of differential distributions in Q^2 ("differential correlation integrals", to be defined below, traditionally used for like-sign pairs for the analysis of Bose–Einstein interference effect) is less significant. Thus the intermittency signals may be observed directly as power-like increase of such ratios. With sufficiently high statistics, one can also investigate the distributions in "components of Q^2", *i. e.* longitudinal-, transverse- and time-like momentum differences and their combination, as often done in BE effect analyses.

As already noted, we will not discuss here other formalisms developed to study the intermittency signals, as *e. g.*

1) The G moments [27] (known in statistics as "frequency moments" [28]), used to investigate (multi)fractal properties of multiparticle production, and in particular to define various fractal dimensions [29,30],

2) The factorial correlators [1] and multivariate cumulants [31] used to investigate correlations between moments in two bins,

3) The bunching parameters [32], allowing to reveal spiky structure of events already from a few points of multiplicity distribution,

4) The wavelet transform [33], analyzing the pattern of fluctuations by multiresolution procedure.

These methods, reviewed *e. g.* in [15], seem at present not yet developed enough to give clear information on the mechanisms of multiparticle production.

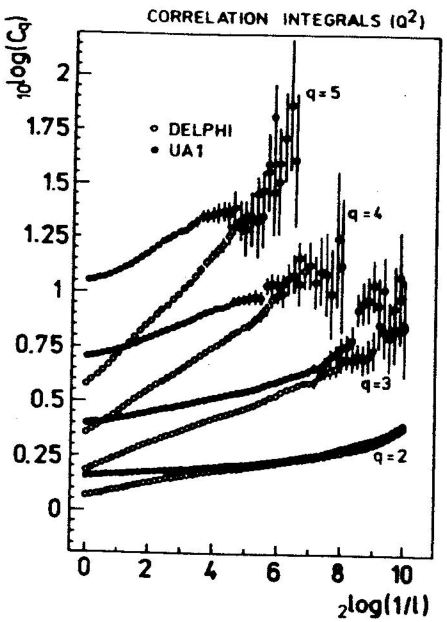

Fig. 5. The correlation integrals vs $l = Q^2$ in a log-log scale for DELPHI data (circles) and UA1 data (dots) [34].

3. Some recent developments

As noted above, investigation of intermittency signals leads to the conclusion that the experiments show strong short-range correlations in momentum space absent in standard descriptions of multiparticle production. For simplicity, we will mainly discuss second order moments and corresponding two-particle correlations.

The analysis of the e^+e^- annihilation data for correlation integrals (8) presented in Fig. 5 shows both similarities and differences from the hadronic data [34]. In both cases the clear increase at small values of Q^2 is seen, and in the restricted range of Q it may be well approximated by a simple formula $\ln c_2 \cong a + b \ln Q$. The increase for e^+e^- data is steeper and extends to higher values of Q, but for the second order integral and Q below 0.25 GeV ($\log_2(1/Q^2) > 4$) the data nearly coincide. For higher orders, the data seem also to converge at lowest available Q^2 .

On the other hand, distinguishing like- and unlike-sign pairs we see more significant differences. In Fig. 6, we show the differential correlation integrals, which for the second order are simply defined by

$$c_2(Q^2) \equiv \frac{\int d^3p_1 d^3p_2 \, \rho_2(p_1, p_2) \, \delta(Q^2 + (p_1 - p_2)^2)}{\int d^3p_1 d^3p_2 \, \rho_1(p_1)\rho_1(p_2) \, \delta(Q^2 + (p_1 - p_2)^2)} \, . \tag{9}$$

For same charges this ratio was often used to present the Bose–Einstein interference

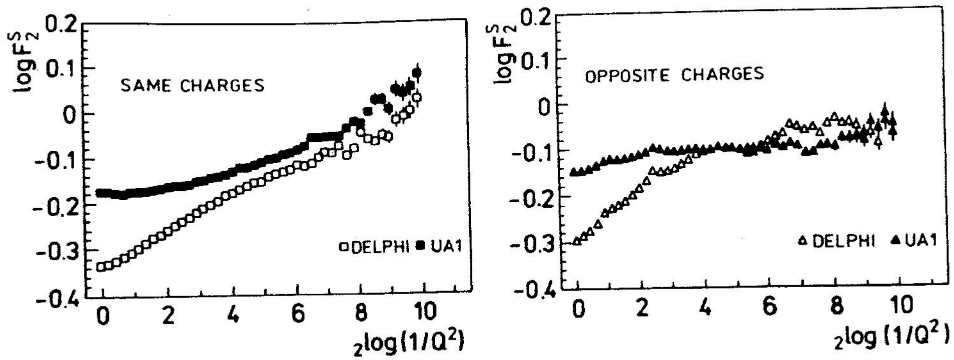

Fig. 6. Second differential correlation integral (9) for DELPHI (open symbols) and UA1 data (black symbols) vs $l = Q^2$ in a log-log scale a) for same charges, b) for opposite charges [34].

effects, since other sources of correlations were expected to be very weak at low Q, and the denominator could be expected to represent well the "data without BE interference" (at high energies, where energy-momentum conservation effects are negligible). In Fig. 6 we see an increase for both sets of data for Q below 0.25 GeV (hadronic data at higher Q were rather flat). For opposite charges the increase of c_2, seen for higher Q in e^+e^- collisions, saturates in the same region, whereas hadronic data show very little dependence on Q in the whole investigated range. This supports the suggestion mentioned earlier: partonic shower effects provide the increase of c_2 in e^+e^- data for a wide range of Q, quite similar for like- and unlike-sign pairs. For Q below 0.25 GeV this increase slows down, but for like-sign pairs the Bose–Einstein effect sets in, resulting in further increase. For hadronic data the Bose–Einstein effect seems dominant.

This picture is further supported by a comparison of data with the parton shower Monte Carlo for e^+e^- collisions (in which no BE effect is present). As seen in Fig. 7, the ratio of the integrals (9) for like- to unlike-sign pairs from MC calculations decreases slowly for low Q, whereas the data show a small but clear increase for Q below 0.25 GeV. We have checked that this slow decrease of the ratio is not a particular feature of "DELPHI MC" used in ref. [34], but occurs as well in a default version of PYTHIA/JETSET Monte Carlo, both with- and without angular ordering. Removing lowest Q pairs of the same charges from data ("BE-cut") one obtains quite similar decrease in data (also shown in Fig. 7).

The analysis of e^+e^- collisions became recently somehow separated from that of hadronic data. This is because the perturbative QCD should be valid in the initial stages of parton cascade (believed to be the hadroproduction mechanism in annihilation), whereas in soft hadronic collisions no such "reference theory" exists.

In fact, various versions of QCD moment calculations for partons (within the double logarithmic approximation [35,36] or modified leading logarithmic approximation [37]) claim a reasonable agreement with the data for hadrons. This effect is known as extension of "local parton-hadron duality" (LPHD), originally formulated as the proportionality of single-particle distributions for hadrons and partons. It seems to be in

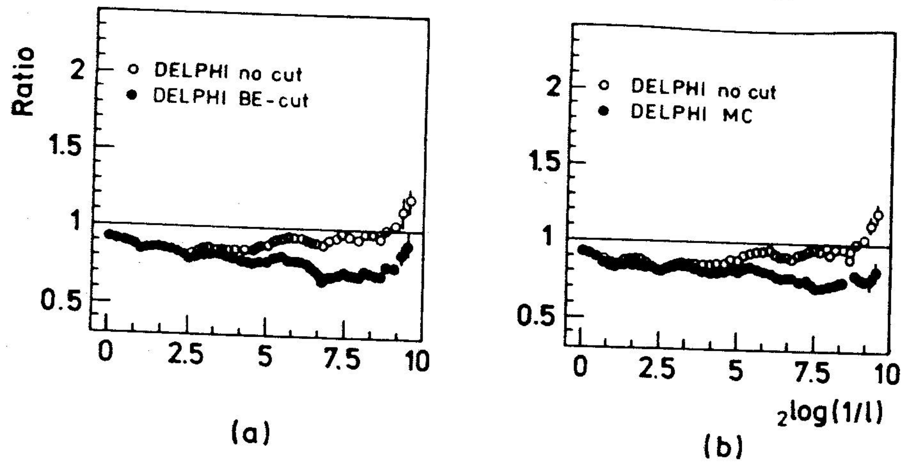

Fig. 7. The ratio of integrals (9) for same and opposite charges vs $-\log(Q^2)$ for DELPHI data (circles) compared to a) data with "BE cut", b) DELPHI Monte Carlo [34].

conflict with the common belief that hadronization effects are crucial for the short range correlations, and with the observation of an important role played by the Bose–Einstein interference effects in hadronic collisions. Thus one can only conclude that there may be various mechanisms producing intermittency signals in annihilation processes, and disentangling them may be very difficult.

On the other hand, the bulk features of "soft" hadronic and nuclear collisions are reasonably well described by various phenomenological models, often equipped with Monte Carlo generators, as JETSET [38], PYTHIA [39], FRITIOF [40], or HERWIG [41]. These models, however, fail to describe intermittency signals. Again, as mentioned above, the main difficulty seems to be the proper inclusion of the Bose–Einstein effect.

Imitating the Bose–Einstein interference in the Monte Carlo generators is not an easy task. The commonly used procedure of shifting final state momenta implemented n JETSET has many drawbacks [42]. The alternative approach of attributing weights o events provides also various problems [43,44]. In any case, the procedures attempt nly to reproduce assumed enhancement of like sign pairs at small Q^2. It seems very ifficult to implement expected complications related to resonances [45] (which may rastically change the effective size and shape of source) or to the space-time develop-nent of source before the hadronization. In particular, the approximately power-like ncrease of integral (9) at small Q^2 seen *e. g.* in UA1 data [46] and NA22 data [47], as nown in Fig. 8, may be obviously imitated by these procedures, but its origin remains nclear. It has been suggested to reflect the size fluctuations of the source [48], or its actal structure (due to resonance contributions?). Continuing these investigations in omponents of Q^2 and for higher order integrals is certainly one of the most important sks for future experiments.

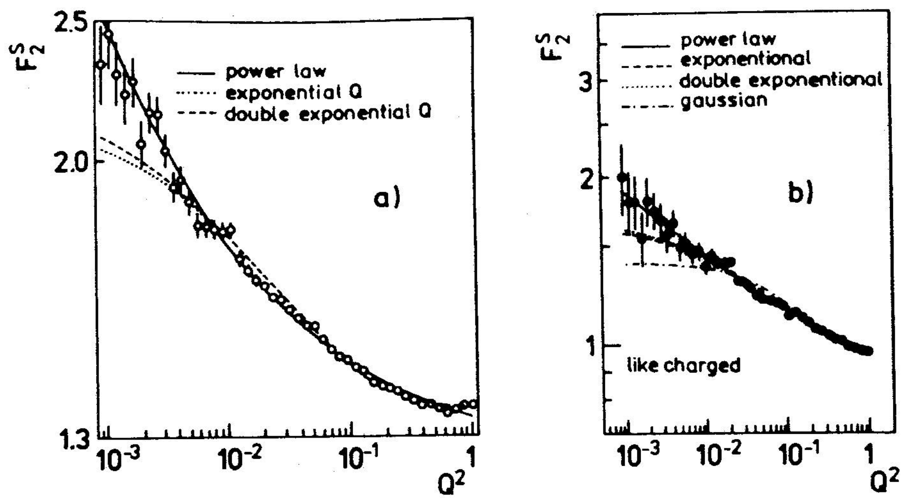

Fig. 8. Second differential correlation integral (9) vs Q^2 in a log-log scale for like-charged pairs a) from UA1 data [46], b) from NA22 data [47].

4. Conclusions and outlook

The investigations of intermittency signals performed over the last decade have brought new information on the multiparticle production and have inspired many new theoretical ideas. We have presented here some of the developments directly related to the more traditional description of correlation effects. In particular, we have summarized the evidence linking the intermittency signals to the parton cascade effects in electron-positron annihilation, and to the Bose–Einstein interference effects in all the processes of multiple production.

Despite many efforts, it still remains unclear how to deduce reliably the source characteristics from the analysis of Bose–Einstein effect. In particular, the resonance effects influence obviously the effective source shape seen in BE analyses. One may hope that further investigations will allow for better understanding of the space-time development of the hadronization process. The other (possibly related) unsolved problem is the proper explanation of the successes of "local parton-hadron duality" (LPHD) idea, apparently contradicting standard pictures of hadronization.

In the near future new data should allow to verify the information on the e^+e^- data, in particular testing the energy dependence of intermittency signal and the applicability limits of LPHD. Another important field are the high energy heavy ion collisions. The present hints of signals for quark-gluon plasma formation should be correlated with event-by-event analysis of intermittency signals, and in particular with the investigations of the Bose–Einstein effect.

One may hope that the second decade of intermittency will be as exciting as the first one, but more fruitful in bringing definite answers about the mechanisms of multiparticle production processes.

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