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THE INFLUENCE OF THE DIMENSIONS OF ELECTRODES ON THE FREQUENCY-TEMPERATURE CHARACTERISTICS OF AT AND BT-CUT QUARTZ RESONATORS[1]

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The comparison of the measured resonant frequency-temperature characteristics of the AT- and BT-cut square and circular quartz resonators with the computed ones is given in the paper. The curves which express the frequency-temperature behaviour of the resonators are compared. The influence of the thickness of the silver and gold electrodes on the first order frequency temperature coefficient is presented. The influence of the dimension ratio of the wafer on the orientation for which the zero first order temperature coefficient occures at the temperature $T_o = 25°\mathrm{C}$ are given.

1. Introduction

The resonant frequency-temperature characteristics of quartz resonators is a subject of the study of many scientensts practicaly from the beginning of the production of quartz resonators and many papers were published in this area. The reason is that the resonant frequency stability of the quartz resonators in the large temperature range is permanently one of their mainly watched parameters. Together with the investigation of the frequency-temperature characteristics of the quartz resonators and their minimalization the attention is also given to the influence of the electrodes in last two decades. Tiersten and Sinha [1], Stevens and Tiersten [2] and Sherman [3] considered the influence of the stresses induced by the change of the temperture in the piezoelectric plate covered on the surface with a thin metal film and the influence of the energy trapping when the electrodes are smaller then the plated surface. The influence of the changes of the inertia and the elastic properties of electrodes caused by the change of the temperature on the frequency-temperature characteristics of electroded quartz resonators was studied by the author and the theoretical results of the study were presented in [4].

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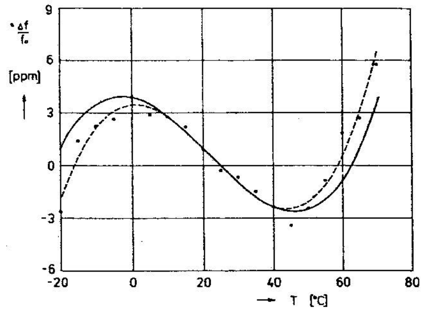

Fig. 1. Frequency-temperature characteristics of AT-cut quartz resonator with orientation $YXl_{35°15'}$ and a/b ratio 90.8. Solid line gives the result of the computation, the blac circles are the measured values and the dashed line express the measured value interpolated curve.

In the present paper the theoretical results given in [4] are compared with the experimental ones obtained by the measurement of the square shape AT- and BT-cuts and circular AT-cut quartz resonators vibrating on the fundamental thickness-shear mode approximately at the frequency 10 MHz.

2. Frequency-temperature characteristic of resonators

Let the f_o is the resonant frequency of the resonator at the temperature T_o. Then the resonant frequency f at the temperature T can be expresed by the relation

$$f = f_o(1 + Tf^{(1)}\Theta + Tf^{(2)}\Theta^2 + Tf^{(3)}\Theta^3), \qquad (1)$$

where $\Theta = T - T_o$ and $Tf^{(n)}$ are n-th order temperature coefficients of the frequency given by the Bechmann's definition

$$Tf^{(n)} = \frac{1}{n!\, f_o} \frac{\partial f}{\partial T^n} |_{T_o}. \qquad (2)$$

The frequency temperature coefficients depend mainly on the orientation of the plate. The first order temperature coefficient of frequency changes a litle with the thickness of electrodes, dimensions of electrodes and with the dimensions ratio of the plate.

3. Theoretical and experimental results

The resonant frequencies of the fundamental thickness-shear mode of vibration of the AT- and BT-cut quartz resonators were measured in the frequency temperature range from -20 to 70 °C. The resonators had the shape of the square plate with the edges 15 x 15 mm and with the edge (a) to the thickness (b) ratio approximately 90.8

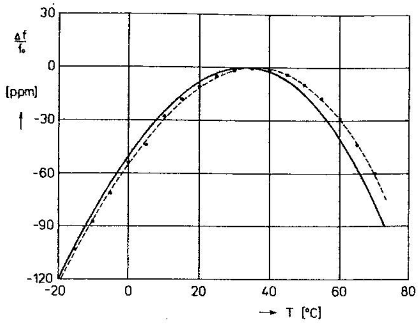

Fig. 2. Frequency-temperature characteristics of BT-cut quartz resonator with a/b ratio 58.7 and with orientation of measured sample $YXl_{-49°00'}$ and $YXl_{-48°50'}$ of computed curve. Solid line gives the result of the computation, the blac circles are the measured values and the dashed line express the measured value interpolated curve.

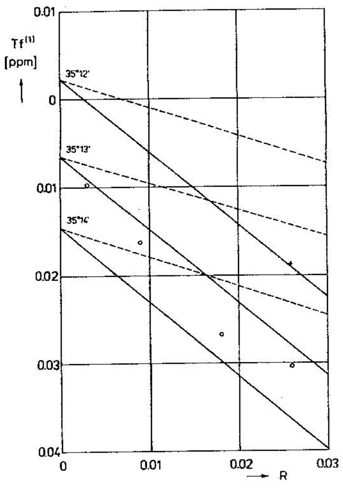

Fig. 3. Calculated dependence of the first order temperature coefficients $Tf^{(1)}$ of the AT-cut quartz resonators on the mass loading R for silver (solid line) and gold (dashed lines) elec- trodes and three cut angles. Circles correspond to the measured values of the AT-cut resonator with silver electrodes and the cross corresponds to the measured values of the same resonator with gold electrodes. The orientation of the measured resonator was $YXa_{-35°13'25''}$.

for AT-cut and 58.7 for BT-cut resonators. The silver electrodes 13 x 13 mm were evaporated on the plates.

The frequency-temperature characteristics measured by Pavlovec [5] on the circular AT-cut quartz resonators with the diameter of the wafer 9 mm and diameter of electrodes 5.2 mm and with the diameter to the thickness ratio of the plate approximately 54.5 were also considered by the investigation.

The measured resonant frequency-temperature characteristics were compared with computed ones. The computed characteristics were obtained from the relations given in [4]. The temperature derivatives of the elastic stiffnesses derived by Sinha and Tiersten

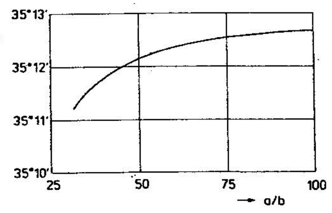

Fig. 4. The dependence of the orientation angle φ of the AT-cut resonator with the orientation YXa_φ and with the zero first order frequency temperature coefficient $Tf^{(1)}$ at the temperature T_o = 25°C on the dimension ratio a/b.

pq	First temperature derivatives C^1_{pq} [10^6Nm^{-2}K^{-1}] *	**	Effective second temperature derivatives C^2_{pq} [10^6Nm^{-2}K^{-2}] *	Efeective third temperature derivatives C^3_{pq} [10^6Nm^{-2}K^{-3}] *
11		1.5752		
12		-8.5296		
13	-2.1983	-2.1271	-18.806	-8.9302
14	0.91675	0.8814	4.2849	85.773
22	1.5976		-12.989	-38.145
33	-6.5255	-7.1395	-20.835	46.255
44	-5.3780	-5.1984	-26.505	-20.468
66	5.0747	5.0528	11.066	-55.824

Table 1. Temperature derivatives of elastic stiffness for the alpha quartz at 25°C given by Yong and Lee [6] (*) and by Sinha and Tiersten [7] (**).

[7] and Yong and Lee [6] (given in Table 1), the thermal expansion coefficients of quartz given by Bechmann, Ballato and Lukaszek [8] and the elastic stiffnesses, density, thermal expansion coeficient and temperature derivatives of electrodes given in [4] were used as input values of the computation.

The computed and measured resonant frequency-temperature characteristics of AT-cut quartz plate with the orientation $YXa_{35°15'}$ and with very thin silver electrodes are given in Fig. 1. The first order temperature coefficients of the elastic stiffnesses given by Yong and Lee [6] were used by the computation. The frequency temperature coefficients $Tf^{(n)}$ are given in Table 2. The excellent agreement of the computed and measured values were obtained in the temperature range from 0 to 50 °C.

The measured resonant frequency-temperature characteristic of the BT-cut quartz resonator with the orientation $YXa_{-49°00'}$ and with very thin silver electrodes is given in Fig. 2. The characteristic is compared with the computed characteristic for the orientation $YXa_{-48°50'}$. The first order temperature derivatives of elastic stiffnesses given by Sinha and Tiersten [7] were used for the computation. The frequency temperature coefficients $Tf^{(n)}$ calculated from the measured and computed values are given in Table 3.

	Computed	Measured
Tf$^{(1)}$	-1.976 $\times 10^{-7}$	-2.015 $\times 10^{-7}$
Tf$^{(2)}$	1.094 $\times 10^{-9}$	1.090 $\times 10^{-9}$
Tf$^{(3)}$	1.109 $\times 10^{-10}$	1.449 $\times 10^{-10}$

Table 2. Computed and measured resonance frequency temperature coefficients for the square AT-cut quartz resonator of the orientation YXl(35°15').

	Computed		Measured
	YXl(-49°00')	YXl(-48°50')	YXl(-49°00')
Tf$^{(1)}$	4.139 $\times 10^{-7}$	7.738 $\times 10^{-7}$	7.784 $\times 10^{-7}$
Tf$^{(2)}$	-4.798 $\times 10^{-8}$	-3.828 $\times 10^{-8}$	-4.750 $\times 10^{-8}$
Tf$^{(3)}$	-1.506 $\times 10^{-10}$	-1.059 $\times 10^{-10}$	1.520 $\times 10^{-10}$

Table 3. Computed and measured resonance frequency temperature coefficients for two orientations of the square BT-cut quartz resonator.

The difference between the measured and computed values can be expressed by means of the difference in the orientation angle approximately 10'. When the first order temperature derivatives of the elastic stiffnesses given by Yong and Lee [6] are used by the computation the difference is approximately three times greater.

The influence of the thickness of electrodes on the AT-cut quartz resonators is expressed for the silver and gold electrodes in Fig. 3. The dependence of the first order resonant frequency temperature coefficient $Tf^{(1)}$ on the mass loading R is given in Fig. 3. The mass loading R is given by the relation

$$R = \frac{\varrho' b'}{\varrho b}, \tag{3}$$

where ϱ' and b' are density and total thickness of electrodes and ϱ and b are density and thickness of quartz plate.

It is clear from Fig. 3, that the change of the thickness of electrodes can compensate small inaccuracy of the orientation angle of the wafer.

The influence of the dimensions ratio a/b of the wafer on the resonant frequency-temperature dependence is given in Fig. 4.

5. Conclusion

The relatively good agreement between measured and computed resonant frequency-temperature characteristics of fully plated quartz resonators was obtained. Yong's and Lee's first order temperature derivatives of the elastic stiffnesses can be used with very good results for the computation of the frequency-temperature behaviour of the AT-cut quartz resonators. Sinha's and Tiersten's first order temperature derivatives can be used for the computation of the frequency-temperature behaviour of the BT-cut quartz resonators when the shift of the orientation angle approximately 10' is considered by the computation.

It follows from the comparison of the measured and computed resonant frequency-temperature characteristics that it is necesssary to correct the values of the first order temperature derivative of the elastic stiffnesses of the quartz when we want to obtain the more precise computation results.

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